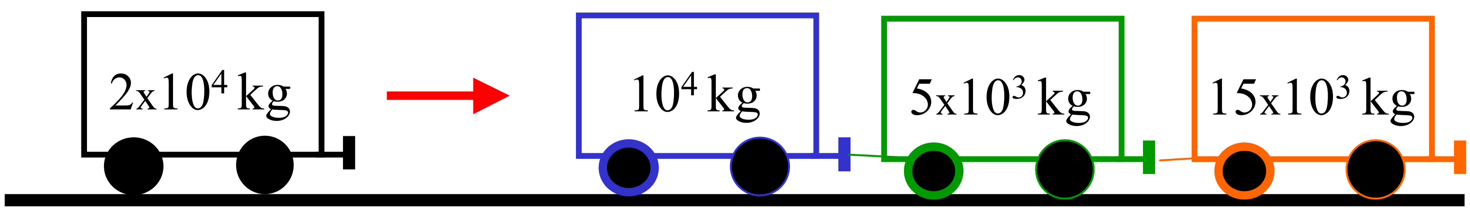
Recap: Collisions (Chapter 7)

- Two main types: **Elastic** and **Inelastic**…
- Different kinds of **collisions** produce different results…e.g. objects **stick** together and other times they **bounce** apart!
- Key to studying collisions is **conservation of momentum** and **energy** considerations…

Coupling train trucks (Perfect Inelastic Collision)



Results: **Energy** is **lost** in an inelastic collision. **Greatest** portion of energy is lost when objects **stick** together!

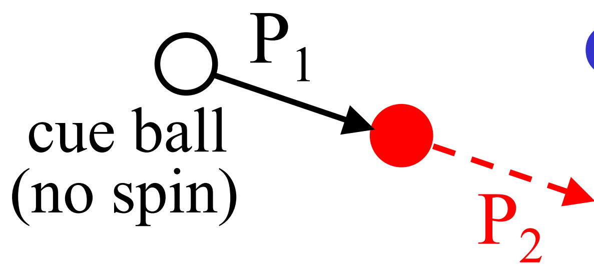
❖**Elastic Collisions:**

- **No energy** is lost in an **elastic collision.**

E.g. A ball bouncing off a wall with **no change** in its speed.

- In an **elastic collision** we need to find the final velocity of **both** colliding objects.
- Use **conservation of momentum** and **conservation of energy** considerations…

Example:



Question: What happens to red ball and cue ball?

- **Answer:** The cue ball **stops dead** on impact and red ball moves forward with the same velocity (magnitude and direction) as that of the cue ball prior to impact!
- Why?...Because both KE($= \frac{1}{2}.m.v^2$) and momentum (m.v) are conserved on impact.
- As the masses of both balls are the same the **only solution** to conserve both KE and momentum is for **<u>all</u>** the energy and momentum to be transferred to the other (red) ball.
- It's a fact…try it for yourself!!!

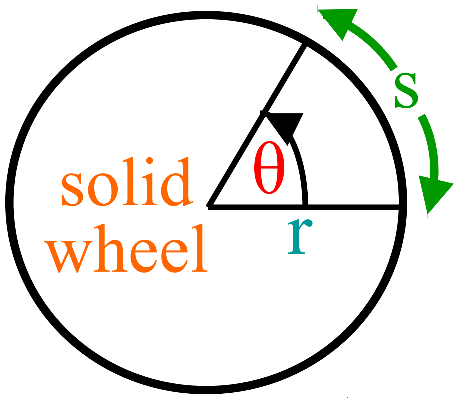
Rotational Motion of Solid Objects (Chapter 8)

- Rotational motion of solid objects forms a very important part of our understanding of natural phenomena, e.g. planets.
- We use rotating objects for tools, transportation, power generation as well as recreation:
  - Motors and generators
  - Wheels
  - Jet engines and propellers
  - Gyroscopes
  - Ice skaters
  - Merry-go-rounds…
- Newton's theory of linear motion can be adapted to help explain what is happening in rotational motion.

Rotational Motion

- An object can rotate about an axis (e.g. a wheel) yet it goes nowhere!
- How can we describe this rotational motion?
  - How **fast** is it rotating?
  - How **far** has it turned?

Rotational Displacement



s = arc length distance
r = radius object
θ = angle rotated

- We can measure (count) the number of whole revolutions an object makes to determine the distance moved (s) at a given radius (r) or we could simply measure the angle turned (θ).
- Rotational displacement is analogous to linear distance moved (d).

❖**Rotational displacement θ is an angle showing how far an object has rotated.**

- θ can be measured in revolutions, degrees or in radians:

$$\textbf{radians} = \frac{s}{r} \quad \text{(dimensionless quantity)}$$

- As 's' is proportional to 'r' (circumference = 2π.r), then ratio (s/r) is same for a given angle θ regardless of radius!

