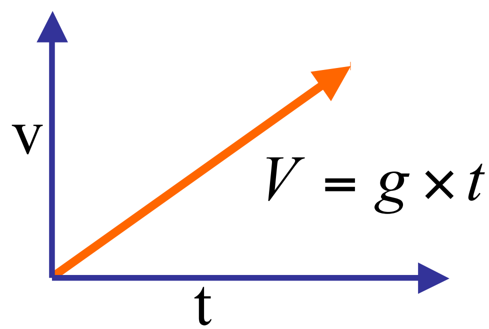
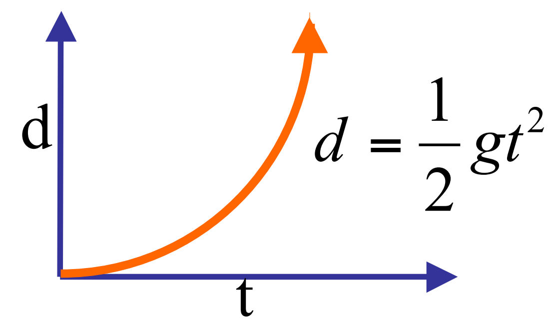
Recap.

1. Acceleration due to gravity "g" near the earth's surface is **CONSTANT** (i.e., **NOT** varying with **TIME**) and has a value of 9.8 m/s^2.
2. An object in free fall will **INCREASE** its **VELOCITY UNIFORMLY** with time. ($v = g.\ t$)
3. The distance fallen in a unit of time will **INCREASE RAPIDLY** with time as the object drops. ($d = \frac{1}{2} g.\ t^2$).
4. The **ACCELERATION** due to gravity is **NOT** dependent on the **MASS** or **SIZE** of the object!
5. "g" is **NOT** a "fundamental" constant!
   - But it does **NOT** vary much near the Earth's surface.





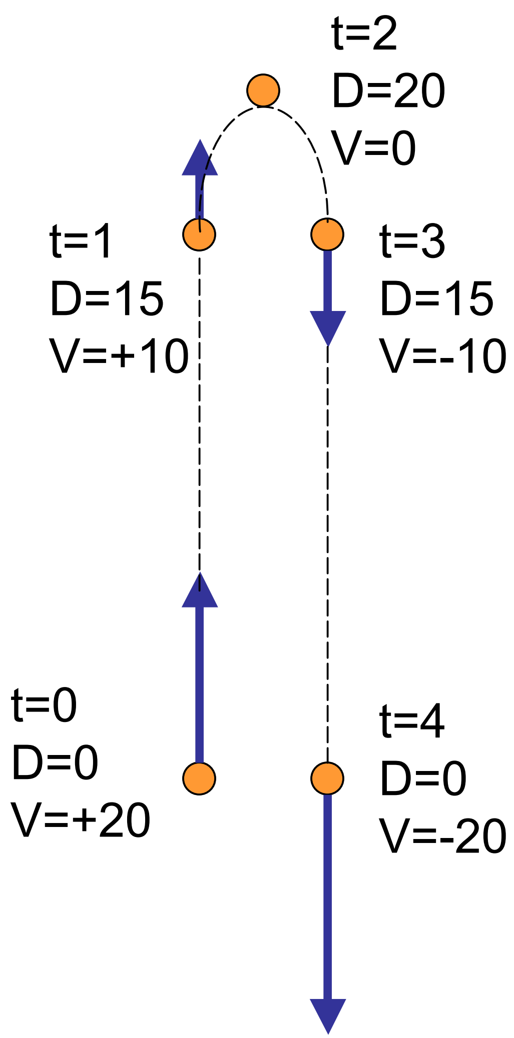
Accn. "g" = constant!

Upward Motion (against gravity)

- What happens when we throw a ball vertically upward?
  (i.e., how does the gravitational attraction affect the ball's motion?)
- We know from every day experience that "what goes up must come down" but we want to know:
  - How far up?
  - How fast it goes up?
  - When will it return?
- To make it move upwards we must first exert a **force** on the ball which will **accelerate** it to a **launch velocity V_0**
- Once it is launched the **primary force** acting on the ball is due to **gravity** (assuming no air resistance) which always produces a **downward acceleration** towards the center of the earth of ~10 m/s^2
- This means that for every second the ball is in the air its velocity will **CHANGE** by ~10 m/s.

