Recap: Rotational Motion of Solid Objects (Chapter 8)

1. Rotational displacement '**θ**' describes how **far** an object has rotated (radians, or revolutions).

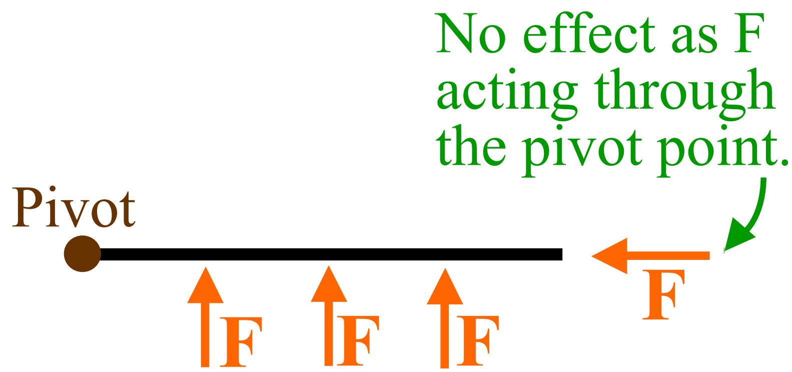
2. Rotational velocity '**ω**' describes how **fast** it rotates ($\boldsymbol{\omega = \theta / t}$) measured in radians/sec.

3. Rotational acceleration '**α**' describes any **rate of change** in its velocity ($\boldsymbol{\alpha = \Delta\theta / t}$) measured in radians /sec^2.

(All analogous to **linear motion** equations.)

Why Do Objects Rotate?

- Need a **force**.
- **Direction** of force and **point of application** are critical…



Question: Which force '**F**' will produce largest effect?

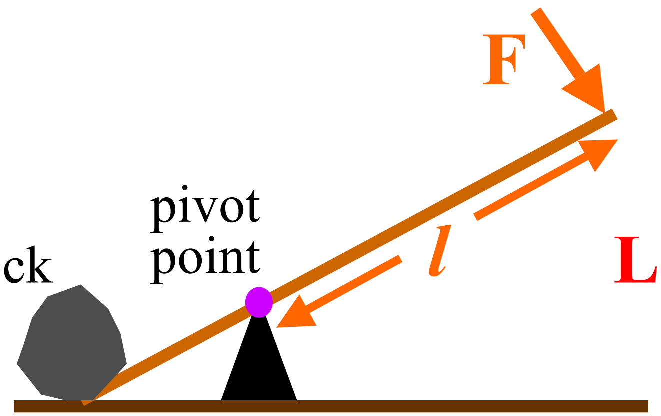
- Effect depends on the **force** and the **distance** from the fulcrum /pivot point.

❖ **Torque 'τ' about a given axis of rotation is the product of the applied force times the lever arm length 'l'.**

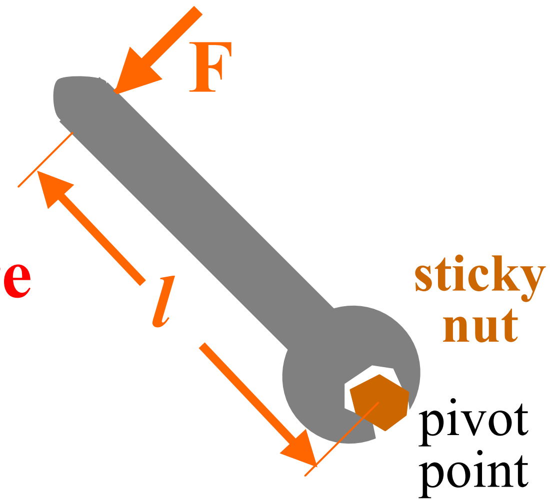
$$\tau = F \cdot l \quad \text{(units: N.m)}$$

- The lever arm 'l' is the **perpendicular distance** from axis of rotation to the **line of action** of the **force**.
- **Result: Torques** (not forces alone) cause objects to **rotate**.

