# Recap: Newton's Gravitational Law

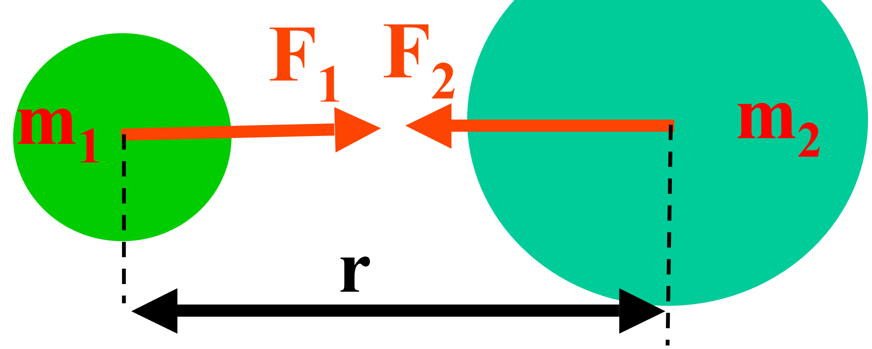
**The gravitational force between two objects is proportional to their masses and inversely proportional to the square of the distance between their centers.**

$$F = \frac{G\, m_1\, m_2}{r^2} \quad \text{(Newtons)}$$

- **F is an attractive force vector acting along line joining the two centers of masses.**

- **G = Universal Gravitational Constant**

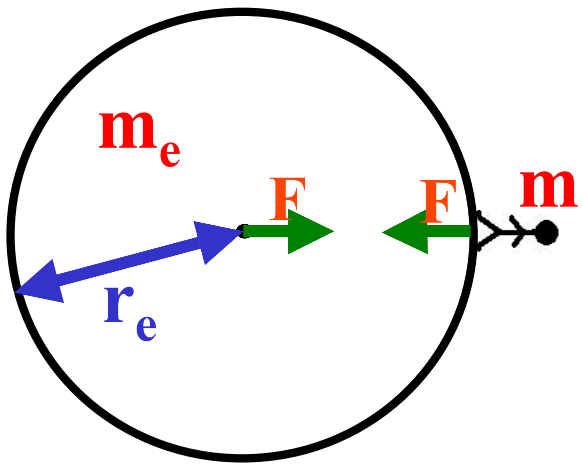
  **$G = 6.67 \times 10^{-11}$ N.m²/kg² (very small)**




**($F_1 = -F_2$)**

**Note: G was not measured until > 100 years after Newton! - by Henry Cavendish (18th cen.)**

# How is Weight Related to Gravitation?




$r_e$ = radius of Earth = 6370 km
m = mass of an object
$m_e$ = mass of Earth = $5.98 \times 10^{24}$ kg

**Gravitational force** of attraction:

$$F = \frac{G\, m_1\, m_2}{r^2} \quad \text{(N)}$$

if m = 150 kg, **F = 1472 N** (or ~ 330 lbs wt)

But this force creates the object's weight:

By Newton's 2nd law (F=ma) we can also calculate weight:

**W = m g** = 9.81 x 150 = **1472 N**

**By equating these expressions for gravitational force:**

$$m\, g = \frac{G\, m_e\, m}{r_e^{\,2}} \qquad \text{or at surface:} \quad g = \frac{G\, m_e}{r_e^{\,2}}$$

**Result: 'g' is independent of mass of object !!**

- Thus acceleration due to gravity 'g' is:
  1. **Constant for a given planet and depends on planets mass and radius.**
  2. **Independent of the mass of the accelerating object! (Galileo's discovery).**
- **However, the gravitational force 'F' is dependent on object mass.**
- **In general, the gravitational acceleration (g) of a planet of mass (M) and radius (R) is:**