- The **direction** of the change in velocity is **downward** opposite to the ball's **original velocity** (i.e., it **subtracts** from it)

Example: Ball projected upward at a velocity of 20 m/s



Velocity at time t is given by:

$$V = V_0 + at$$

but accn. g = -10 m/s^2 (as opposite to initial motion)

t = 1	V = 20-10x1 = 10 m/s
2	V = 20-10x2 = 0 m/s
3	V = 20-10x3 = -10 m/s
4	V = 20-10x4 = -20 m/s

Distance at any time is:

$$d = V_0 t + \frac{1}{2} a t^2$$

t = 1	d = 20x1-0.5x10x1^2 = 15 m
2	d = 20x2-0.5x10x2^2 = 20 m
3	d = 20x3-0.5x10x3^2 = 15 m
4	d = 20x4-0.5x10x4^2 = 0 m

Results:

- Ball took the **same time** (2 sec) to travel **upward** as it did to **return downward** to its original position.
- **Velocity** of ball on its return was of **same size** as original velocity but in **opposite direction**.
- The "turn around" point of the ball's motion occurred at 2 sec when its **velocity** was momentarily **ZERO**.
- However, the **acceleration** at this point (and throughout the motion) was still $-10 m/s^2$ as "g" is **CONSTANT.** (remember acceleration is the **rate of change** of velocity and is not dependent on its size.)

* The **larger** the **initial velocity** the **higher** the **altitude** and the **longer** the flight time.

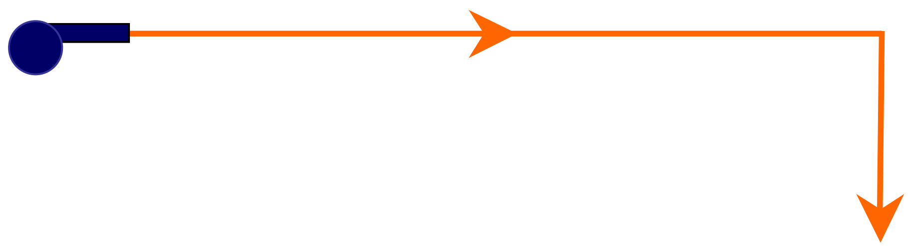
Exp: "Hang time"

Near the top of the trajectory the **velocity** is **VERY LOW** and the ball **SPENDS MORE TIME** there than during the rest of flight!

Projectile Motion

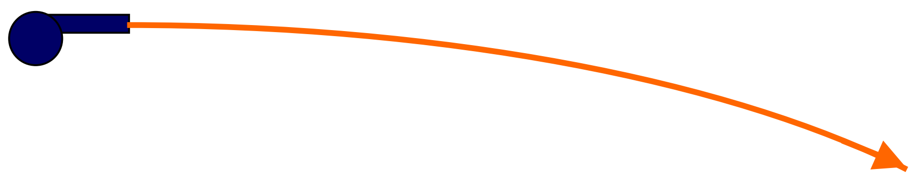
Question:

What happens to a projectile's trajectory (path) when it is launched horizontally?



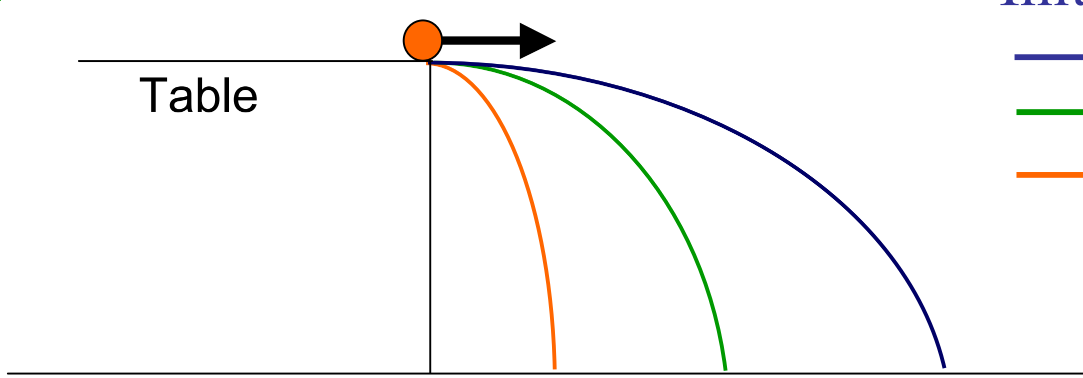
Does it continue in a straight line until it looses all its horizontal velocity and then fall?

or

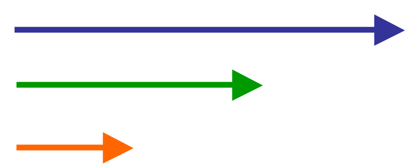


Does it begin to fall immediately after launch?

