Recap: Energy Accounting

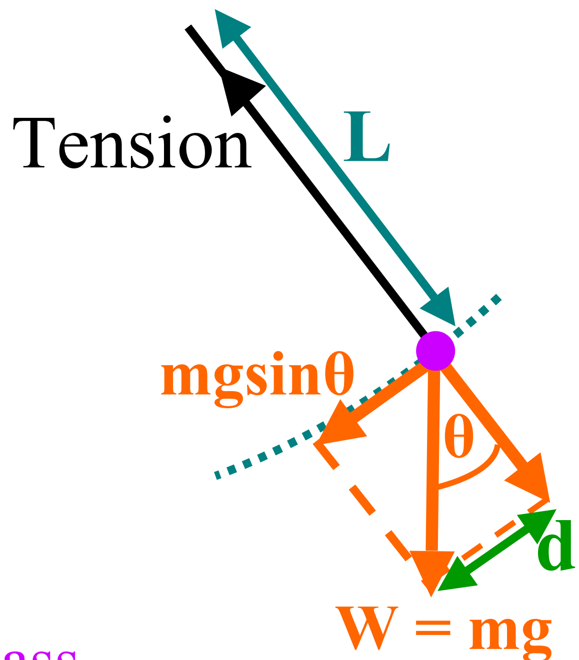
- Energy accounting **enables complex systems** to be studied.

  Total Energy = KE + PE = conserved

- Even the simple pendulum is not easy to study using Newton's laws of motion, as the **force** (hence acceleration) is **constantly changing** in a cyclic manner.
- **Energy considerations enable us to predict things:**
  - Height is same on both sides of swing
  - Where KE = max
  - Where velocity = 0, PE = max etc.
- Energy accounting can also be used to **compute velocity at any point** in the motion.

Simple Harmonic Motion (SHM)

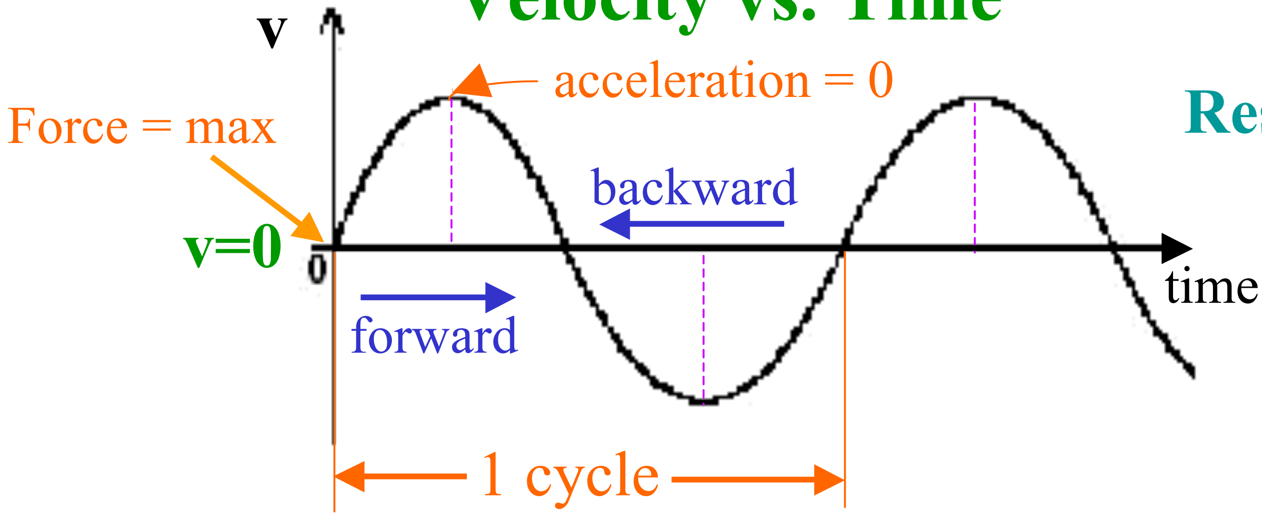
- The pendulum is an example of **conservation of energy**, where PE and KE are transformed back and forth.
- SHM occurs whenever the "**restoring force**" F is **proportional** to object's **displacement** (d).
- **For SHM to occur: $F \propto -d$**



