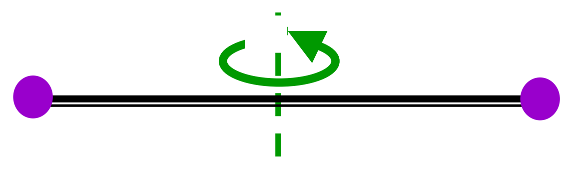
Recap: Solid Rotational Motion (Chapter 8)

- We have developed equations to describe rotational **displacement** ‘θ’, rotational **velocity** ‘ω’ and rotational **acceleration** ‘α’.
- We have used these new terms to modify **Newton’s 2nd law** for rotational motion:

$$\tau = I.\alpha \quad \text{(units: N.s)}$$

‘τ’ is the applied torque (**$\tau = F.l$**) , and ‘I’ is the **moment of inertia** which depends on the **mass**, **size** and **shape** of the rotating body (‘I’ ~ $m\,r^2$)

Example: Twirling a baton:



- The **longer** the baton, the **larger** the moment of inertia ‘I’ and the **harder** it is to rotate (i.e. need bigger torque).

Eg. As ‘I’ depends on r^2, a doubling of ‘r’ will quadruple ‘I’!!!

Angular Momentum (L)

- Linear momentum 'P' is a very important property of a body:

$$P = m.v \qquad (kg.\ m/s)$$

- An increase in **mass** or **velocity** of a body will **increase** its linear momentum (**a vector**).
- Linear momentum is a measure of the **quantity of motion** of a body as it can tell us how much is moving and how fast.

Angular Momentum (L):

❖ **Angular momentum is the product of the rotational inertia 'I' and the rotational velocity 'ω':**

$$\boxed{L = I.\ \omega} \qquad (\text{units: } \mathbf{kg.\ m^2/s})$$

- 'L' is a **vector** and its **magnitude** and **direction** are **key quantities**.
- Like linear momentum, **angular momentum** 'L' can also be **increased** - by increasing **either** '**I**' or '**ω**' (or both).

Angular Momentum (L)

$L = I.\,\omega$ (units: $kg.m^2/s$)

- As '**I**' can be different for **different shaped objects of same mass** (e.g. a sphere or a disk), the angular momentum will be different.

Example: What is angular momentum of the Earth?

$$\omega = \frac{2\pi}{T} = 0.727 \times 10^{-4} \text{ rad/sec}$$

For a solid sphere: $I = \frac{2}{5}\, m.r^2$

$I = 9.8 \times 10^{37}\ kg.m^2$

$T = 24$ hrs,
$r = 6400$ km
$m = 6 \times 10^{24}$ kg

Thus: $L = I.\,\omega = 7.1 \times 10^{33}\ kg.m^2/s$

(If Earth was flat, 'L' would be **even larger** as 'I' is larger)

Conservation of Angular Momentum (L)

- Linear momentum is **conserved** when there is **NO net force** acting on a "system"…likewise…

❖ **The total angular momentum of a system is conserved if there are NO net <u>torques</u> acting of it.**

- **Torque** replaces **force** and **angular momentum** replaces **linear momentum**.
- Both **linear momentum** and **angular momentum** are very important **conserved** quantities (magnitude and direction).

Rotational Kinetic Energy:

- For **linear** motion the **kinetic energy** of a body is:

$$\mathbf{KE_{lin} = \tfrac{1}{2}\, m.\, v^2} \quad \text{(units: J)}$$

- By analogy, the kinetic energy of a rotating body is:

$$\mathbf{KE_{rot} = \tfrac{1}{2}\, I.\, \omega^2} \quad \text{(units: J)}$$

- A rolling object has both linear and rotational kinetic energy.

Example: What is **total KE** of a rolling ball on level surface?

Let: m = 5 kg, linear velocity v = 4 m /s, radius r = 0.1 m, and angular velocity ω = 3 rad /s (0.5 rev/s)

Total KE = KE_{lin} + KE_{rot}

$KE_{lin} = \frac{1}{2} m. v^2 = \frac{1}{2}.5.(4)^2 = 40$ J

$KE_{rot} = \frac{1}{2} I. \omega^2$

Need: $I_{solid\ sphere} = \frac{2}{5} m. r^2 = \frac{2}{5} \times 5 \times (0.1)^2 = 0.02\ kg.m^2$

Thus: $KE_{rot} = \frac{1}{2} \times (0.02) \times (3)^2 = 0.1$ J

Total KE = 40 + 0.1 = 40.1 J

Result: The **rotational KE** is **usually much less** than the **linear KE** of a body.