$$g = \frac{G\,M}{R^2}$$

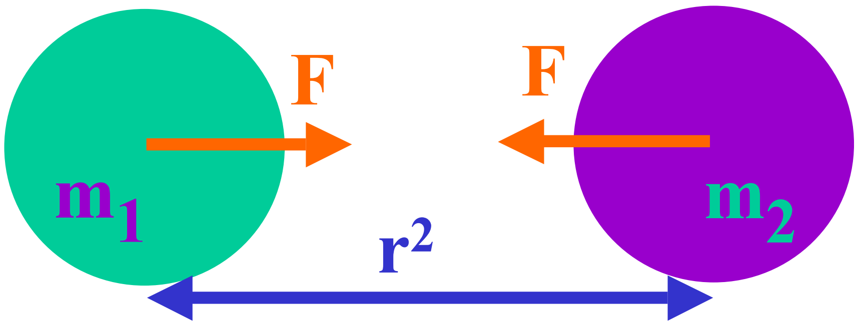
**This equation also shows that 'g' will decrease with altitude:**

e.g. At 100 km height $g = 9.53\ m/s^2$

At moon's orbit $g = 2.7 \times 10^{-3}\ m/s^2$

| Planet | 'g' $m/s^2$ |
|---|---|
| Mercury | 3.7 |
| Venus | 8.9 |
| Earth | 9.8 |
| Moon | 1.6 |
| Mars | 3.7 |
| Jupiter | 26 |
| Saturn | 12 |
| Uranus | 11 |
| Neptune | 12 |
| Pluto | 2 |

}no solid surface (Jupiter, Saturn, Uranus, Neptune)




- **Newton's 3rd law: Each body feels same force $F = \frac{G m_1 m_2}{r^2}$ acting on it (but in opposite directions)**
- **Thus each body experiences an acceleration!**

Example: Boy 40 kg jumps off a box:

Force on boy: $F = m\,g = 40 \times 9.81 = 392$ N

Force on Earth: $F = m_e\,a = 392$ N

or $a = \dfrac{392}{5.98 \times 10^{24}} = 6.56 \times 10^{-23}$ m/s$^2$ ie. almost zero!

**Example:** 3 billion people jumping off boxes all at same time (mass 100 kg each)

$$a = \frac{3 \times 10^9 \times 100 \times 9.81}{5.98 \times 10^{24}} = 5 \times 10^{-13} \text{ m/s}^2$$

**Conclusion: The Earth is so massive, we have essentially no effect on its motion!**

# Planetary Motions & Orbits (Chapter 5)

- **Heavenly bodies: sun, planets, stars…**How planets move? **Greeks:**
- Stars **remain in the same** relative position to one another as they move across the sky.
- Several bright "stars" **exhibit motion** relative to other stars.
- Bright "**wanderers**" called **planets**.
- **Planets roam** in regular but curious manner.