- **Long** lever arms can produce **more torque** (turning motion) than shorter ones for **same applied force**.



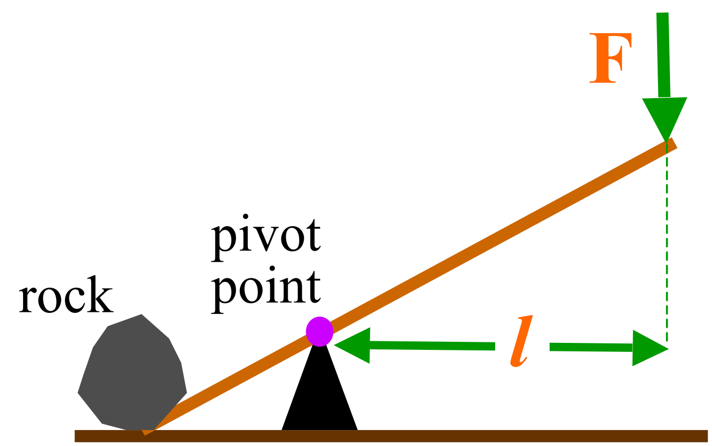
Larger '*l*' more torque…



- **For maximum effect the force should be <u>perpendicular</u> to the lever arm.**

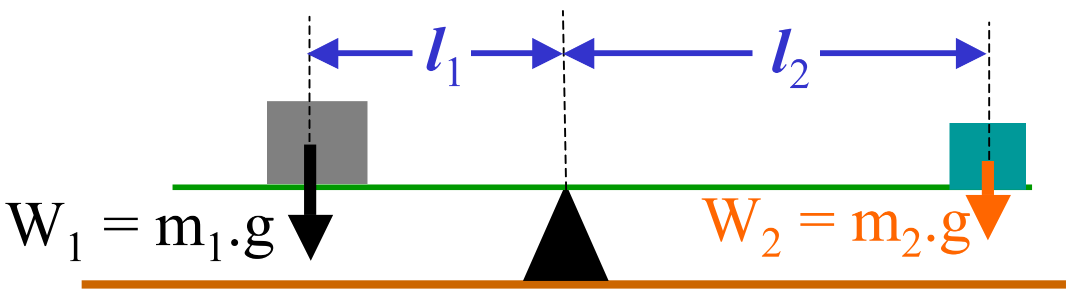
- If '**F**' **not perpendicular**, the effective '***l***' is **reduced**.

  Example: Easier to change
  wheel on a car..



Balanced Torques

- **Direction of rotation** of applied **torque** is very **important** (i.e. clockwise or anticlockwise).
- Torques can **add** or **oppose** each other.
- If two **opposing torques** are of **equal magnitude** they will **cancel** one another to create a **balanced** system.



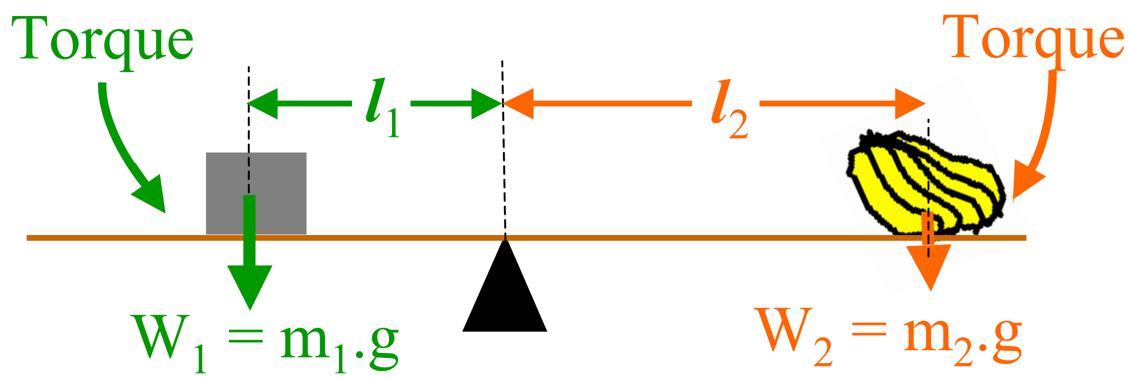
(Torque = F.l) $\quad W_1.l_1 = W_2.l_2$

or $\quad m_1.\cancel{g}.l_1 = m_2.\cancel{g}.l_2$

Thus at balance: $\mathbf{m_1.l_1 = m_2.l_2}$

(This is the principle of weighing scales.)

