Conservation of Momentum & Collisions
(Chapter 7)

Recap:

- For situations involving an **impact** or a **collision,** where large forces exists for a very small time we define:

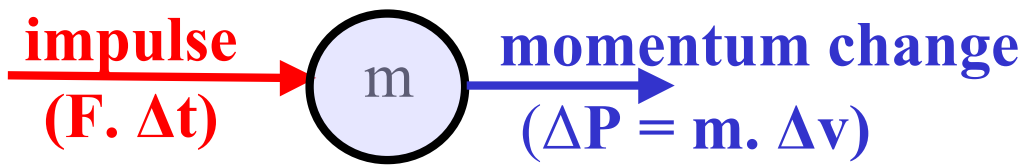
❖ **Impulse = F x Δt** (units: **N.s**) - a **vector**

where **F** is the **force** and **Δt** is the **time of action.**

- By Newton's 2nd law (F = m.a) we determined:

❖ **Impulse = F. Δt = m. Δv**

or **Impulse = Change in momentum (ΔP)**



- Result: **Large** impulses cause **large** changes in motion!

Conservation of Momentum

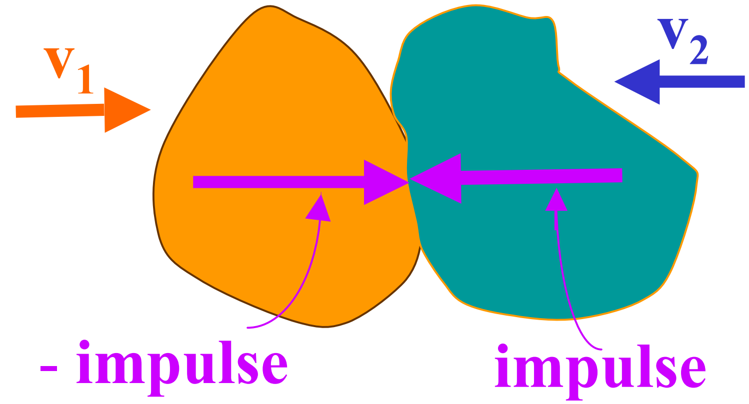
- A new principle for studying collisions which results from **Newton's 3rd law** applied to the **impulse/ momentum** equation.
- Conservation of momentum enables us to **understand collisions** and to **predict** many results **without** a detailed knowledge of **time varying** forces.

Consider: Two **sticky objects** moving towards each other…they meet in mid-air and after colliding stick together and **move as one body**.

- During the moment of impact there is a **strong force** acting for a **time Δt**.
- By Newton's 3rd law, an **equal** and **opposite** force **'F'** acts back (remember forces occur in pairs).

Conservation of Momentum

- As Δt is the same for both forces, the impulses they produce (= F.Δt) are the same magnitude but in opposite directions.



- **By Newton's 2[nd] law:**

Impulse = Change in momentum $(\Delta P = m.\ \Delta v)$

- Thus the **change in momentum** experienced by both objects must be the **same**…but in **opposite** directions.

- The **total change in momentum** of the **system** (i.e. both objects combined) is therefore **ZERO**!

- In other words , the **total momentum** of the **system** is **conserved** (i.e. changes of momentum within system cancel each other out).

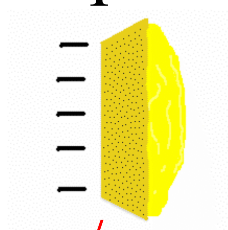
Conservation of Momentum

- **The total momentum of the system is conserved (if no other external forces acting).**
- However, different parts of the system **can** exchange momentum (but the total remains the same).

Note: If a net external force acts on system, then it will accelerate and its momentum will change.

- **Conservation of momentum** allows us to examine interesting impact situations…

Example: Custard pie fights!



