

Recap: Artificial Satellites

- Kepler's 3rd Law: $T^2 \propto r^3$
- Thus the lower the altitude, the shorter the period.
- Example: Sputnik (1958)
Launched into a very low altitude orbit of ~ 270 km.
 $\Rightarrow T = 90$ min (1.5 hrs to orbit Earth)
- Example: Geosynchronous Orbit
- Orbital period, $T = 24$ hours
- Permits satellites to remain “stationary” over a given equatorial longitude.
 $\Rightarrow r = 42,000$ km (to center Earth)
i.e. altitude $\approx 7 R_e$
(compared with $60 R_e$ for Moon.)

Orbital Velocity

- By equating the centripetal force to the gravitational attraction force we can determine orbital speed (v_{or}).

For circular motion:

Centripetal force = gravitational force ($F_C = F_G$)

$$\frac{m v_{or}^2}{r} = \frac{G M m}{r^2}$$

M = planet's mass
 m = satellite's mass
 $M \gg m$

$$v_{or} = \sqrt{\frac{G M}{r}}$$

Results:

- Any satellite regardless of its mass (provided $M \gg m$) will move in a circular orbit of radius r and velocity v_{or} .
- The larger the orbital altitude, the lower the required tangential velocity!

Example: Low Earth orbit (LEO): altitude 600 km

$$v_{\text{or}} = \sqrt{\frac{GM}{r}} = \sqrt{\frac{(6.67 \times 10^{-11}) \times (5.98 \times 10^{24})}{6970 \times 10^3}}$$

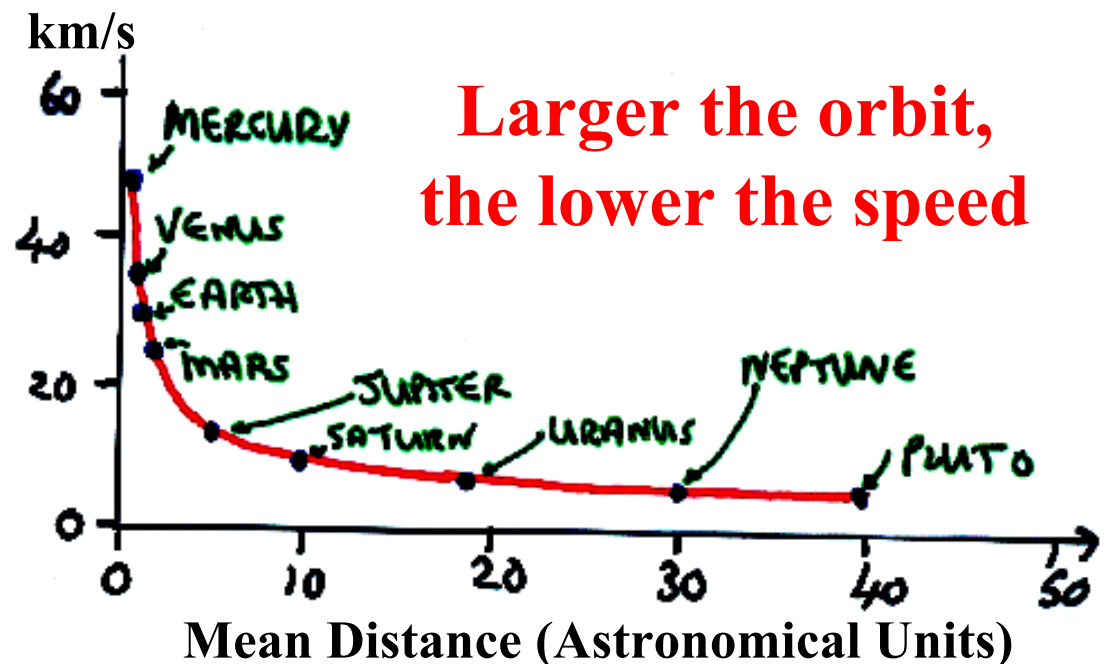
= 7.6 km/s (or 27×10^3 km/hr)