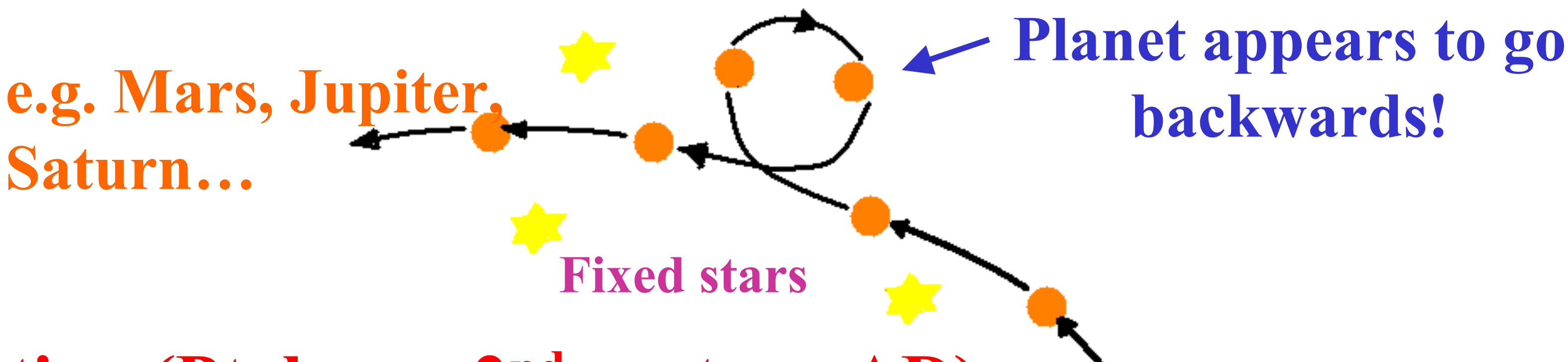
**Hypothesis:**

- **Geocentric "Earth-centered"** universe!
- **Sun** moves **around the Earth** - like on a long rope with Earth at its center.
- **Stars – lying on a giant sphere** with Earth at center.
- Moon too – exhibits phases as it orbits Earth.

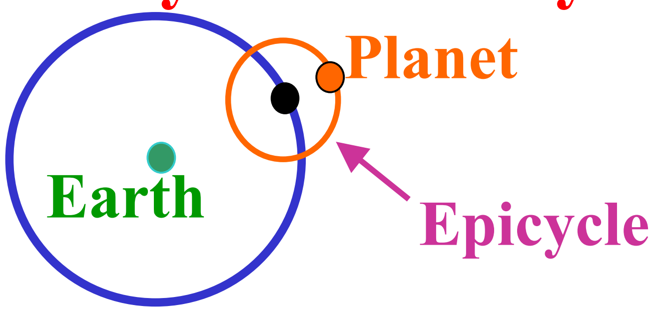
**Plato: Concentric spheres** – sun, moon and 5 known planets each move on a sphere centered on the Earth.

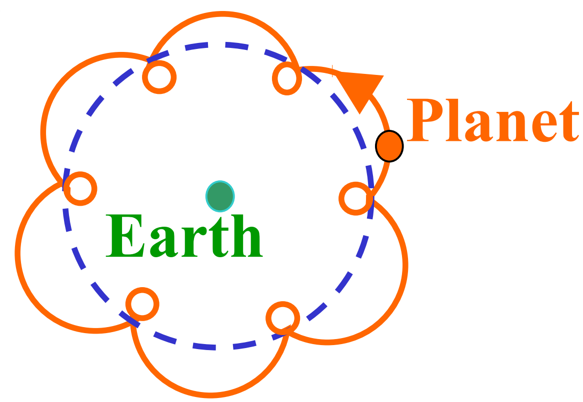
**Big problem** – Planets do not always behave as if moving continuously on a sphere's surface.