Exp: roll ball.



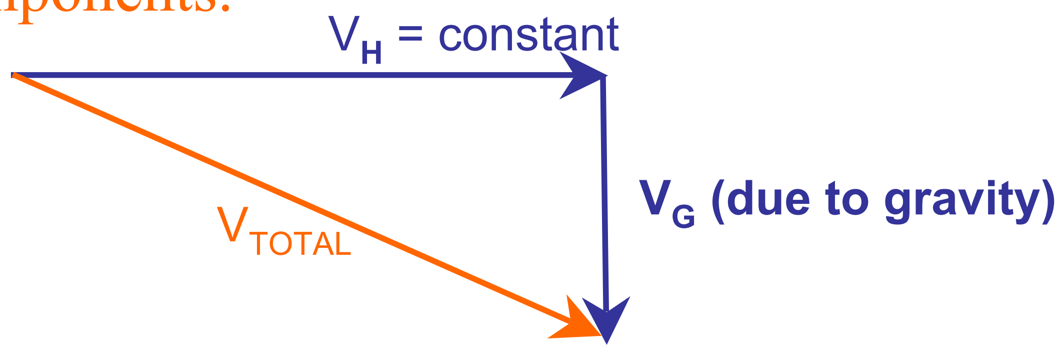
Initial velocity:



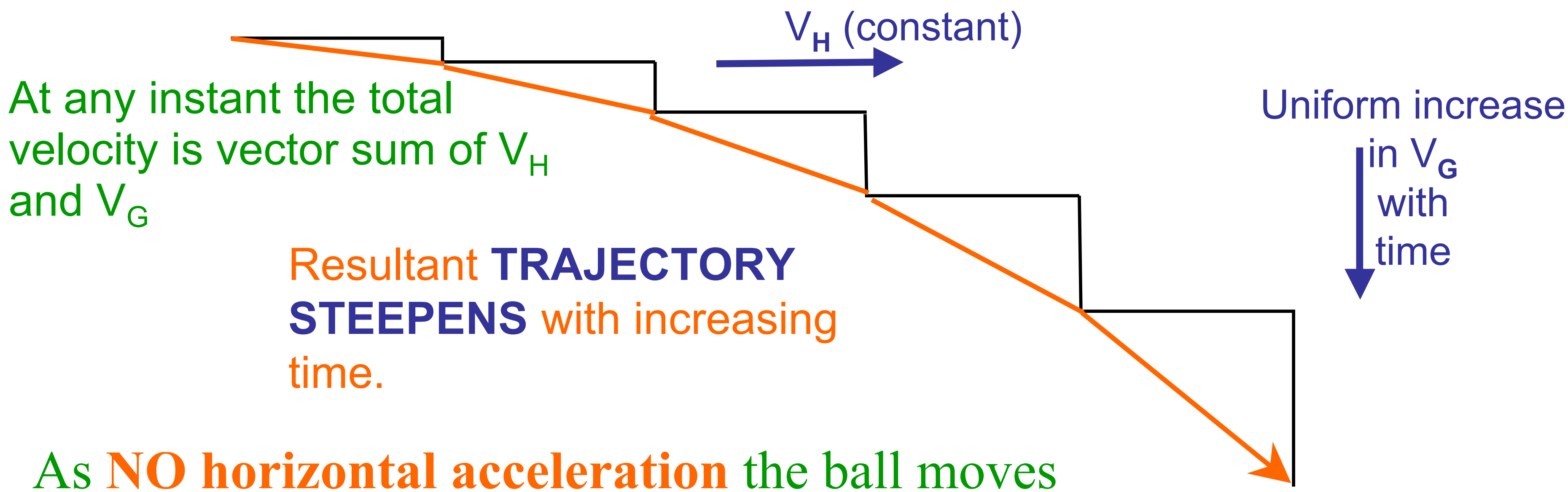
Lower speed the less distance moved.

How does this trajectory happen?