Note: At lunar orbit (~400,000 km) orbital speed ~ 1 km/s

**Planetary Orbital
Velocity:**

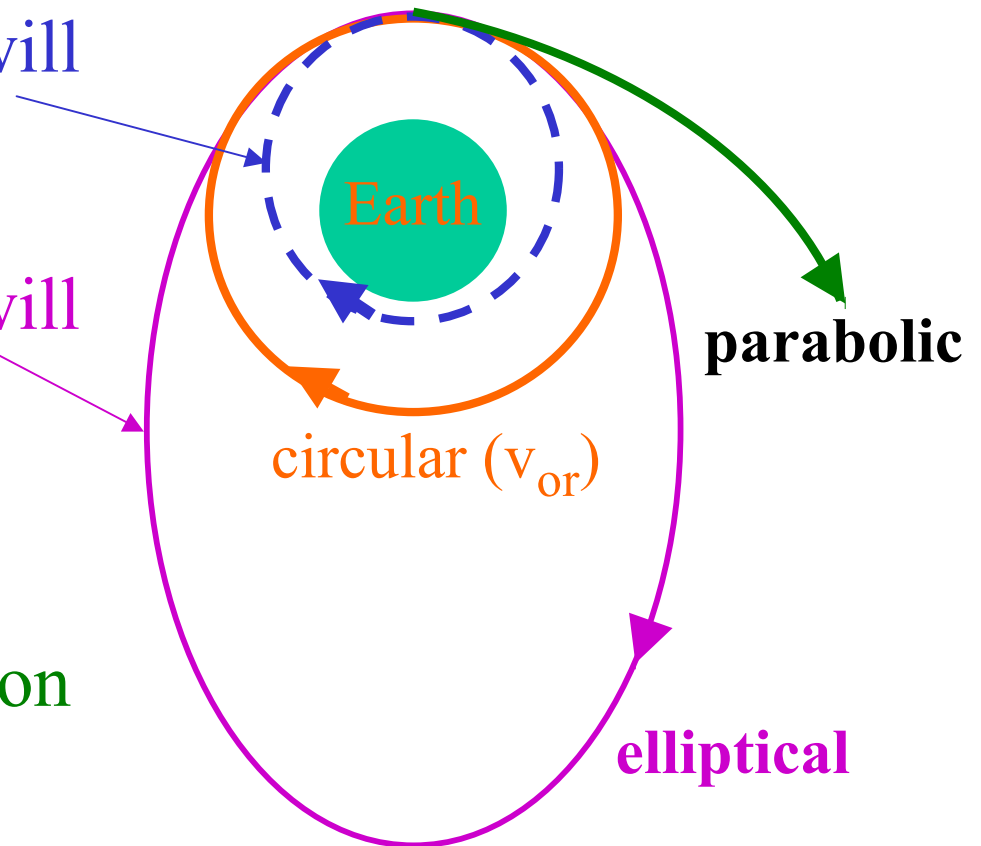
$$v_{\text{or}} = \sqrt{\frac{GM}{r}}$$

M = mass of the
Sun



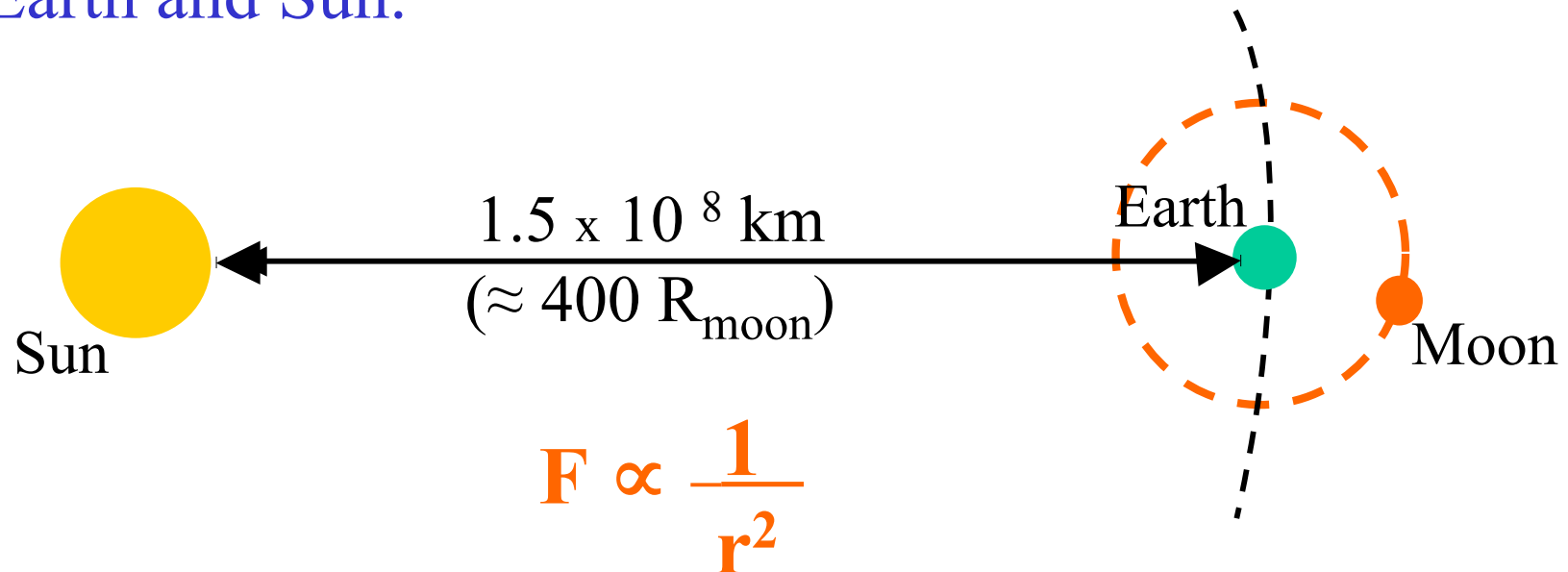
How to Achieve Orbit

- Launch vehicle initially rises vertically (minimum air drag).
- Gradually rolls over and on separation of payload is moving tangentially at speed v_{or} produces circular orbit.
- If speed less than v_{or} , craft will descend to Earth in an (decaying) elliptical orbits.
- If speed greater than v_{or} it will ascend into a large elliptical orbit.