- Pendulum: restoring force is: $F = - m.g.\sin\theta$
- For small angles $\sin\theta \approx \theta \approx \frac{d}{L}$
- Thus: $F = -\left(\frac{m \times g}{L}\right) \times d$
- And its period $T = 2\,D\sqrt{\frac{L}{g}}$ mass independent

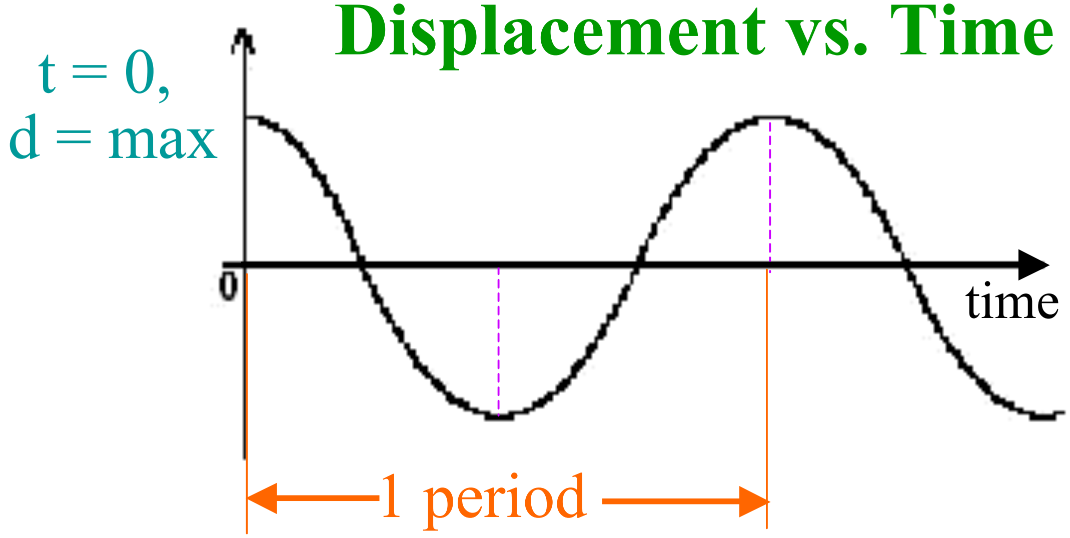
Analysis of Pendulum SH Motion

Velocity vs. Time



Result: acceleration ∝ displacement

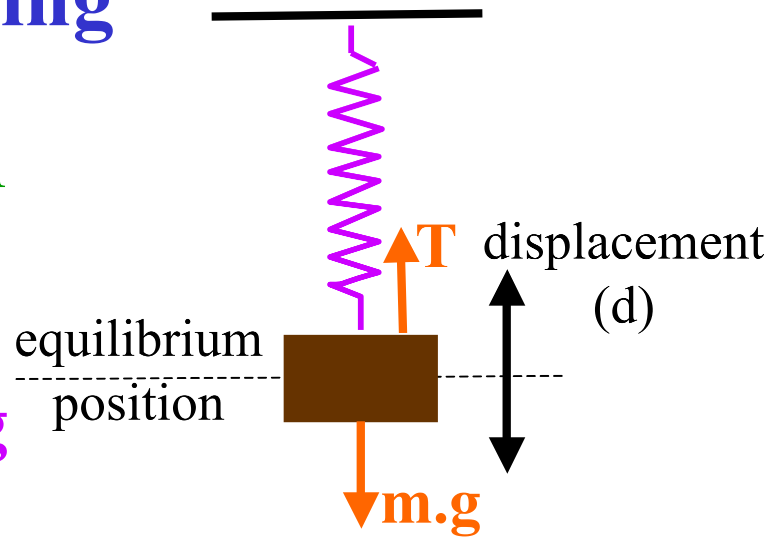
Displacement vs. Time



Result: velocity and displacement ¼ cycle out of phase

Many systems exhibit SHM (or more complex oscillations).

SHM Example: Mass on a Spring



At equilibrium position the tension in spring **balances** the weight (m.g).

- When we pull on mass to displace it, the spring will exert a **"restoring force"** to pull it back towards equilibrium position.