Key: - resolve motion into its **HORIZONTAL** and **VERTICAL** components.



But we know **V_G increases** with **time** due to gravity acceleration!

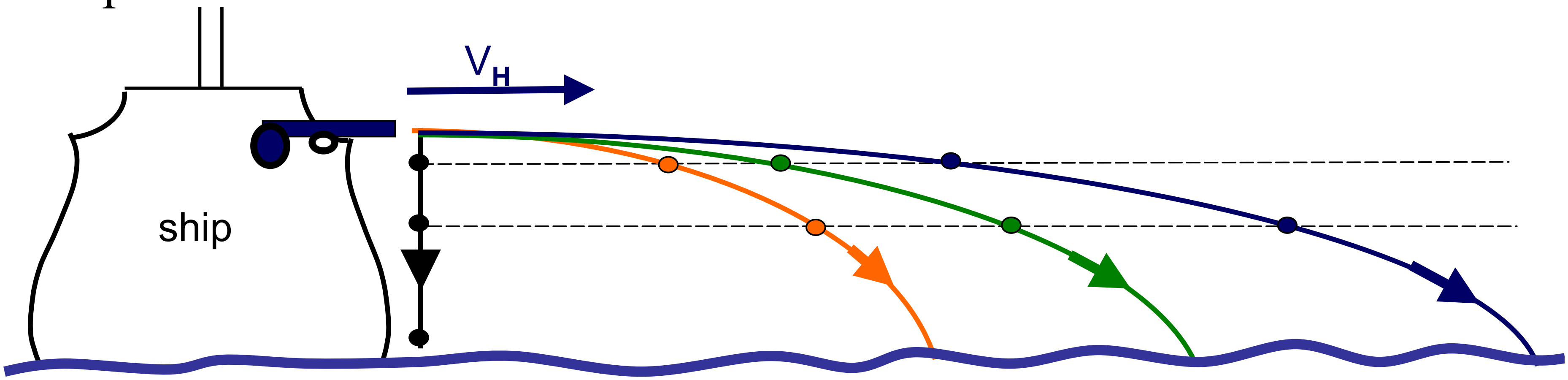


As **NO horizontal acceleration** the ball moves equal distances horizontally in equal time (assuming NO air resistance).

Results:

- Projectile travels **HORIZONTALLY** at a **CONSTANT VELOCITY** but at the same time it experiences a **downward** gravitational **acceleration**.
- The downward gravitational component of velocity **INCREASES UNIFORMLY** with time. (in exactly the same way as a ball dropped vertically from a roof.)

Unexpected result:



- If **increase** the cannon ball's initial **velocity** it will travel **further** from ship. However, which one will hit the sea first?
- Intuitively we expect the ball that travels **furthest** to have the **longest** time of flight……..

Answer = all hit sea at SAME TIME!

WHY?

- Because they **all** experience the **same vertical acceleration** due to gravity, $g = -10 m/s^2$.
- In other words the **vertical component** of their **velocities are equal** at any given **moment** in time.
- This is true even for a cannon ball accidentally dropped over the side!
- This surprising result is due to the fact that the **vertical motion** depends only on gravity and is **independent** of horizontal motion.

Conclusion:

The horizontal distance traveled is therefore given by the time of flight (which is dictated by the vertical motion) and the initial velocity V_H.

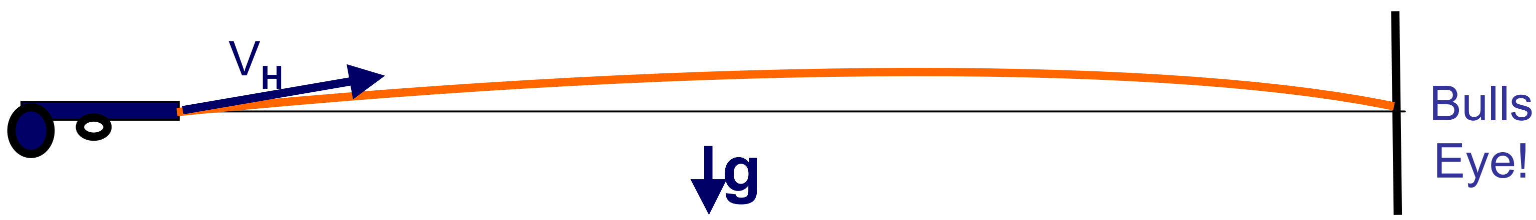
Thus, taller ships could therefore **fire further** but they were much more unstable!