Example: Find balance point for a lead mass of 10 kg at 0.2 m using 1 kg bananas.



At balance: Torques are of equal size and opposite in rotation.

$$W_1.l_1 = W_2.l_2 \quad \text{or} \quad m_1.l_1 = m_2.l_2$$

$$l_2 = \frac{m_1.l_1}{m_2} = \frac{10 \times 0.2}{1} = 2.0 \text{ m}$$

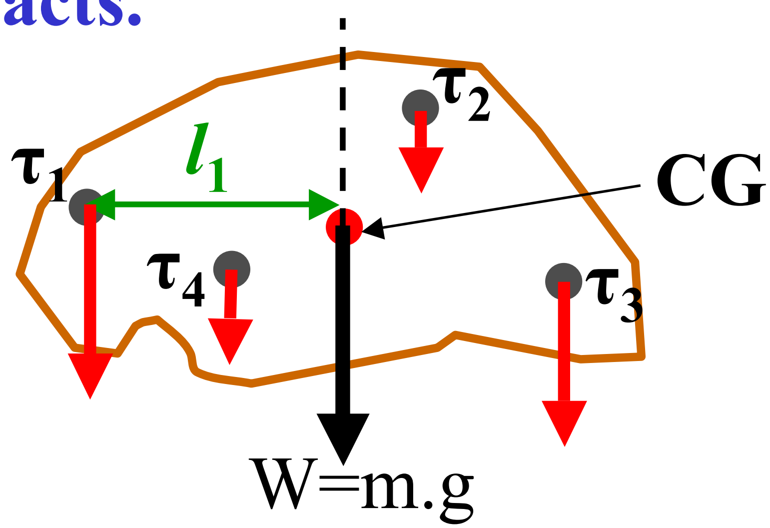
- Balances use a **known** (standard) weight (or mass) to determine another, simply by measuring the **lengths of the lever arms** at balance.
- **Important note:** There is **NO torque** when force goes through a pivot point.

Center of Gravity

- The **shape** and **distribution** of **mass** in an object determines whether it is **stable** (i.e. balanced) or whether it **will rotate.**
- Any ordinary object can be thought of as composed of a large number of **point-masses** each of which experiences a **downward force** due to gravity.
- These individual forces are parallel and combine together to produce a **single resultant force** (W = m.g) weight of body.

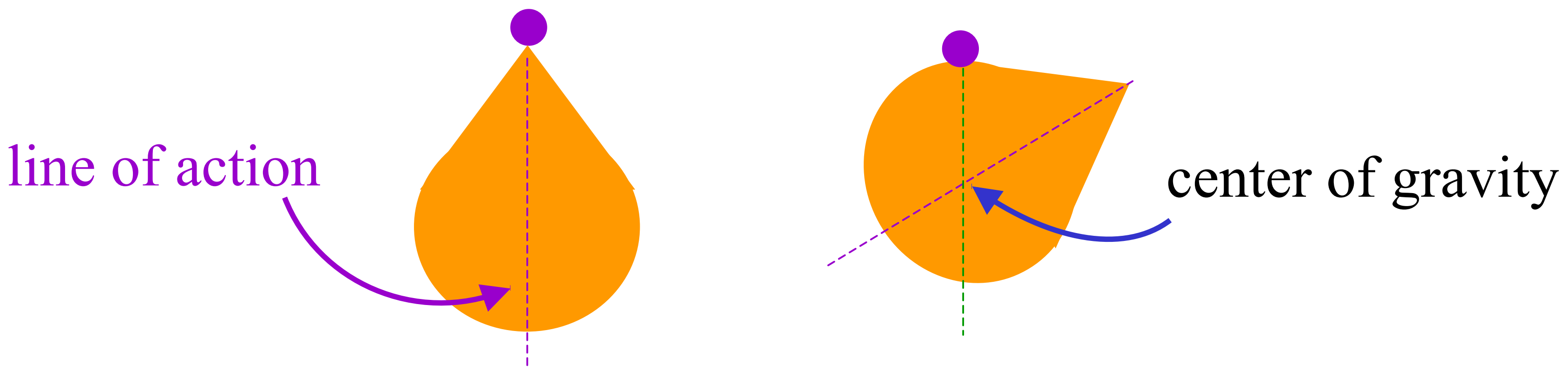
❖ **The center of gravity of an object is the point of balance through which the total weight acts.**

