Recap: Rotational Motion of Solid Objects (Chapter 8)

1. Rotational displacement ‘θ’ describes how far an object has rotated (radians, or revolutions).

2. Rotational velocity ‘ω’ describes how fast it rotates (ω = θ / t) measured in radians/sec.

3. Rotational acceleration ‘α’ describes any rate of change in its velocity (α = Δθ / t) measured in radians / sec².

(All analogous to linear motion equations.)
Why Do Objects Rotate?

• Need a **force**.

• **Direction** of force and **point of application** are critical…

**Question:** Which force ‘F’ will produce largest effect?

• Effect depends on the **force** and the **distance** from the fulcrum /pivot point.

❖ **Torque** ‘τ’ about a given axis of rotation is the product of the applied force times the lever arm length ‘l’.

\[ \tau = F \cdot l \quad \text{(units: N.m)} \]

• The lever arm ‘l’ is the **perpendicular distance** from axis of rotation to the **line of action** of the force.

• **Result:** Torques (not forces alone) cause objects to **rotate**.
• **Long** lever arms can produce **more torque** (turning motion) than shorter ones for **same applied force**.

![Diagram of torque with lever arm](image1)

Larger ‘\(l\)’ more torque...

• **For maximum effect the force should be perpendicular to the lever arm.**

• If ‘\(F\)’ not perpendicular, the effective ‘\(l\)’ is reduced.

Example: Easier to change wheel on a car..

![Diagram of torque with perpendicular force](image2)
Balanced Torques

- **Direction of rotation** of applied **torque** is very important (i.e. clockwise or anticlockwise).
- Torques can **add** or **oppose** each other.
- If two **opposing torques** are of **equal magnitude** they will **cancel** one another to create a **balanced** system.

\[
W_1 = m_1 \cdot g \\
W_2 = m_2 \cdot g
\]

\[
W_1 \cdot l_1 = W_2 \cdot l_2
\]

or

\[
m_1 \cdot g \cdot l_1 = m_2 \cdot g \cdot l_2
\]

Thus at balance: \( m_1 \cdot l_1 = m_2 \cdot l_2 \)

(This is the principle of weighing scales.)
**Example:** Find balance point for a lead mass of 10 kg at 0.2 m using 1 kg bananas.

At balance: Torques are of equal size and opposite in rotation.

\[ W_1 \cdot l_1 = W_2 \cdot l_2 \quad \text{or} \quad m_1 \cdot l_1 = m_2 \cdot l_2 \]

\[ l_2 = \frac{m_1 \cdot l_1}{m_2} = \frac{10 \times 0.2}{1} = 2.0 \text{ m} \]

• Balances use a **known** (standard) weight (or mass) to determine another, simply by measuring the lengths of the lever arms at balance.

• **Important note:** There is **NO torque** when force goes through a pivot point.
**Center of Gravity**

- The **shape** and **distribution of mass** in an object determines whether it is **stable** (i.e. balanced) or whether it will **rotate**.
- Any ordinary object can be thought of as composed of a large number of **point-masses** each of which experiences a **downward force** due to gravity.
- These individual forces are parallel and combine together to produce a **single resultant force** \( W = m.g \) weight of body.

ót The center of gravity of an object is the point of balance through which the total weight acts.

- As **weight** is a force and acts through the **center of gravity** (CG), no torque exists and the object is in **equilibrium**.
How to Find the CG of an Object

• To find CG (balance point) of any object simply suspend it from any 2 different points and determine point of intersection of the two “lines of action”.

• The center of gravity does not necessarily lie within the object…e.g. a ring.

• Objects that can change shape (mass distribution) can alter their center of gravity, e.g. rockets, cranes…very dangerous.

• Demo: touching toes!
Stability

- If CG falls outside the line of action through pivot point (your feet) then a torque will exist and you will rotate!
- Objects with center of gravity below the pivot point are inherently stable e.g. a pendulum…

Summary:
- Center of gravity is a point through which the weight of an object acts. It is a balance point with NO net torque.
Dynamics of Rotation

- Rotational equivalent of Newton’s 1st law: A body at rest tends to stay at rest; a body in uniform rotational motion tends to stay in motion, unless acted upon by a torque.

Question: How to adapt Newton’s 2nd law (F = m.a) to cover rotational motion?

- We know that if a torque ‘τ’ is applied to an object it will cause it to rotationally accelerate ‘α’.
- Thus torque is proportional to rotational acceleration just as force ‘F’ is proportional to linear acceleration ‘a’.
- Define a new quantity: the rotational inertia (I) to replace mass ‘m’ in Newton’s 2nd law:

\[ \tau = I.\alpha \] (analogous to F = m.a)

- ‘I’ is a measure of the resistance of an object to change in its rotational motion.

(Just as mass is measure of inertial resistance to changes in linear motion)
So What Is ‘I’?

- Unlike mass ‘m’, ‘I’ depends not only on constituent matter but also the object’s shape and size. Consider a point mass ‘m’ on end of a light rod of length ‘r’ rotating.

The applied force ‘F’ will produce a tangential acceleration ‘a_t’

**By Newton’s 2\(^{nd}\) law:** \(F = m.a_t\)

But tangential acceleration = \(r\) times angular acceleration (i.e. \(a_t = r.\alpha\)) by analogy with \(v = r.\omega\).

So: \(F = m.r.\alpha\)  (but we know that \(\tau = F.r\))

So: \(\tau = m.r^2.\alpha\)  (but \(\tau = I.\alpha\))

Thus: \(I = m.r^2\)  (units: kg. m\(^2\))

- This is moment of inertia of a point mass ‘m’ at a distance ‘r’ from the axis of rotation.

- In general, an object consists of many such point masses and \(I = m_1r_1^2 + m_2r_2^2 + m_3r_3^2\)…equals the sum of all the point masses.
Now we can restate Newton’s 2\textsuperscript{nd} law for a rotating body:

- The net torque acting on an object about a given axis of rotation is equal to the moment of inertia about that axis times the rotational acceleration.

\[ \tau = I \alpha \]

- Or the rotational acceleration produced is equal to the torque divided by the moment of inertia of object. \( (\alpha = \frac{\tau}{I}) \).

- Larger rotational inertia ‘I’ will result in lower acceleration. ‘I’ dictates how hard it is to change rotational velocity.

**Example:** Twirling a baton:

- The **longer** the baton, the **larger** the moment of inertia ‘I’ and the **harder** it is to rotate (i.e. need bigger torque).

Eg. As ‘I’ depends on \( r^2 \), a doubling of ‘r’ will quadruple ‘I’!!!

(Note: If spin baton on axis, it’s much easier as ‘I’ is small.)
Example: What is the moment of inertia ‘I’ of the Earth?

For a solid sphere: \( I = \frac{2}{5} \, m \cdot r^2 \)

\[
I = \frac{2}{5} \, (6 \times 10^{24}) \times (6.4 \times 10^6)^2
\]

\[
I = 9.8 \times 10^{37} \, \text{kg} \cdot \text{m}^2
\]

The rotational inertia of the Earth is therefore enormous and a tremendous torque would be needed to slow its rotation down (around \( 10^{29} \, \text{N} \cdot \text{m} \))

Question: Would it be more difficult to slow the Earth if it were flat?

For a flat disk: \( I = \frac{1}{2} \, m \cdot r^2 \)

\[
I = 12.3 \times 10^{37} \, \text{kg} \cdot \text{m}^2
\]

So it would take even more torque to slow a flat Earth down!

In general the larger the mass and its length or radius from axis of rotation the larger the moment of inertia of an object.