**Retrograde motion** (happens over several months)




**Solution (Ptolemy 2nd century AD)**



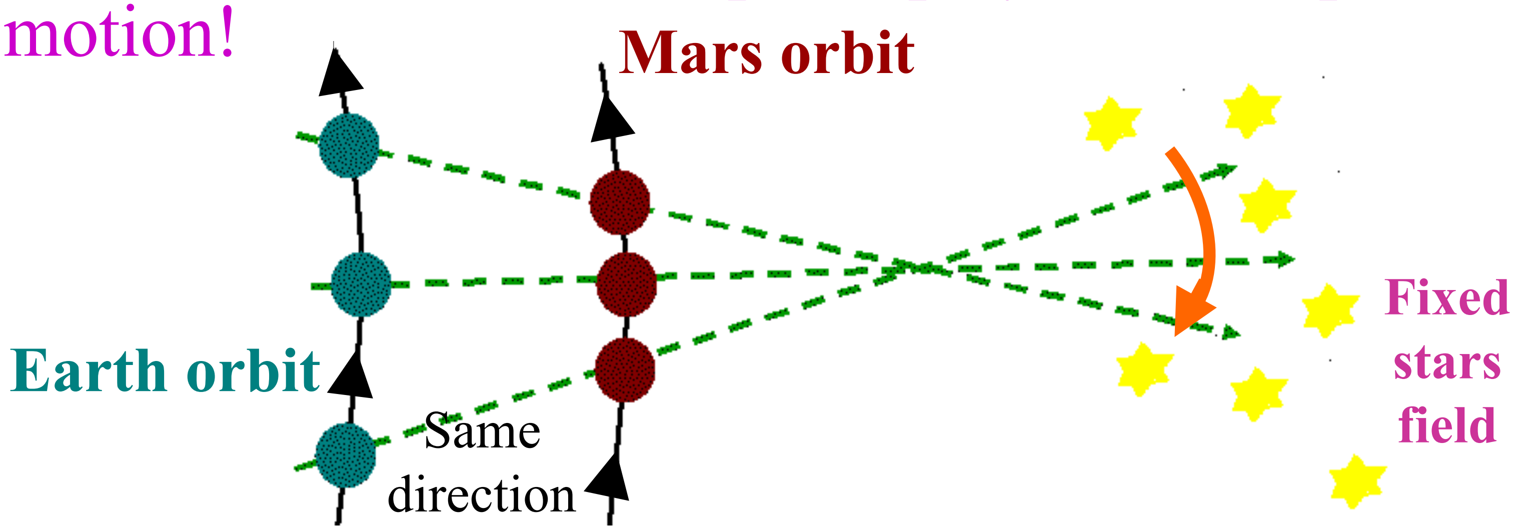




**Epicycles –circular orbits –not on spheres.**

**Planets moved in circles that rolled around larger orbits - still centered on Earth.**

# Heliocentric Model: Copernicus (16th century)

- **Sun centered** view that was later proven by **Galileo** – using telescope observations of Jupiter and its **satellite moons**.
- **Bad news**: **demoted Earth** to status of just another planet!
- **Revolutionary concept** – required the **Earth** to **spin** (to explain Sun's motion).
- If Earth spinning why are we not thrown off ? (at 1000 mph).
- **Good news**: no more need for complex epicycles to explain retrograde motion!

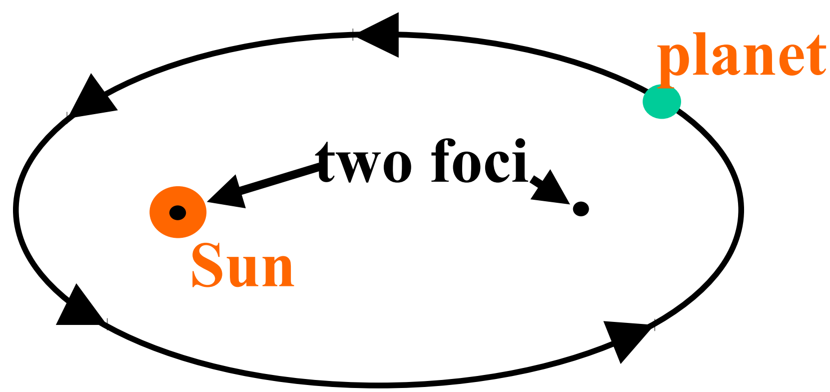



**Result: Earth moves faster in orbit and Mars <u>appears</u> to move backwards at certain times.**

- Copernicus heliocentric model assumed circular orbits – but careful observations by **Tycho Brahe** (the last great "naked eye" astronomer) showed not true…
- **Kepler** (17th century, Brahe's student) developed three laws based on emperical analysis of Brahe's extensive data…

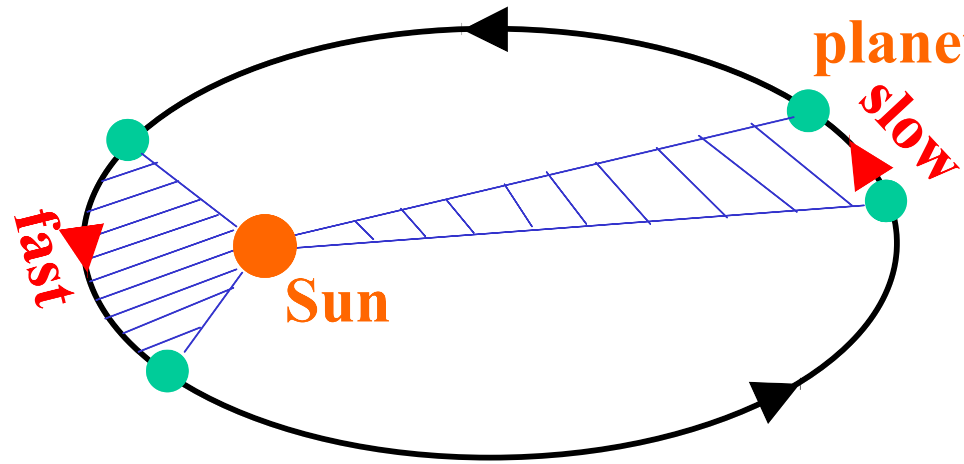
**1. Orbits of planets around the sun are ellipses with Sun at one focus.**