Question: How many radians are in a circle?

$$\text{radians} = \frac{s}{r} = \frac{2\pi . r}{r} = 2\pi$$

$$1 \text{ radian} = \frac{360}{2\pi} = 57.3^{o}$$

$$1 \text{ degree} = \frac{2\pi}{360} \text{ radians}$$

Fraction	Degrees	Radians
1	360 °	2π
1/2	180 °	π
1/3	120 °	2π / 3
1/4	90 °	π / 2
1/8	45 °	π / 4

Example: How many radians in 720° of rotation?

Answer: $\frac{720}{57.3} = 13.8$ or $720^{o} \times \frac{2\pi}{360^{o}} = 4\pi$ radians

Rotational velocity (ω):

- **Rotational velocity is the rate of change of rotational displacement.**

$$\omega = \frac{\theta}{t}$$

units: rev/sec, deg/sec, rad/sec

Note: ω is analogous to linear velocity ($v = d / t$).

Rotational Acceleration (α)

- By applying a force we can cause a rotating object to accelerate and change its rotational velocity.

❖ **Rotational acceleration is the rate of change in rotational velocity.**

$$\alpha = \frac{\Delta\omega}{t} \quad \text{units: rev / sec}^2 \text{ or rad / sec}^2$$

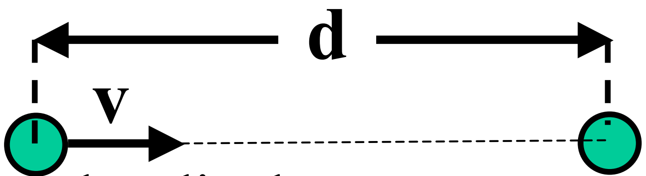
- Note: 'α' is analogous to linear acceleration ($a = \frac{\Delta v}{t}$).

  Example: Spinning up a wheel will cause its velocity to increase as it accelerates.

- If no force, then ω = constant and α = 0.
- In general, these definitions for 'ω' and 'α' yield average values. (Just as we did with the linear equations.)
- To determine instantaneous value for 'ω' and 'α' need to use very small time interval. (Again, just as in linear motion application.)

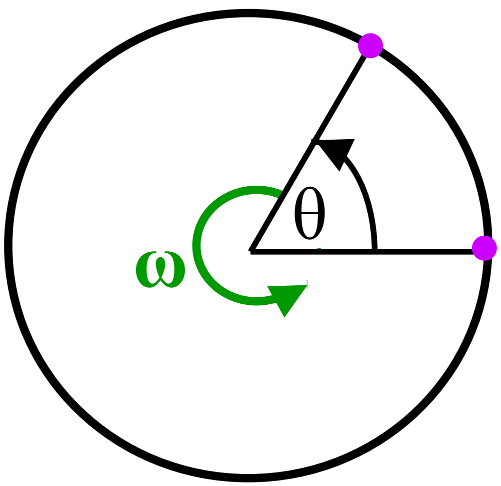
Comparison

Linear Motion



d = displacement
v = velocity = d /t
a = acceleration = $\frac{\Delta v}{t}$

Rotational Motion



θ = rotational displacement
ω = rotational velocity = θ /t
α = rotational acceleration = $\frac{\Delta\omega}{t}$

- We can extend analogy to cover linear and rotational motion under constant acceleration conditions (Galileo's equations of motion):

Linear motion	Rotational motion
$v = v_0 + a.t$	$\omega = \omega_0 + \alpha.t$
$d = v_0.t + ½.a.t^2$	$\theta = \omega_0.t + ½.\alpha.t^2$

Note: In many cases v_0 and ω_0 (initial values) are zero, simplifying equations.

Example: A rotating drive shaft accelerates at a constant rate of 0.1 rev /sec^2 starting from rest. Determine:

(a) Rotational velocity after 20 sec?

$\omega = \omega_0 + \alpha.t$ $\omega_0 = 0$, $t = 20$ sec, $\alpha = 0.1$rev /sec^2

$= 0 + 0.1 \times 20$

$= 2$ rev /sec

(b) Rotational velocity after 1 min? $\omega = 0.1 \times 60 = 6$ rev /sec

(c) Number of revolutions in 1 min?

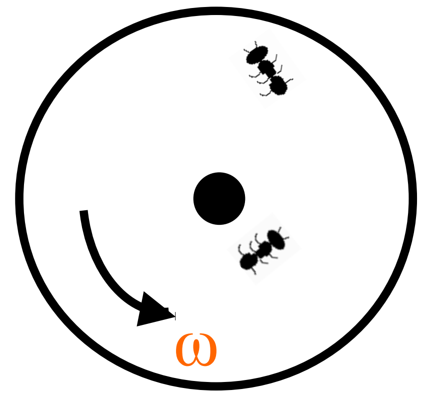
$\theta = \omega_0.t + ½.\alpha.t^2$ $\omega_0 = 0$, $t = 60$ sec, $\alpha = 0.1$ rev /sec^2

$= 0 + ½.(0.1).(60)^2$

$= 180$ rev

Note: The answer is not 6 rev/sec x 60 sec = 360 rev, as shaft is accelerating from zero rotational velocity and **NOT** running at constant rotational velocity.