$m_c = 1$ kg

$\rightarrow$ $v_c = -10$ m/s

$\rightarrow$ $P_c = m_c.v_c$

positive direction $\leftarrow$

$m_t = 100$ kg

$v_t = 0.5$ m/s $\leftarrow$

$P_t = m_t.v_t$ $\leftarrow$

$P_c = -1 \times 10 = -10$ kg.m/s

(negative sign as opposite direction)

$P_t = 100 \times 0.5 = 50$ kg.m/s

Total momentum = $P_{target} + P_{custard\ pie} = 50 - 10 = 40$ kg.m/s

Question: What is the velocity of target and pie after impact?

Total mass = $m_t + m_c = 101$ kg

Total momentum = 40 kg.m/s (P = m.v)

$v = P / m = 40 / 101 \approx 0.4$ m/s

Result: The unsuspecting target has a larger initial momentum, so his direction of motion prevails but the pie reduces his forward velocity (briefly)!

Recoil

- A **special case** of **conservation of momentum** when the **initial velocity** of the interacting bodies is often **zero.**

  E.g.: - two ice skaters "pushing off" - firing a gun

  - rocket propulsion…

- We have already looked at what happens to the ice skaters motion (using Newton's 3rd law), but now we can use conservation of momentum to determine their velocities…

Example: Initial momentum = 0

Thus: total momentum after "push off" = 0

The momentum of each person must therefore be **equal** but **opposite** in direction and $P_2 = -P_1$.

But, as $\mathbf{P_1 = m_1.v_1}$ and $\mathbf{P_2 = m_2.v_2}$, the velocities will be in opposite directions and will depend on their masses.

Eg. If $m_1 = 3m_2$ then v_2 will be $3v_1$ in opposite direction!

Firing a Gun (initial momentum = zero)

Momentum of bullet = Momentum of gun

$$m_1 v_1 = - m_2 v_2$$

- Mass of bullet is small but its velocity is high…creating a large recoil.
- To reduce velocity of recoil (v_2), hold gun with locked arms so the mass m_2 becomes mass of gun + your body.
- Similarly a very massive cannon will "jump back" much less than a light one…for the same shot.

Rocket propulsion:

- Exhaust gasses have **large momentum** (light molecules but **very high velocity**).
- Momentum gained by rocket in **forward direction** equals momentum of exhaust gasses in **opposite direction**.
- This is why **rockets** (i.e. recoil) **work in outer space**… as gasses and rocket **push** against each other as gasses expelled.

Collisions

- Two main types: **Elastic** and **Inelastic**…
- Different kinds of collisions produce different results…e.g. sometimes objects **stick** together and other times they **bounce** apart!
- Key to studying collisions is **conservation of momentum** and **energy** considerations…

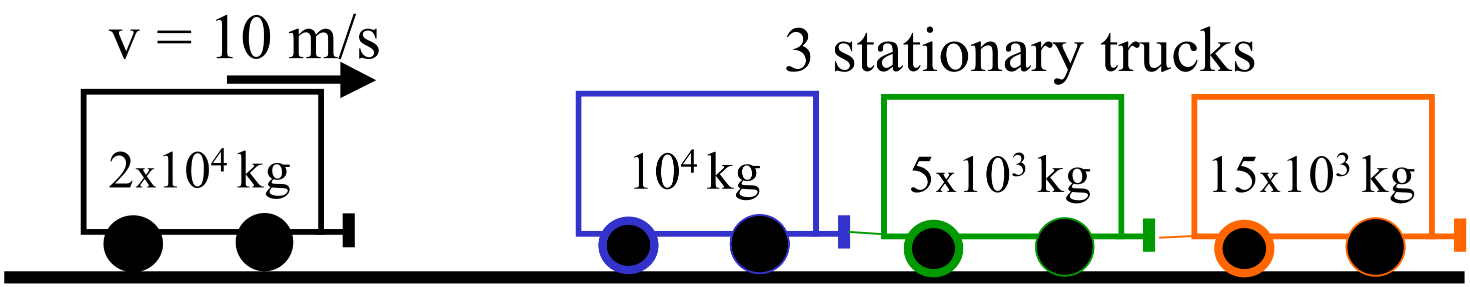
Questions:

- What happens to **energy** during a collision?
- Is **energy** conserved as well as **momentum**?