Note: A circle is a special case of an ellipse with 2 foci coincident.




In reality, the **planets' orbits are very close to circular** but nevertheless are slightly elliptical.

2nd law: Describes how a planet moves faster when nearer the Sun.




**2. A radius vector from Sun to planet sweeps out equal areas in equal times.**

3rd law: Lots of numbers later (trial and error) Kepler discovered that to a very high approximation, the period of orbit is related to average radius of orbit:

3. $$\frac{T^2}{r^3} = \text{constant}$$

**Same value** for constant for all planets (except the Moon)

- This means that the **"outer" planets** (i.e. further from the Sun than Earth) all have **much larger orbital periods** than Earth (and vice versa).

i.e. $\mathbf{T^2 \propto r^3}$

- So if we know T (by observations) we can **find "r" for each planet!**

Conclusion:

- These careful observations and new formula set the scene for Newton's theory of gravitation…

$$\mathbf{F = \frac{G\, m_1\, m_2}{r^2}}$$

- Using **Kepler's 3rd law**, Newton calculated:

$$\frac{\hat{O}^2}{r^3} = \frac{4\,D^2}{G\,m} = \text{constant (for a given 'm')}$$

where: **m = mass of Sun** for the planetary motions, but
**m = mass of Earth** for the Moon's motion.

- Hence Kepler's **different result** for the constant

$$\frac{T^2}{r^3} = \text{a constant number}$$

for moon compared with other planets!

# Artificial Satellites

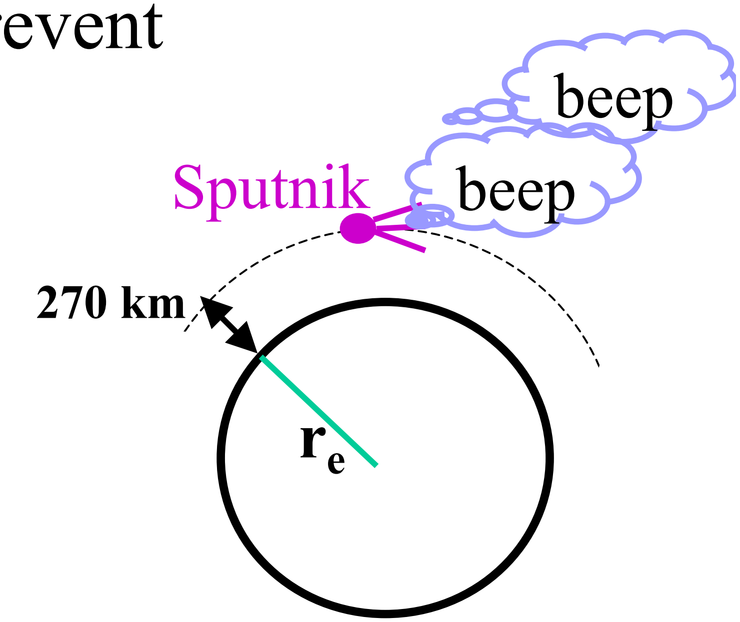
- Each of the **planets** will have its own value of $\frac{T^2}{r^3}$ for its satellites (as constant has a 1/m dependence).
- But in each case as $T^2 \propto r^3$ the **lower** the altitude of the satellite the **shorter** its orbital period.
- **Example**: 1st Earth satellite "Sputnik" (1957)