- The spring's restoring force **F = - k. d** (where 'k' is a constant of spring – its stiffness).
- **SHM results as F ∝ -d** and the mass will oscillate about its equilibrium position.
- SHM for a mass on spring: $\hat{O} = 2\,D\sqrt{\frac{m}{k}}$ **(mass dependent)**
- Thus, we can use an oscillating spring to **determine mass!**

Note: At any time PE+KE = constant

PE = sum of gravity PE + elastic PE (PE = mgh + $\frac{1}{2}kd^2$)

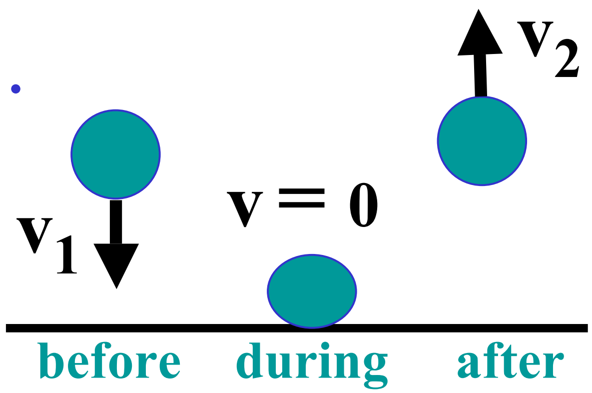
Impulse and Momentum (Chapter 7)

- What happens when **strong forces** are at work? E.g. during a collision…
  - Forces act only **briefly**…
  - Produce **large accelerations**…
  - Large **changes in direction**!

Example: What happens when a bat hits a ball?

- At impact the balls **velocity** suddenly **changes in direction** and it is then **accelerated** in the new direction (often nearly opposite to its original motion).
- Brief impacts, termed "**impulses**" therefore **cause rapid changes** in an object's **velocity**.
- **Forces** responsible for such rapid changes in motion must be **very large** but only **act** for a **short time**.
- Such forces / situations are **very difficult to measure**, especially as the forces may **change** during the collision!

Example: Dropping a tennis ball…



- Initially the ball is **accelerated downwards** by gravitational force.
- On impact the ball behaves like a **spring**: **compressing** as it moves downward **absorbing kinetic energy** and then **expanding** (springing back) a moment later to begin upward motion.
- At the ground the **velocity** of the ball suddenly **changes direction** – it "bounces up".
- A **strong force** must therefore have been exerted **on the ball by the floor** (but for a very short time).
- This force produced a rapid **deceleration** of ball to zero velocity and then **increased** its **velocity** equally rapidly in **opposite direction**!

Newton's 2nd Law Revisited

- It is very **difficult** to apply Newton's 2nd law ($F = m.a$) in this situation as forces act only briefly and can vary a lot.
- Easier for us to look at the **total change in motion** during the impact.

Newton's 2nd law: $\mathbf{F = m. a}$; $a = \frac{\text{change in velocity}}{\text{time interval}} = \frac{\Delta v}{t}$

- During impact **'t' is very small** indicating that **'a'** must be **very large** (for a given change in velocity).

For **impact situation** we can write:

$$a = \frac{\Delta v}{\Delta t} \qquad \text{and} \qquad F = m.\frac{\Delta v}{\Delta t}$$

Re-arranging gives:

$\mathbf{F. \Delta t = m. \Delta v}$ (still Newton's 2nd law)

where $\mathbf{F. \Delta t}$ is defined as the **"impulse".**

Impulse

- Impulse is the **average force** acting on an object **multiplied** by its **time interval of action**.

$$\text{Impulse} = F \cdot \Delta t \quad (\text{units} = \text{N.s})$$

Note: Since instantaneous force may vary during impact we must use **average force**.

- Impulse is a **vector** acting in the **direction** of **average force**
- The **larger** the **force** (F) and the **longer** it **acts** (Δt) the **larger** the **impulse**. (Impulse is therefore a measure of the overall effect of the force.)

However: Impulse = $F \cdot \Delta t = m \cdot \Delta v$

- So an impulse causes a **change in velocity** (Δv) in **magnitude** and **direction.**
- In Newton's words the product "**$m.\Delta v$**" is the change in the "**quantity of motion**".
- We now term this the **change in momentum**.