Other Implications:

- Hitting a distant target

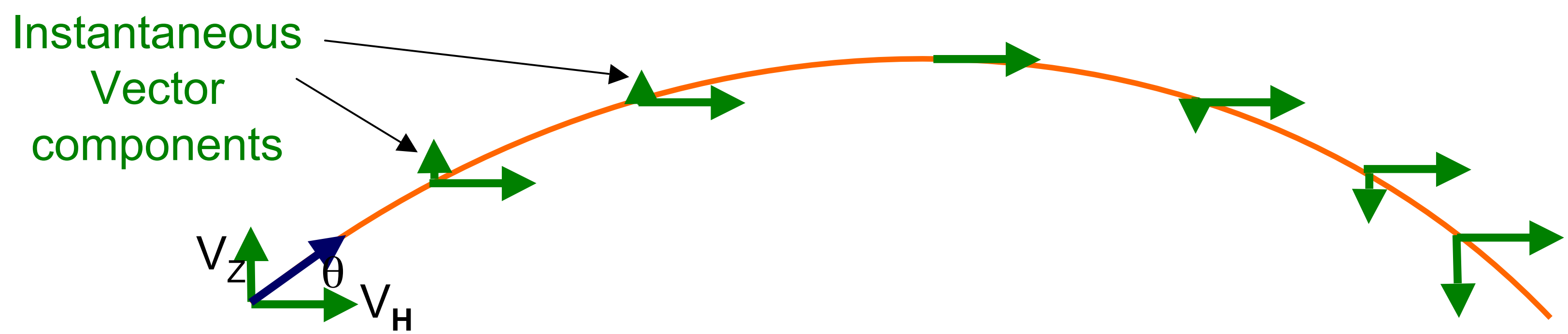
- If no account is taken of the pull of gravity on a projectile it will miss its target.
- The magnitude of the "miss" will depend on the **distance** to the target and the **initial velocity** (i.e., time to target)!
- The longer the time the larger the error.

Result - Compensate –"aim high" to balance out gravitational effect.



- Archers and sharp shooters need to be very skilled at the correction.
- Increasing velocity V_H significantly **reduces** this error.

General Result - Ballistics

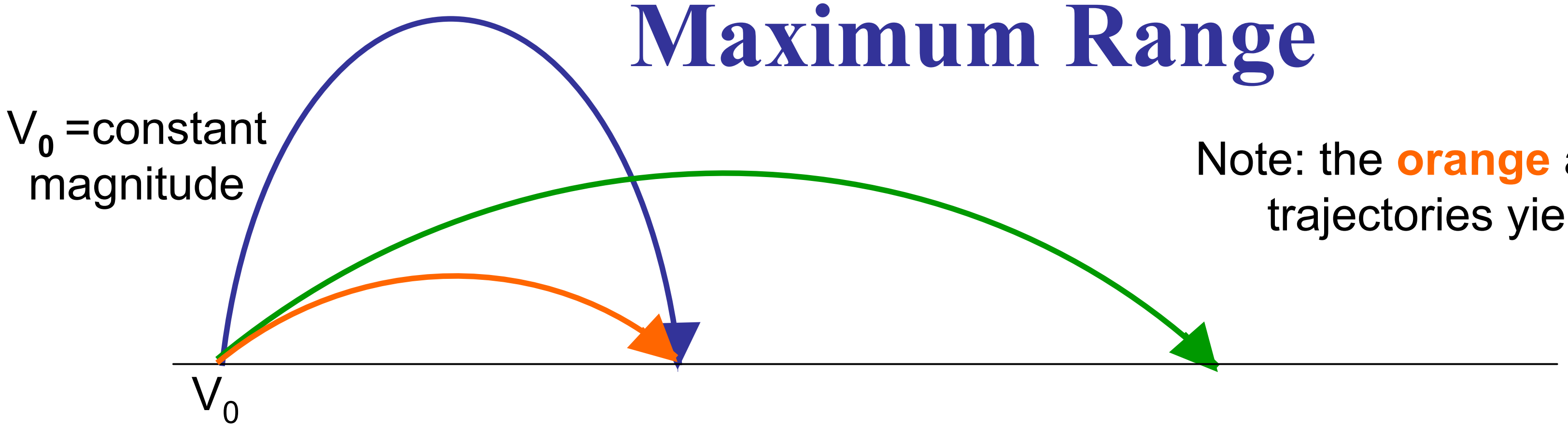


- To project an object any significant distance it is necessary to launch it at an **ANGLE θ** to the horizon.
- This provides a controlled horizontal and vertical vector component.
- The V_H component will **remain unaltered** during flight (in practice air resistance will reduce it).
- The **vertical component** is highest initially and steadily **reduces to zero** at the **highest point.**
- At this **center point** of the trajectory the **velocity vector** is completely **horizontal**.
- The **trajectory is symmetrical** – time to target equals **twice** time to maximum altitude.

