Recap: Work and Energy

- **Work** is the **applied force** times the **distance moved** (in direction of applied force).
- **Work done: $W = F . d$** Units: Joules

Power is the **rate of doing work** – the faster its done, the greater the power. Units = **Watts** (1 hp = 746 W)

- **Two types of energy transfer can occur:**
  - **Kinetic energy** (due to a change in its motion)
  - **Potential energy** (due to its change in position while acted upon by a force)

❖ Work done due to **change in kinetic energy**

$$W = KE = \frac{1}{2} m.v^2 \quad \text{(Joules)}$$

Kinetic energy is the result of an object's motion and is **proportional** to its **velocity squared.**

Energy Loss "Negative Work"

- To reduce the KE of an object (e.g. a car), we also need to perform work, called "Negative Work".
- In this case work is done by friction when braking. (Brake drums heat up or tires skid.)
- Negative work **reduces energy** of a system.

Example: Stopping distance for a car, etc…

Kinetic energy of a vehicle is proportional to v^2 - if speed is doubled, the KE is quadrupled!

i.e. A bus traveling at 80 km/hr has four times as much kinetic energy than one at 40 km/hr.

Doubling the speed requires four times as much "negative work" to stop it.

Result: Stopping distance is around **four times longer** for the 80 km/hr vehicle!!! (assuming a constant frictional force).

Summary: The KE gained or lost by an object is equal to the work done by the net force.

Potential Energy

Question: What happens to the work done when e.g. lifting a box onto a table?

If we lift it so that our applied force is equal and opposite to its weight force, there will be no acceleration and no change in its Kinetic Energy. (F = m.g)

Yet work is clearly done…

Answer: The work done increases the **gravitational potential energy** of the box.

- Potential energy is **stored energy** (for use later) e.g. a rock poised to fall…
- Potential energy involves changing the position of an object that is being acted upon by a specific force (e.g. gravity).
- **Gravitational potential energy (PE) equals work done to move object a vertical distance (h).**

PE = W = F . d

PE = m.g.h (units = Joules)

Potential Energy

- Gravitational PE = m.g.h
- Thus, the **further** we move an **object** away from the **center of the Earth**, the **greater** its **potential energy**.
- In practice, we measure the **change** in potential energy.
- The **larger** the **height change**, the **larger** the **change in PE**

Example: What is PE of a 50kg box lifted 10m?

Gravitational PE = m.g.h

PE = 50 x 9.8 x 10 = 4,900 J

- If height is doubled, PE is doubled…

- Other kinds of PE involving **other forces** also exist (e.g springs).
- For a spring, the elastic PE = $\frac{1}{2}$ $k.x^2$

where x = spring extension, and k = constant

Summary

- **Work done = Change in kinetic energy**

  $W = KE = \frac{1}{2}\, m.v^2$ (Joules)

- **Potential energy is stored energy associated with an object's position rather than its motion.**

  $PE = m.g.h$ (Joules)

- **The "system" is poised to release energy converting it to KE or work done on another system.**

- **Potential energy can result from work done against a variety of <u>conservative forces</u> eg. gravity and springs.**

Conservative Forces

- PE can result from **work done** against a variety of **conservative forces** (eg. gravity…)
- Forces that can result in **stored PE** are called **conservative forces** – the **energy gained** by work done against a conservative force is **completely recoverable.**
- I.e. for conservative forces **there is no energy loss**.
- Work done against **friction** does **not** increase the **potential energy** of a system (as heat is generated and energy is transferred out of system).
- Friction is **not** a conservative force!