- If speed greater than $\sqrt{2}v_{or}$ it will escape earths gravity on parabolic orbit!



Lunar Orbit

- Not a simple ellipse... due to gravitational force of Earth and Sun.



- Gravitational attraction between Earth and Moon provides **centripetal acceleration** for orbit.
- **Sun's** gravitation **distorts** lunar **orbital ellipse**. (Orbit oscillates about true elliptical path.)

Properties of the Moon

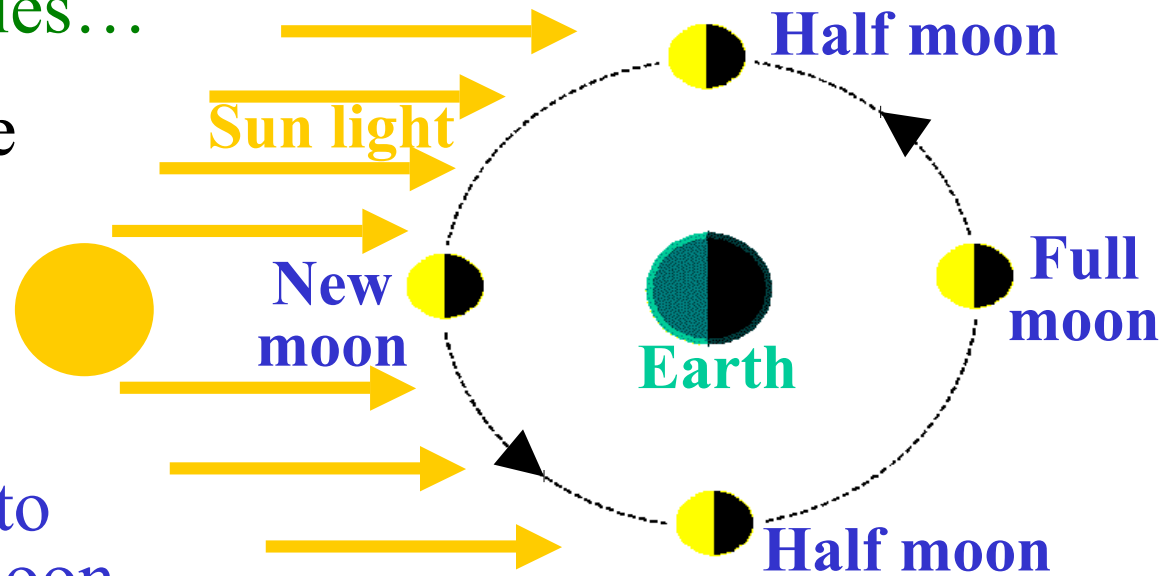
- The only satellite Newton could study and played a key role in his discoveries...

- Phases known since prehistoric times...

- Moonlight is reflected sunlight.

- Phases simply due to geometry of sun, moon and Earth.