E.g. In this example The **rotational velocity** '**ω**' would need to be **increased** by a factor of ~ $\sqrt{400}$ = **20 times**, to **equal** the linear momentum (i.e to 10 rev /s).

Summary: Linear vs. Rotational Motion

Quantity	Linear Motion	Rotational Motion
Displacement	d (m)	θ (rad)
Velocity	v (m/s)	ω (rad /s)
Acceleration	a (m/s^2)	α (rad / s^2)
Inertia	m (kg)	I ($kg.m^2$)
Force	F (N)	τ (N.m)
Newton's 2nd law	F = m.a	$\tau = I.\ \alpha$
Momentum	P = m.v	$L = I.\ \omega$
Kinetic Energy	$KE_{lin} = ½.m.v^2$	$KE_{rot} = ½.I.\ \omega^2$
Conservation of momentum	P = constant (if $F_{net} = 0$)	L = constant (if $\tau_{net} = 0$)

- Conservation of angular momentum requires both the magnitude and direction of angular momentum vector to remain constant.
- This fact produces some very interesting phenomena!

Applications Using Conserved Angular Momentum

Spinning Ice Skater:

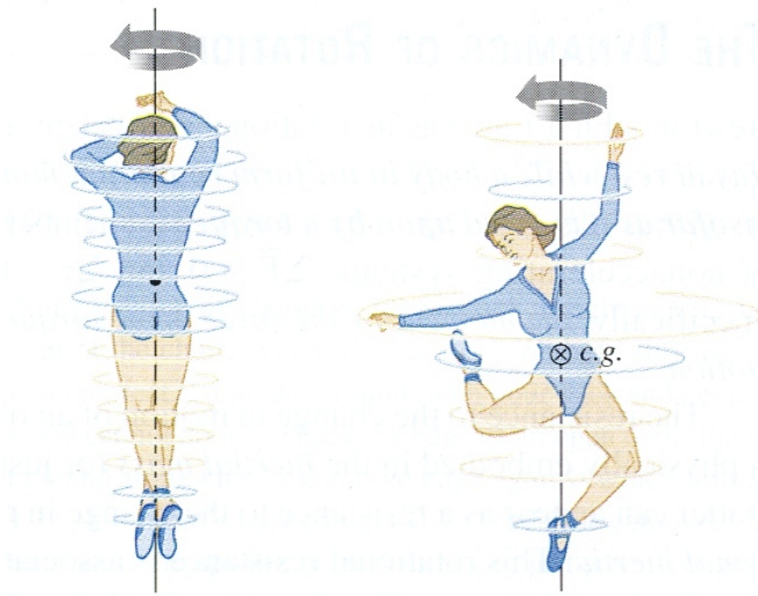
- Starts by pushing on ice - with both arms and then one leg fully extended.
- By pulling in arms and the extended leg closer to her body the skater's rotational velocity 'ω' increases rapidly.

Why?

- Her **angular momentum is conserved** as the external torque acting on the skater about the axis of rotation is very small.
- When both arms and 1 leg are extended they contribute significantly to the moment of inertia 'I'…
- This is because '**I**' depends on **mass distribution** and **distance**^{2} from **axis of rotation** ($I \sim m.r^2$).
- When her arms and leg are pulled in, her **moment of inertia reduces** significantly and to conserve angular momentum her **rotational velocity increases** (as $L = I.\ \omega$ = conserved).
- To **slow down** the skater simply **extends** her arms again…

Example: Ice skater at S.L.C. Olympic games

Initial $I = 3.5\ kg.m^2$,
Initial $\omega = 1.0$ rev /s,
Final $I = 1.0\ kg.m^2$,
Final $\omega = ?$



As L is conserved:

$$L_{final} = L_{initial}$$

$$I_f\,.\omega_f = I_i.\omega_i$$

$$\omega_f = \frac{I_i.\ \omega_i}{I_f} = \frac{3.5 \times 1.0}{1.0}$$

$$\boldsymbol{\omega_f = 3.5\ \text{rev /s.}}$$

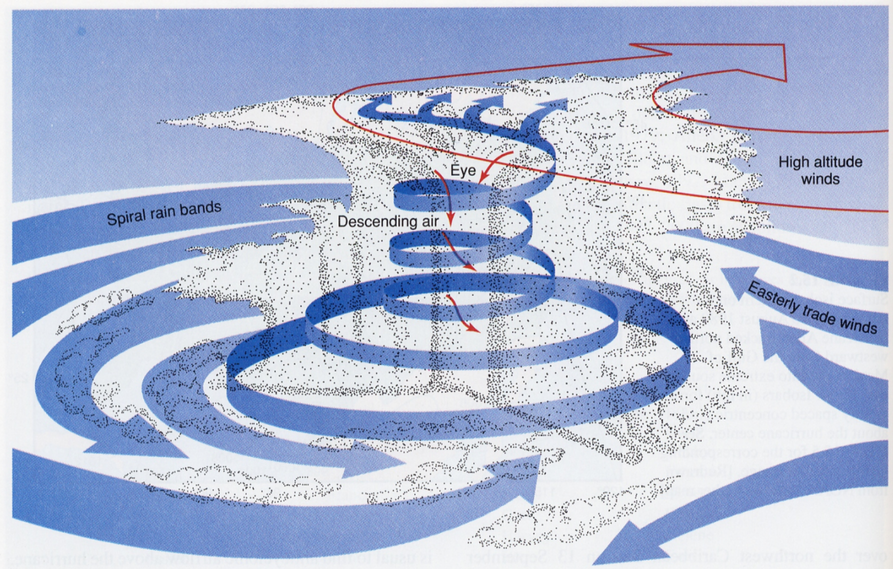
Thus, for spin finish ω has increased by a factor of 3.5 times.

Other Examples

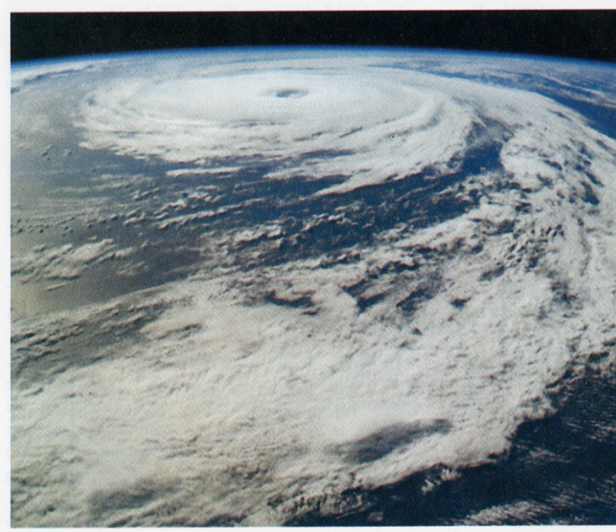
Acrobatic Diving:

- Diver initially extends body and starts to rotate about **center of gravity.**
- Diver then goes into a "tuck" position by pulling in arms and legs to **drastically reduce** moment of inertia.
- **Rotational velocity** therefore **increases** as no external torque on diver (gravity is acting on CG).
- Before entering water diver extends body to reduce 'ω' again.

Hurricane Formation:



(A)



(B)

FIGURE 15.4
(A) Schematic diagram showing the internal structure of a hurricane. Note that the vertical dimension is greatly exaggerated. (B) Hurricane Florence over the Atlantic Ocean as photographed by crew members aboard the Space Shuttle Atlantis in November 1994. Note the spiral cloud bands surrounding the eye of the storm. Earth's curvature is visible in the background. (C) A photograph of the eye of Hurricane Emilia in the Pacific Ocean in July 1994. Space Shuttle crew members used a handheld 70-mm camera to take this vertical view. [Diagram from NOAA, *Hurricane*. Washington, DC: Superintendent of Documents, 1977, p. 11; NASA photographs courtesy of the Johnson Space Center, Houston, Texas]

Pulsars: Spinning Neutron Stars!

- When a star reaches the end of its active life gravity causes it to collapse on itself (as insufficient radiant pressure from nuclear fusion to hold up outer layers of gas).
- This causes the **moment of inertia** of the star to **decrease drastically** and results in a **tremendous increase** in its **angular velocity**.

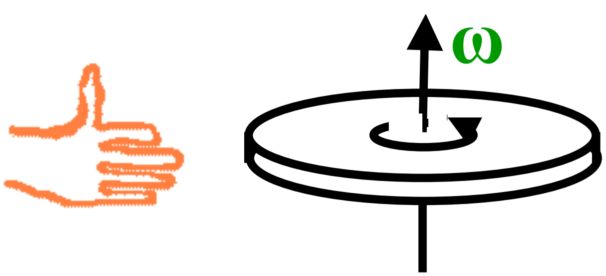
Example: A star of similar size & mass to the Sun would shrink down to form a very dense object of diameter ~25 km! Called a 'neutron' star!

- A neutron star is at the center of the Crab nebula which is the remnant of a supernova explosion that occurred in 1054 AD.
- This star is **spinning at 30 rev /sec** and emits a dangerous **beam of x-rays** as it whirls around (like a light house beacon) **30 times each second**. (~73 million times faster than the Sun!).
- Black holes are much more exotic objects that also have **tremendous angular momentum**.