Conservation of Energy

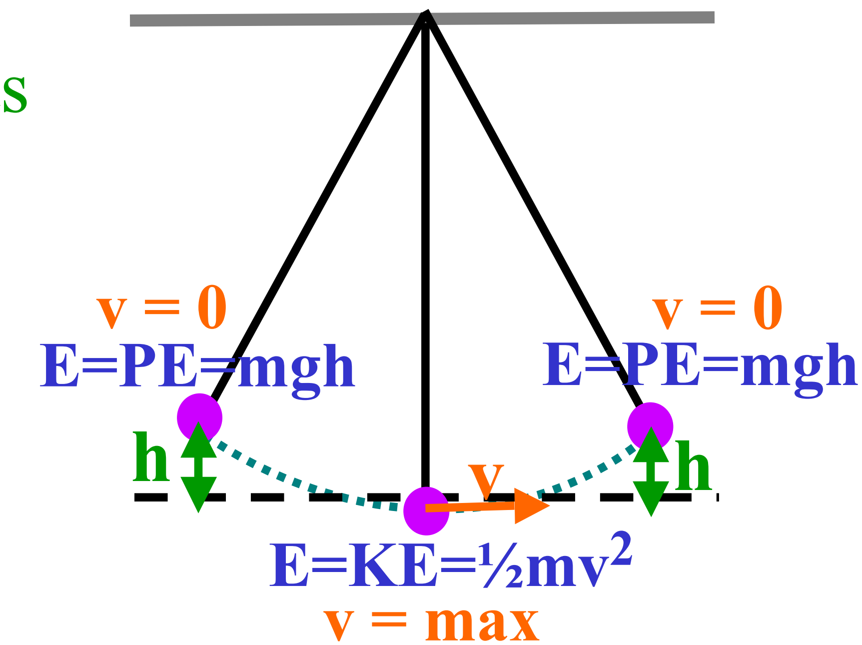
- **Total energy** equals the **sum** of **PE** and **KE** and is **constant** (ie. conserved) in many situations.

Total energy, $E = PE + KE = constant$

- This is true provided **no** additional work is done on the system i.e. no energy is **added** or **subtracted.**

Example: A pendulum swinging is a conservative system where the PE and KE are **constantly** transforming from **one form into the other** as the pendulum bob oscillates.

Simple Pendulum: Energy change considerations



1. Work is done to raise ball, so its **PE = m.g.h**
2. Release ball and PE begins to **convert** to KE.
3. At equilibrium (vertical position) **PE = 0** and all energy = KE (max velocity).
4. Balls **inertia** carries it through this point and KE is transferred back to PE again. **KE = 0** at height 'h'.
5. Motion then **repeats – perfectly conserved** (if no friction).

- At any intervening point the total mechanical energy is **sum of PE and KE and is constant.**
- Thus pendulums demonstrate **"conservation of energy".**

Energy Accounting

- Energy accounting enables complex systems to be studied.
- Even the simple pendulum is not easy to study using Newton’s laws of motion, as the **force** (hence acceleration) is **constantly changing** in a cyclic manner.
- Energy considerations enable us to predict things:
  - Height is same on both sides of swing
  - Where KE = max
  - Where velocity = 0, PE = max etc.
- Energy accounting can also be used to compute velocity at any point in the motion.

Example: "Big Bob" has mass of 2 kg and displaced a height h=30 cm above its equilibrium value. Determine its max speed.

$$PE = m.g.h = 2 \times 9.8 \times 0.3 = 5.9 \text{ J}$$

At lowest point PE = 0 and all energy is KE:

$$KE = \tfrac{1}{2} m.v^2 \quad \text{or} \quad v_{max} = \sqrt{\frac{2\times KE}{m}} = \sqrt{\frac{2\times 5.9}{2}} = 2.4 \text{ m/s}$$

Qu: What is the velocity at a point where bob height = 10 cm?