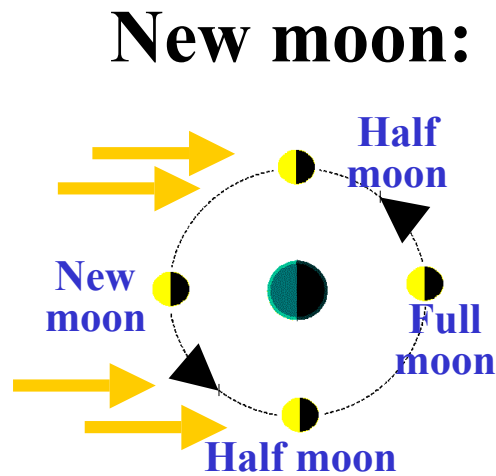
Phases repeat every 27.3 days (lunar orbit).



- Full moon:**
- Moon on opposite side of Earth from Sun.
 - Fully illuminated disk.
 - Rises at sunset and sets at sunrise (due to Earth's 24 hr rotation).
 - Lunar eclipse only during full moon.

Properties of the Moon (cont'd)

- At other times during moon's orbit of Earth, we see only a part of illuminated disk.



New moon:

- Moon on same side of Earth as the Sun.
- Essentially invisible (crescent moons seen either side of New moon).
- Rises at sunrise and sets at sunset (i.e. up all day).
- Solar eclipse condition.
- When moon is in-between full and new phase, it can often be seen during daylight too.
- Example: half moon rises at noon and sets at midnight (and vice versa).
- Under good observing conditions (at sunset or sunrise) you can see dark parts of moon illuminated by Earthshine!

Energy (Chapter 6)

What is energy?

- Energy comes in many forms:

Mechanical

Electrical

Chemical

Atomic

Thermal

Acoustic...

- Energy can be converted from one form to another...

Example: A car converts chemical energy into mechanical (motion) energy and heat.

Question: How can a “system” change its energy?

Answer: By doing “**work**” on it!

What is work and what does it depend on?

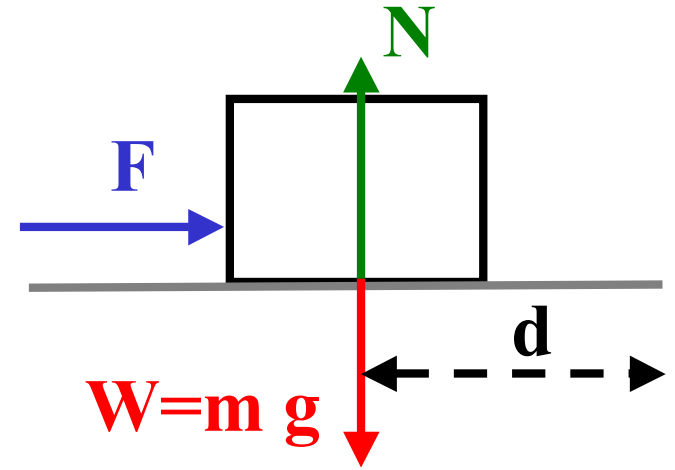
- When you **push** your car or **lift** a box, you are doing **work** (against friction or against gravity).
- When we do **work** on a system, we **raise** its **energy** (eg. lift a pendulum bob).
- This excess **energy** can then be used for **motion** (eg. starting a pendulum) or it can be **stored** (eg. in a battery) for later use.
- **Example:** When you **lift** a **book** up and put it on a high shelf, you are doing work **against** gravity and **increasing** the **potential energy** of the book.
- **Work:** - **Depends on** **strength** of applied force.
- **Distance** object is **moved** by the force.

$$\text{Work} = \text{Force} \times \text{Distance}$$
$$= F \times d$$

Units: Work = N.m or Joules (J)

- 1 Joule is the work expended by an applied force of 1 Newton acting over a distance of 1 meter.
- Work is the same as energy (work is mechanical energy).
- The work done by a given force is the product of the component of force acting along the line of motion of object, multiplied by the distance moved under that force.

Thus the further you move an object or larger the applied force, the larger the work done.



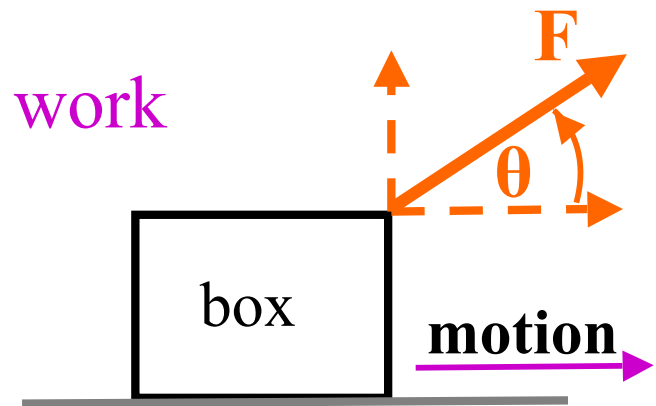
Example: What is the work done to move a box by 10 m using a force of 100 N?

$$W = F \times d = 100 \times 10 = 1,000 \text{ J}$$

i.e. 1000 J of **energy are expended** by you and used to **raise the energy** of box and its surroundings by 1,000 J.