Momentum

- **Momentum (P)** is the product of the **mass** of an object and its **velocity**:

$$P = m.v \quad \text{(units: kg.m/s)}$$

- **Momentum** is a **vector** acting in same **direction** as **velocity vector.**

Example: A 100kg boulder rolling towards a castle gate at 3m/s.

Momentum boulder: $P = m.v = 100 \times 3 = 300$ kg.m/s

But different objects can have the same momentum:
e.g. A 1 kg missile flying towards the castle gate at 300 m/s (speed of sound).

Momentum missile: $P = m.v = 1 \times 300 = 300$ kg.m/s

- For an object of mass 'm' the change in momentum (Δp) is:

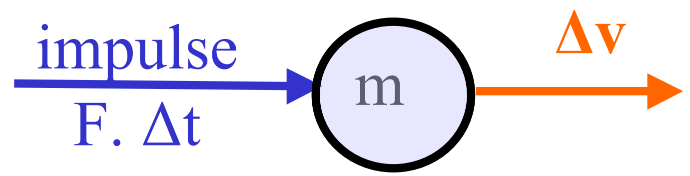
$$\Delta P = m.\,\Delta v = \text{impulse}$$

Result: impulse = change in momentum.

Impulse - Momentum Principle

- An **impulse** acting on an object causes a **change** in its **momentum**.

- The change in momentum is **equal** in **magnitude** and **direction** to the applied **impulse**.



- **Impulse = m. Δv**
- **A new way of looking at Newton's 2nd law!**

Example: A 50kg rock is hurled by a giant catapult with a force of 4000 N applied for 0.5 sec.

Impulse = F. Δt = 4000 x 0.5 = 2000 N.s

Thus: **Change in momentum:** = 2000 Ns = **m. Δv**

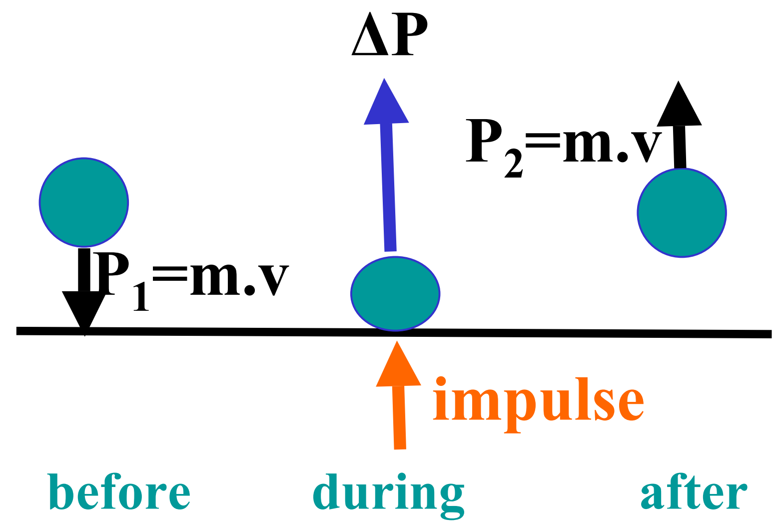
or $\Delta v = \frac{2000}{50} = 40$ m/s (≈ 140 km/hr)

Note: When the initial velocity is zero:

- change in momentum = objects momentum
- change in velocity = objects velocity

Ball's Change in Momentum

1. Assume **no** energy is lost, therefore KE of the ball is the same before and after impact. (KE = ½m.v²)



2. On **impact** the momentum of ball is decreased to **zero**.

3. The **total change** in momentum ΔP is:

$$\Delta P = P_2 - P_1 = m.v - (- m.v)$$

$$\mathbf{\Delta P = 2\ m.v}$$

(As P_1 opposite to P_2)

- The impulse required to **change** the direction of ball is therefore equal to **twice momentum** of the impacting ball.

(ie. Twice as large as what is needed to simply stop the ball.)

Summary:

- Rewriting Newton's 2nd law shows that:

  The change in momentum is equal to the impulse.

- **Large impulses** produce **large changes** in **momentum -** resulting in **large velocity changes.**

- Next we will talk about the **conservation of momentum** and how to use it to study impacts.

Read chapter 7.