Angular Momentum and Stability

Key:

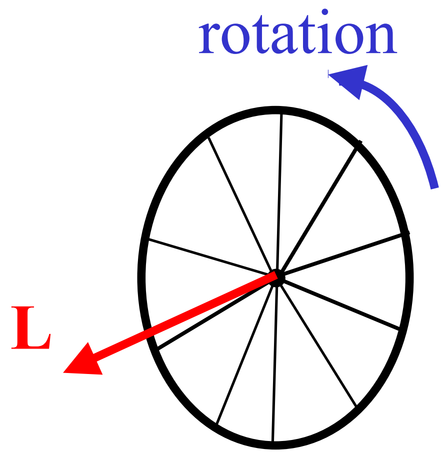
- Angular momentum is a **vector** and both its **magnitude** and **direction** are **conserved** (…as with linear momentum).
- Recap: Linear momentum 'P' is in <u>same direction</u> as velocity.
- Angular momentum is due to angular velocity 'ω'.



Right hand rule: The angular velocity for <u>counter clockwise</u> rotation is <u>directed upwards</u> (and vice versa).

- i.e. 'ω' and 'L' in direction of extended thumb.
- Thus, the **direction of 'L'** is important as it requires a **torque** to change it.
- **Result:** It is difficult to **change** the axis of a spinning object.

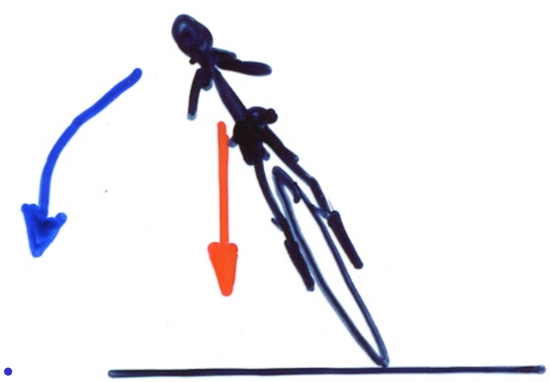
Stability and Riding a Bicycle



- At rest the bicycle has **no angular momentum** and it will fall over.
- Applying **torque** to rear wheel produces angular momentum.
- Once in motion the **angular momentum** will **stabilize** it (as need a torque to change).

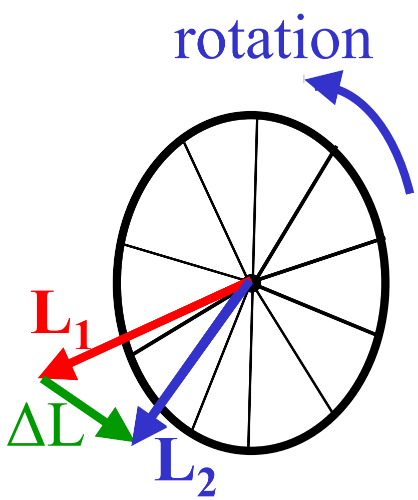
How to Turn a Bicycle

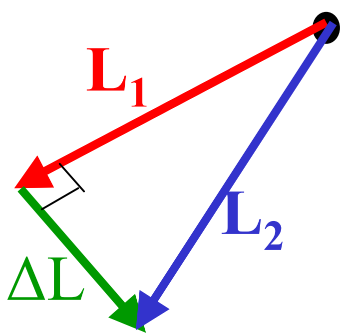
- To turn bicycle, need to **change direction of angular momentum vector** (i.e. need to introduce a **torque**).
- This is most efficiently done by **tilting** the bike over in direction you wish to turn.



- This introduces a **gravitational torque** due to shift in center of gravity no longer over balance point which causes the bicycle to **rotate** (start to fall).

How to Turn Bicycle…





Left turn

- The **torque** which causes the bicycle to rotate (fall) downwards generates a second angular momentum component ($\Delta L = I.\omega_{fall}$)
- **Total angular momentum** $L_2 = L_1 + \Delta L$
- ΔL points **backwards** if turning left of **forwards** if turning right.
- **Result:** We use gravitational torque to change direction of angular momentum to help turn a bend.
- **The larger the initial 'L'** the smaller the ΔL needed to stay balanced (slow speed needs large angle changes).

Summary

- Many examples of **changing rotational inertia** 'I' producing interesting phenomena.
- Angular momentum (and its conservation) are key properties governing **motion and stability** of **spinning bodies -** ranging from atoms to stars and galaxies!
- Many practical uses of spinning bodies for stability and for energy storage / generation:
  - Helicopters
  - Gyroscopes
  - Spacecraft reaction wheels
  - Generators and motors
  - Engines, fly wheels etc.