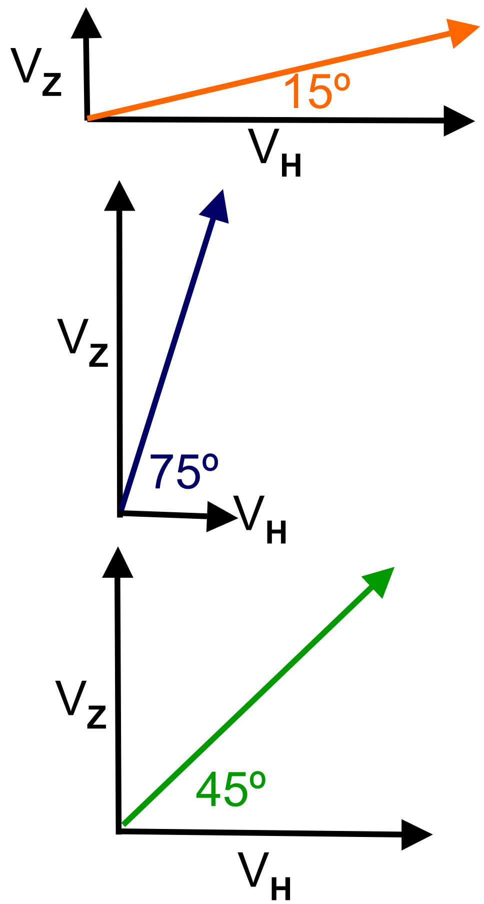
Can we CONTROL the Trajectory?

- By altering the **initial velocity** or by changing the **launch angle-** we can dictate where object will land.
- A large range of trajectories are therefore possible.
- A **faster projectile** will reach the target sooner and will be **less affected** by "g". So the object can be launched at a **fairly shallow** angle (e.g. a bullet)
  (note: "flat" basket ball shots are more difficult to make as the ball "sees" a smaller opening.)
- By **increasing the angle** of the trajectory (i.e., lower V_H) the **time of flight increases** but not necessarily the range (e.g., if point gun vertically its range = 0..aagh!!)
- **Longer** time of flight introduces **larger errors** due to variations in any winds.

Maximum Range



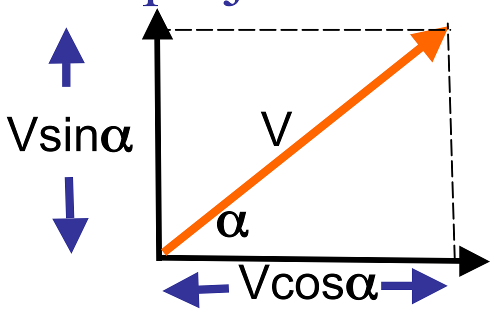
Note: the **orange** and **blue** trajectories yield similar ranges



- V_H is much larger than V_Z – vertical motion therefore limited and **time of flight short**.
- Large V_Z – object travels much higher and stays aloft longer but **does not travel far horizontally.**
- Intermediate **angle of 45º** maximizes the horizontal and vertical components providing **maximum flight time** and hence **maximum distance** covered in that time.

Summary - Ballistics

- The **trajectory** of a projectile is described by a **symmetric curve** called a **parabola**.
  - The altitude and **range** of an object's trajectory depend on its **initial launch speed** and **angle**.
- For a projectile of launch angle "**α**" the initial velocity can be broken down into its **VERTICAL** and **HORIZONTAL** velocity components.



- Throughout the flight the **vertical cpnt.** is subject to a constant downward **gravitational acceleration** which determines **how long** it will stay in the air.
- The **horizontal component** is constant during the flight (if no air resistance) and determines **how far** it will travel in that time.

In practice: - a range of launch angles are possible