We use the fact that total energy is conserved.

$$PE + KE = const = 5.9 \text{ J}$$

But PE (at 10 cm height) = m.g.h = 2 x 9.8 x 0.1 = 1.96 J

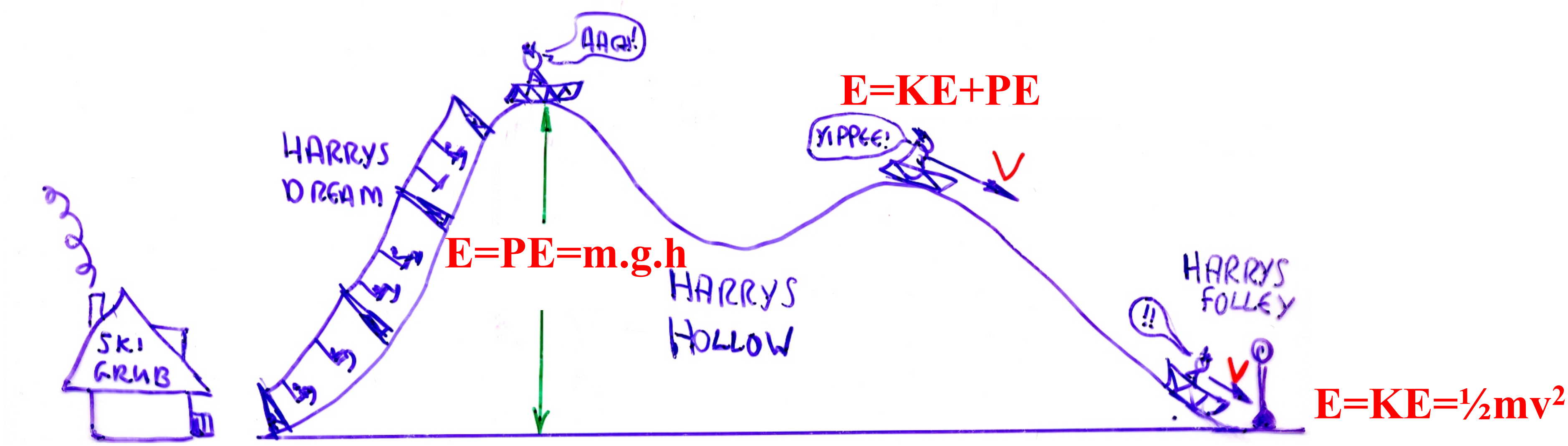
Therefore, KE = 5.9 – 1.96 = 3.94 J

As $KE = \tfrac{1}{2} m.v^2 = 3.94$ J => $v = 1.98$ m/s (i.e. $< v_{max}$)

Thus, we can compute velocity at any point using simple energy considerations…energy accounting.

More complicated situations can also be studied:

- Drawing a string on a bow,
- Plucking a guitar,
- Riding a roller coaster,
- Sled on a hill at Beaver!



- As long as PE at the **initial point is greater** than at other points, the ride will continue…
- This neglects friction, however, any work done by friction ($W=F_f \times d$) will reduce system energy as **distance increases.**
- Thus **frictional work accumulates** (as negative work) and should **not be ignored** in practical applications.

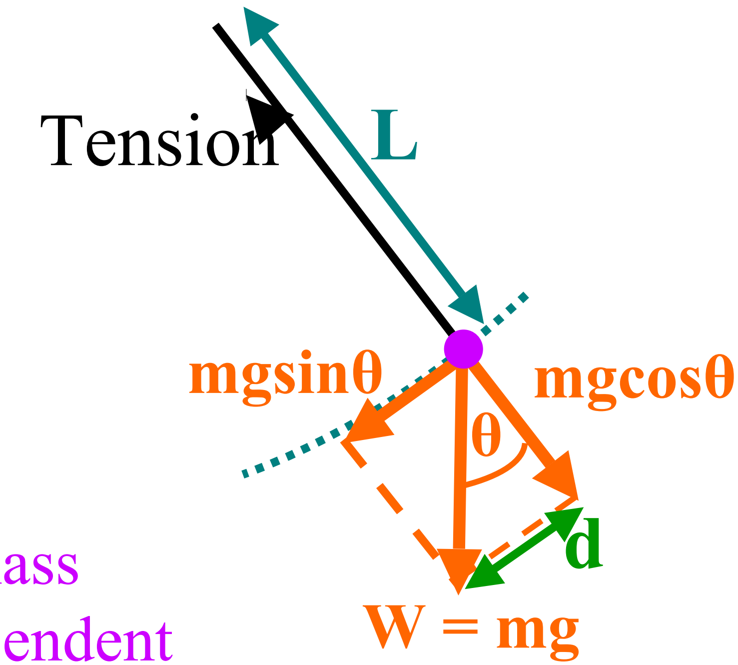
Simple Harmonic Motion (SHM)