- As **weight** is a force and acts through the **center of gravity** (CG), **no torque** exists and the object is in **equilibrium**.



How to Find the CG of an Object

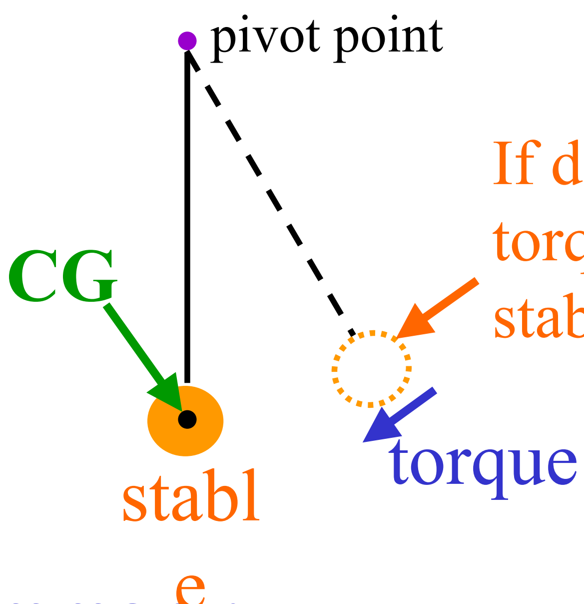
- To find CG (balance point) of any object simply suspend it from any 2 different points and determine point of intersection of the two “lines of action”.



- The center of gravity does not necessarily lie within the object…e.g. a ring.
- Objects that can change shape (mass distribution) can alter their center of gravity, e.g. rockets, cranes…very dangerous.
- Demo: touching toes!

Stability

- If **CG** falls **outside** the **line of action** through **pivot point** (your feet) then a torque will exist and you **will rotate**!
- Objects with center of gravity **below** the **pivot point** are **inherently stable** e.g. a pendulum…



If displaced the object becomes unstable and a torque will exist that acts to return it to a stable condition (after a while).

Summary:

- **Center of gravity** is a **point** through which the **weight** of an object **acts**. It is a **balance point** with **NO net torque.**

Dynamics of Rotation

- **Rotational equivalent of Newton's 1st law:** A body at rest tends to stay at rest; a body in **uniform rotational motion** tends to **stay in motion**, unless acted upon by a **torque**.

Question: How to adapt Newton's 2nd law (F = m.a) to cover rotational motion?