- In general we use the component of applied force causing motion, to find work done:

e.g. $W = F \cos \theta \cdot d$



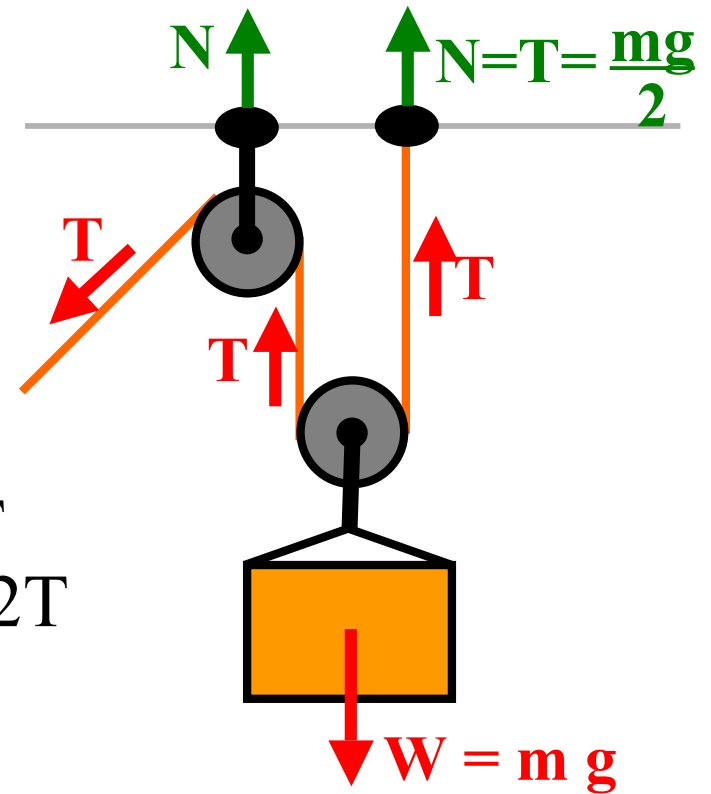
- No work is done by forces acting perpendicular to the motion.** (e.g. by box weight or normal force for horizontal motion).

Simple machines:

e.g. a lever or a pulley

Mechanical devices designed to **multiply** effects of applied force.

In this example a tension force of $1T$ can be used to lift a weight equal to $2T = W = m g$.



Key: The **work done** is the same for both the **lifter** and the **object lifted** as **lifter** needs to **move $2d$** to lift **object** up **$1d$** in height.

i.e. **Work output = Work input**
(assuming no losses due to friction etc.)

Power and Work

- The **larger** the **applied force** (F) the **quicker** we will move the object - e.g. a car accelerating (as $F = m.a$).
- The **rate at which work** is done on a system depends on its **power** – more powerful engines can accelerate a car to a given velocity in a shorter time.

❖ **Power is the rate of doing work:**

$$P = \frac{\text{Work}}{\text{Time}} = \frac{W}{T} = \frac{\text{Force} \times \text{Distance}}{\text{Time}}$$

- Units: Power = J / s = Watt (W)

Example: What is the power required to move the 50 kg box 10 m using a force of 100 N in (a) 10s and (b) 2s.

$$\text{Power} = \frac{F \times d}{t} \quad \begin{array}{ll} \text{(a) } P = \frac{100 \times 10}{10} & \text{(b) } P = \frac{100 \times 10}{2} \\ & = 100 \text{ W} \qquad \qquad \qquad = 500 \text{ W} \end{array}$$

Summary

1. **Work** is the applied force times the distance moved (in direction of applied force). Units = **Joules**
2. **Work output** cannot exceed **work input** (but forces can be multiplied at expense of distance moved).
3. **Power** is the rate of doing work – the faster its done, the greater the power. Units = **Watts** (1 hp = 746 W)
4. **Work is energy** and doing work on a system increases the total energy of the system.

Question: How does “**doing work**” on a system raise its energy?