Perfectly Inelastic Collisions (Sticky ones!)

- E.g. Two objects collide head on and stick together, moving as one after collision (only one final momentum / velocity to compute).
- Ignoring external forces (which are often low compared with large impact forces), we use conservation of momentum.

E.g. Coupling train trucks (low rolling friction)



Before: System momentum: $mv = 2 \times 10^4 \times 10 = 2 \times 10^5$ kg.m/s

After: Final system mass $= (20+10+5+15) \times 10^3$ kg $= 50 \times 10^3$ kg

As final momentum = initial momentum

$$v_{final} = \frac{P_{final}}{\text{total mass}} = \frac{2 \times 10^5}{5 \times 10^4} = 4 \text{ m/s}$$

- Thus total momentum of system has remained **constant** but the colliding truck's velocity has reduced (i.e. momentum shared).

Question: What happens to the energy of this system?

Total energy = Kinetic Energy $= \frac{1}{2}.m.v^2$ (i.e. no PE change)

Before impact: $KE_{tot} = KE_{truck} + KE_{3trucks}$

$= \frac{1}{2}(2 \times 10^4)(10)^2 + 0$

$= 10^6$ Joules (1MJ)

After impact: $KE_{tot} = \frac{1}{2}.m_{tot}.v^2_{tot} = \frac{1}{2}(5 \times 10^4)(4)^2$

$= 2 \times 10^5$ J

Energy differences = $(10 - 2) \times 10^5$ J = 8×10^5 J (**i.e. 80% loss**)

Results: Energy is **lost** in an **inelastic collision** (heat, sound…) and the **greatest** portion of energy is lost in a **perfectly inelastic collision** when objects **stick** together!

Extreme example:

Pie (or bullet) hitting a wall… **All KE is lost** on impact!

Bouncing Collisions

- If objects **bounce** off one another rather than sticking together, **less energy is lost** in the collision.
- Bouncing objects are called either **"elastic"** or **"partially inelastic"**. The distinction is based on **energy**.

❖ Elastic Collisions:

- **No energy** is lost in an **elastic collision.**

E.g. A ball bouncing off a wall / floor with **no change** in its speed (only direction).

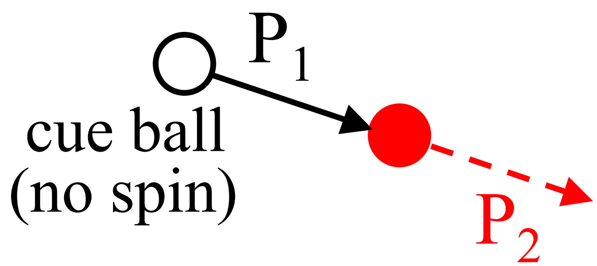
❖ Partially Inelastic Collisions:

- In general **most collisions** are "**partially inelastic**" and involve some **loss** of **energy**… as they bounce apart.
- **Playing pool:**
- **Very little energy** is lost when balls hit each other and the collision is essentially **elastic**. In such cases:

Momentum and **energy** are **conserved.**

- In an **elastic collision** we need to find the final velocity of **both** colliding objects.
- Use **conservation of momentum** and **conservation of energy** considerations…

Example:



Question: What happens to red ball and cue ball?

- Answer: The cue ball **stops dead** on impact and red ball moves forward with the same velocity (magnitude and direction) as that of the cue ball prior to impact!
- Why?...Because both KE($= \frac{1}{2}.m.v^2$) and momentum (m.v) are conserved on impact.
- As the masses of both balls are the **same** the only solution to conserve both KE and momentum is for **all** the energy and momentum to be transferred to the other (red) ball.
- It's a fact…try it for yourself!!!