- We know that if a **torque** 'τ' is **applied** to an object it will cause it to **rotationally accelerate** 'α'.
- Thus **torque** is proportional to **rotational acceleration** just as **force** 'F' is proportional to **linear acceleration** 'a'.
- Define a new quantity: the **rotational inertia (I)** to replace **mass** 'm' in **Newton's 2nd law**:

$$\tau = I.\alpha \qquad \text{(analogous to } F = m.a\text{)}$$

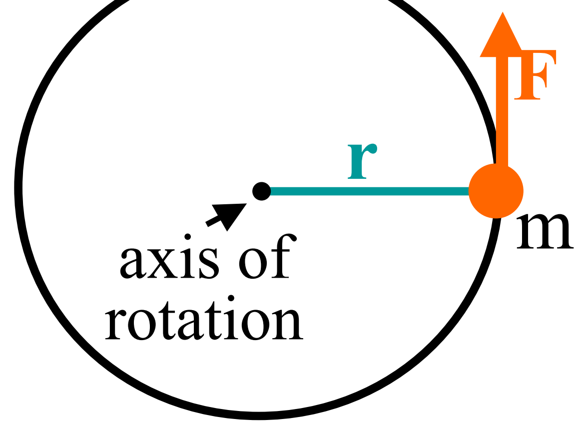
- 'I' is a measure of the **resistance** of an object to **change in its rotational motion.**

(Just as mass is measure of inertial resistance to changes in linear motion)

So What Is 'I'?

- Unlike mass 'm', **'I' depends** not only on constituent matter but also the object's **shape** and **size**.

Consider a **point mass** 'm' on end of a light rod of length '**r**' rotating.



The applied force **'F'** will produce a tangential acceleration **'a_t'**

By Newton's 2nd law: $F = m.a_t$

But tangential acceleration = r times angular acceleration (i.e. $a_t = r.\alpha$) by analogy with $v = r.\omega$.

So: $F = m.r.\alpha$ (but we know that $\tau = F.r$)

So: $\tau = m.r^2.\alpha$ (but $\tau = I.\alpha$)

Thus: **$I = m.r^2$** (units: **$kg.\ m^2$**)

- This is **moment of inertia** of a **point mass 'm'** at a **distance 'r'** from the axis of rotation.
- In general, an object **consists of many** such point masses and $I = m_1r_1^2 + m_2r_2^2 + m_3r_3^2 \ldots$ equals the sum of all the point masses.

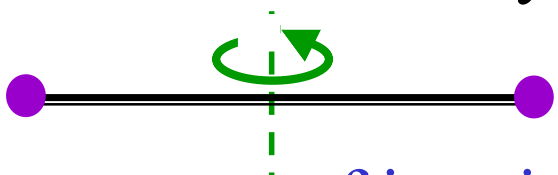
Now we can restate Newton's 2nd law for a rotating body:

❖ **The net torque acting on an object about a given axis of rotation is equal to the moment of inertia about that axis times the rotational acceleration.**

$$\tau = I.\alpha$$

- Or the rotational acceleration produced is equal to the torque divided by the moment of inertia of object. ($\alpha = \frac{\tau}{I}$).
- Larger rotational inertia 'I' will result in lower acceleration. 'I' dictates how hard it is to change rotational velocity.

Example: Twirling a baton:



- The **longer** the baton, the **larger** the moment of inertia 'I' and the **harder** it is to rotate (i.e. need bigger torque).

Eg. As 'I' depends on r^2, a doubling of 'r' will quadruple 'I'!!!

(Note: If spin baton on axis, it's much easier as 'I' is small.)

Example: What is the moment of inertia 'I' of the Earth?

For a solid sphere: $I = \frac{2}{5} m.r^2$

$I = \frac{2}{5} (6 \times 10^{24}) \times (6.4 \times 10^6)^2$

$I = 9.8 \times 10^{37}$ kg.m^2

Earth:
$r = 6400$ km
$m = 6 \times 10^{24}$ kg

The **rotational inertia** of the **Earth** is therefore **enormous** and a **tremendous torque** would be needed to **slow** its rotation down (around 10^{29} N.m)

Question: Would it be more difficult to slow the Earth if it were flat?

For a flat disk: $I = ½ m.r^2$

$I = 12.3 \times 10^{37}$ kg.m^2

So it would take **even more torque** to slow a flat Earth down!

In general the larger the **mass** and its **length** or **radius** from axis of rotation the **larger** the **moment of inertia** of an object.