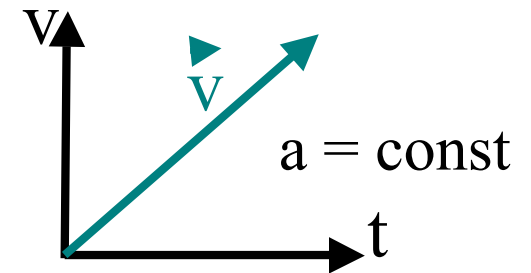
Two types of energy transfer can occur:

1. **Kinetic energy** (due to a change in its motion)
2. **Potential energy** (due to its change in position while acted upon by a force)

Kinetic Energy

- An applied force (F) will cause a body of mass (m) to accelerate ($F = m \cdot a$) and hence its velocity will increase uniformly with time.
- Doing work on an object in this manner causes its kinetic energy to increase i.e.

- In time as the velocity increases you need to run faster to apply the force.



i.e. In equal time intervals you will move larger distances as your velocity increases ($d \propto v^2$).

- Thus work done is proportional to velocity squared.

❖ **Work done = Change in kinetic energy**

$$W = KE = \frac{1}{2} m \cdot v^2 \quad (\text{Joules})$$

- Kinetic energy is the result of an object's motion and is proportional to its velocity squared.
- **Note:** Under constant acceleration the KE (and work done) increases rapidly with time and hence the power needed increases with time.

Example: Compare energy gained by a system by doing work on it with its resultant KE. (Box at rest, mass = 50 kg, applied force = 100N (net force), distance moved = 10 m.)

Work done $W = F \cdot D = 100 \times 10 = 1,000 \text{ J}$

KE $= \frac{1}{2} m.v^2$ but 'v', is unknown, and so are 't' and 'a'.

So, use 1. $F = m.a$ or $a = \frac{F}{m} = \frac{100}{50} = 2 \text{ m/s}^2$

2. $d = \frac{1}{2} a.t^2$ or $t = \sqrt{\frac{2d}{a}} = \sqrt{\frac{2 \times 10}{2}} = \sqrt{10} \text{ sec}$

3. $v = a.t$ or $v = 2\sqrt{10} \text{ m/s}$

Thus, $KE = \frac{1}{2} m.v^2 = \frac{1}{2} \times 50 \times (2\sqrt{10})^2 = 1,000 \text{ J}$

Conclusion: The work done equals the change (increase) in Kinetic Energy.

Energy Loss “Negative Work”

- To **reduce the KE** of an object (e.g. a car), we also need to perform work, called “**Negative Work**”.
- In this case work is done by friction when braking. (Brake drums heat up or tires skid.)
- Negative work **reduces energy** of a system.

Example: Stopping distance for a car, etc...

Kinetic energy of a vehicle is proportional to v^2 - if speed is doubled, the KE is quadrupled!

i.e. A bus traveling at 80 km/hr has four times as much kinetic energy than one at 40 km/hr.

Doubling the speed requires four times as much “negative work” to stop it.

Result: Stopping distance is around **four times longer** for the 80 km/hr vehicle!!! (assuming a constant frictional force).

Summary: The **KE gained or lost** by an object is **equal to the work done by the net force**.

Potential Energy

Question: What happens to the work done when e.g. lifting a box onto a table?

If we lift it so that our **applied force** is **equal and opposite** to its **weight force**, there will be **no acceleration and no change** in its **Kinetic Energy**. ($F = m.g$)

Yet work is clearly done...

Answer: The work done increases the **gravitational potential energy** of the box.

- Potential energy is **stored energy** (for use later) e.g. a rock poised to fall...
- Potential energy involves **changing the position** of an object that is being **acted upon** by a **specific force** (e.g. gravity).
- **Gravitational potential energy (PE) equals work done to move object a vertical distance (h).**

$$PE = W = F \cdot d$$

$$PE = m.g.h \text{ (units = Joules)}$$

- Thus, the further we move an object away from the center of the Earth, the greater its potential energy.
- And, the larger the height change, the larger the change in PE.

Example: What is PE of a 50kg box lifted 10m?

$$PE = m.g.h = 50 \times 9.8 \times 10 = 4,900 \text{ J}$$

- If height is doubled, PE is doubled...
- Other kinds of PE involving other forces also exist (eg. springs).

Summary:

- Potential energy is stored energy associated with an object's position rather than its motion.
- The “system” is poised to release energy converting it to KE or work done on another system.
- Potential energy can result from work done against a variety of conservative forces eg. gravity and springs.