- The pendulum is an example of conservation of energy, where PE and KE are transformed back and forth in a cyclic manner.
- It is also an excellent example of "simple harmonic motion".
- SHM occurs whenever the "restoring force" F is proportional to object's displacement (d) from its equilibrium position.

For SHM to occur: $F \propto -d$

- This also means the acceleration of a SHM oscillator is proportional to displacement (as F = m.a).
- During SHM the force and acceleration are constantly changing as 'd' changes cyclically!

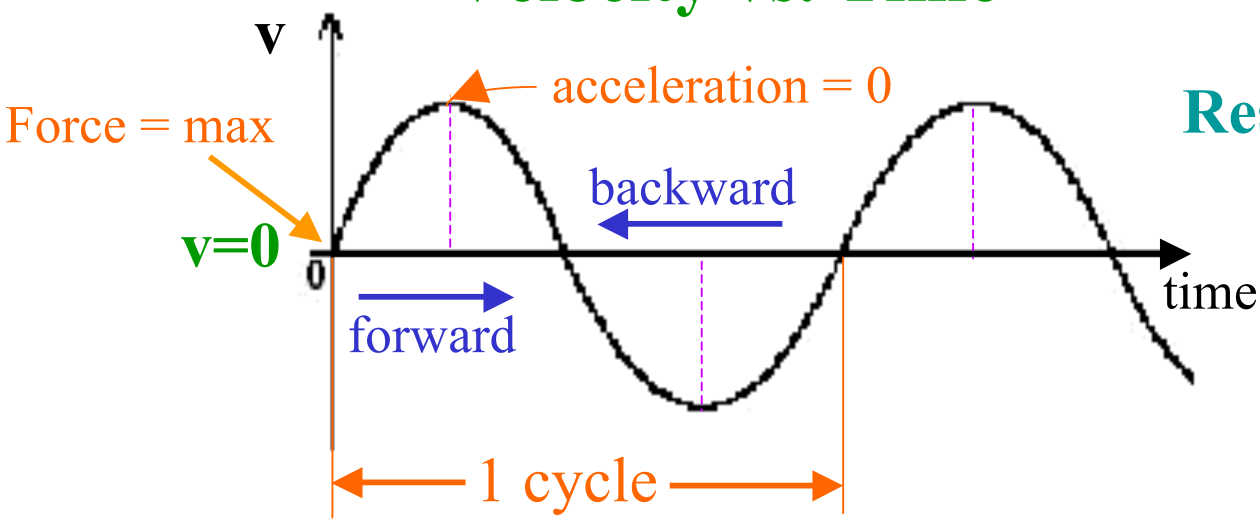
Example: SHM for a Simple Pendulum



- Pendulum: restoring force is: $F= - m.g.\sin\theta$
- For small angles $\sin\theta \approx \theta \approx \frac{d}{L}$
- Thus: $F = -\left(\frac{m \times g}{L}\right) \times d$ ie. SHM
- And its period $T = 2 D \sqrt{\frac{L}{g}}$ mass independent
- Thus for a pendulum the restoring force is due to gravity alone ($F = - m.g.\sin\theta$).
- The tension force is perpendicular to the motion and has no contribution.
- Air resistance dissipates its energy in time.