Relation Between Linear & Rotational Velocity



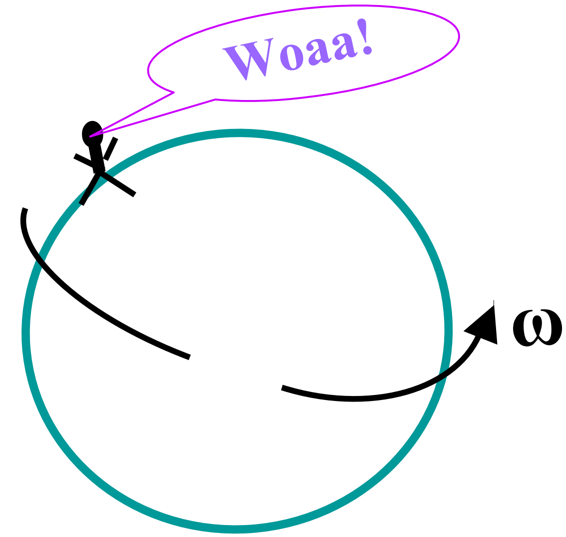
- Consider two ants on an old fashioned record player…
- Their rotational velocity '**ω**' will be the same but…
- Their linear velocity will depend on where they are on the record (i.e. how far from the center /axis of rotation).
- If the ant sat at center then it would have zero linear velocity.
- The ant farthest from center has the largest linear velocity as it travels a greater circular distance in 1 revolution than the other ant.
- Thus the larger the radius 'r', the higher the linear velocity 'v' for a given 'ω'.

$$\mathbf{v = r.\omega}$$

ω measured in **radians /sec**

- This is why you get the biggest thrill when you are on the outside of a merry-go-round.

Example: What is the linear velocity at the surface of the Earth at the equator?



radius = 6400 km

period = 24 hrs

using: $\omega = \frac{\theta}{t} = \frac{2\pi}{24 \times 3600}$

$\omega = 7.27 \times 10^{-5}$ rad /sec

using $v = r.\,\omega$ (ω must be in rad /sec)

$v = (6400 \times 10^{3})\,(7.27 \times 10^{-5})$

v = 465 m/sec (i.e. 0.5 km/sec)!!

Note: Tangential velocity depends on latitude.

Question: So why are we not "blown off" the Earth if we are whirling around so fast?

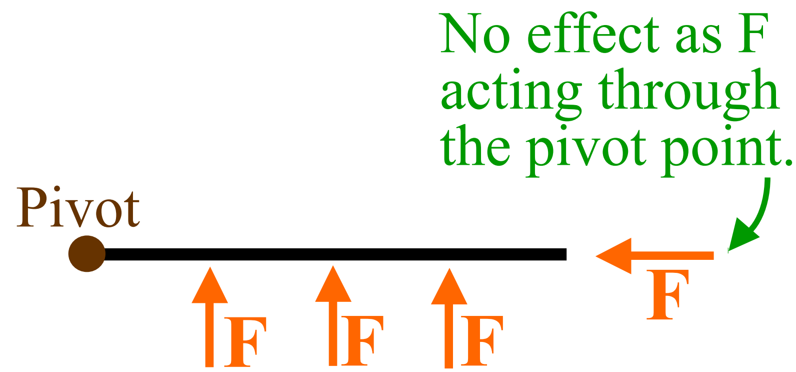
Summary

1. Rotational displacement '**θ**' describes how **far** an object has rotated.
2. Rotational velocity '**ω**' describes how **fast** it rotates (**ω = θ /t**) measured in radians.
3. Rotational acceleration '**α**' describes any **rate of change** in its velocity ($\alpha = \Delta\theta / t$) measured in radians /sec^2.

(All analogous to **linear motion** concepts.)

Why Do Objects Rotate? (Chapter 8)

- Need a **force**.
- **Direction** of force and **point of application** are critical…



Question: Which force '**F**' will produce largest effect?

- Effect depends on the force and the distance from the fulcrum /pivot point.

❖ **Torque 'τ' about a given axis of rotation is the product of the applied force times the lever arm length 'l'.**

$$\tau = F.\ l \quad \text{(units: N.m)}$$

- The lever arm 'l' is **perpendicular distance** from axis of rotation to the line of action of the force.
- Result: Torques (not forces alone) cause objects to rotate.