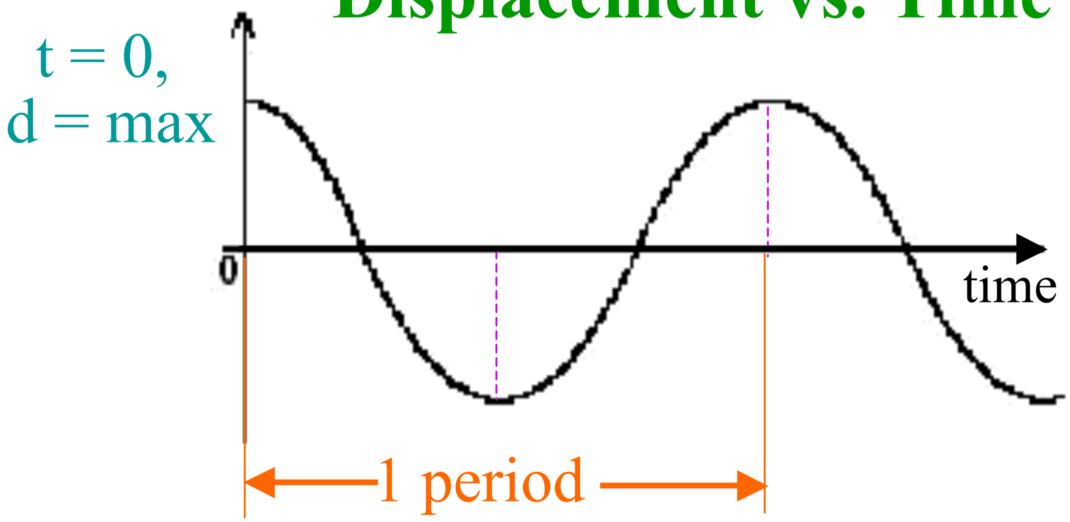
Analysis of Pendulum SH Motion

Velocity vs. Time



Result: acceleration ∝ displacement

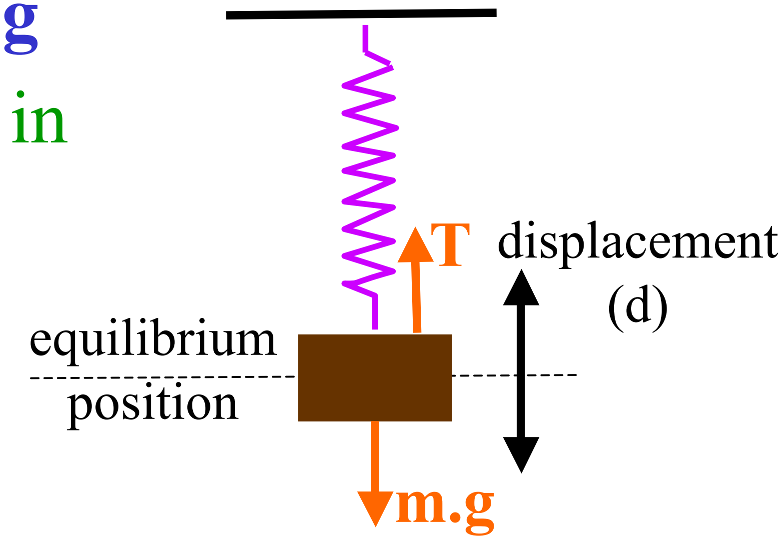
Displacement vs. Time



Result: velocity and displacement ¼ cycle out of phase

Many systems exhibit SHM (or more complex oscillations).

SHM Example: Mass on a Spring



- At equilibrium position the tension in spring balances the weight (m.g).
- When we pull on mass to displace it, the spring will exert a "restoring force" to pull it back towards equilibrium position.
- The spring's restoring force $F = - k. d$ (where 'k' is a constant of spring – its stiffness).
- SHM results as $F \propto d$ and the mass will oscillate about its equilibrium position.
- SHM for a mass on spring: $\hat{O} = 2 D \sqrt{\frac{m}{k}}$ **(mass dependent)**
- Thus, we can use an oscillating spring to **determine mass!**

Note: At any time PE+KE = constant

PE = sum of gravity PE + elastic PE (PE = mgh + ½kd^2)