Launched into a very low altitude orbit of ~ 270 km (any lower and atmospheric drag would prevent orbital motion).

r = 6370+270=6640 km

=> T = 90 min

(1.5 hrs to orbit Earth)



# Example: Geosynchronous Orbit

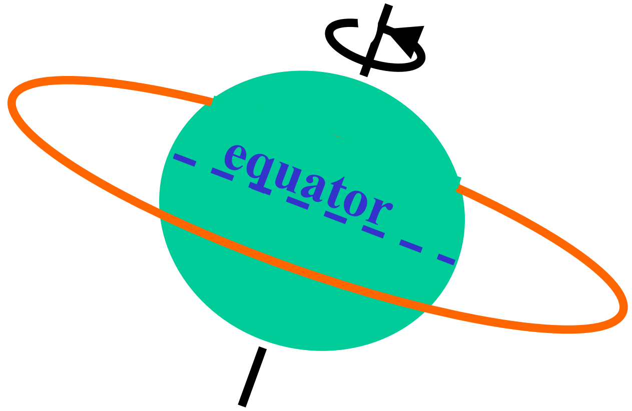
- Orbital period = 24 hours
- Permits satellites to remain "stationary" over a given equatorial longitude.

  For T = 24 hrs

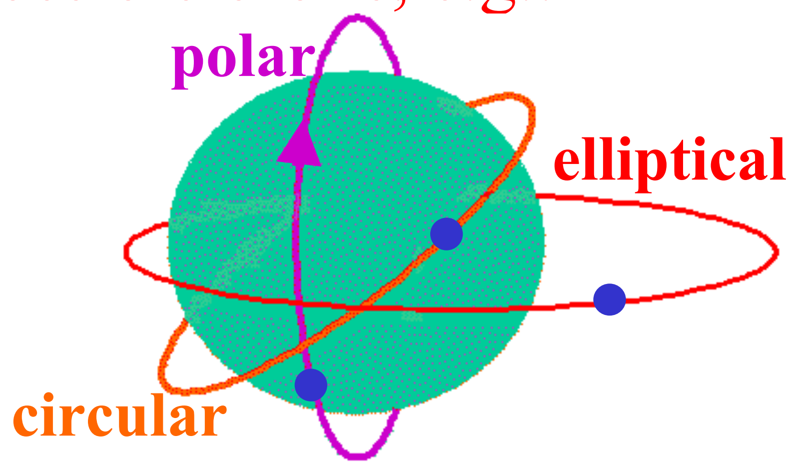
  => r = 42,000 km (to center Earth)

  i.e. altitude ≈7 $R_e$

  (compared with 60 $R_e$ for Moon.)




- Geostationary orbit is very important for Earth observing and communications satellites – a very **busy** orbit!
- In general there are many, many possible orbits, e.g.:
  - –Circular and elliptical
  - –Low Earth orbits (LEO)
  - –Geostationary orbit
  - –Polar orbit
  - –e.g. GPS system uses many orbiting satellites.



# Orbital Velocity

- We can equate centripetal force to gravitational attraction force to determine orbital speed ($v_{or}$).

**For circular motion:**

**Centripetal force = gravitational force ($F_C = F_G$)**

$$\frac{m\,v_{or}^2}{r} = \frac{G\,M\,m}{r^2}$$

**M = planet's mass**
**m = satellite's mass**
**M » m**

$$v_{or} = \sqrt{\frac{GM}{r}}$$

**Results:**

- Any satellite regardless of its mass (provided M » m) will move in a circular orbit or radius r and velocity $v_{or}$.
- The larger the orbital altitude, the lower the required tangential velocity!

## Example: Low Earth orbit: altitude 600 km)

$$v_{or} = \sqrt{\frac{G\,M}{r}} = \sqrt{\frac{\left(6.67\times10^{-11}\right)\times\left(5.98\times10^{-24}\right)}{6970\times10^{3}}}$$

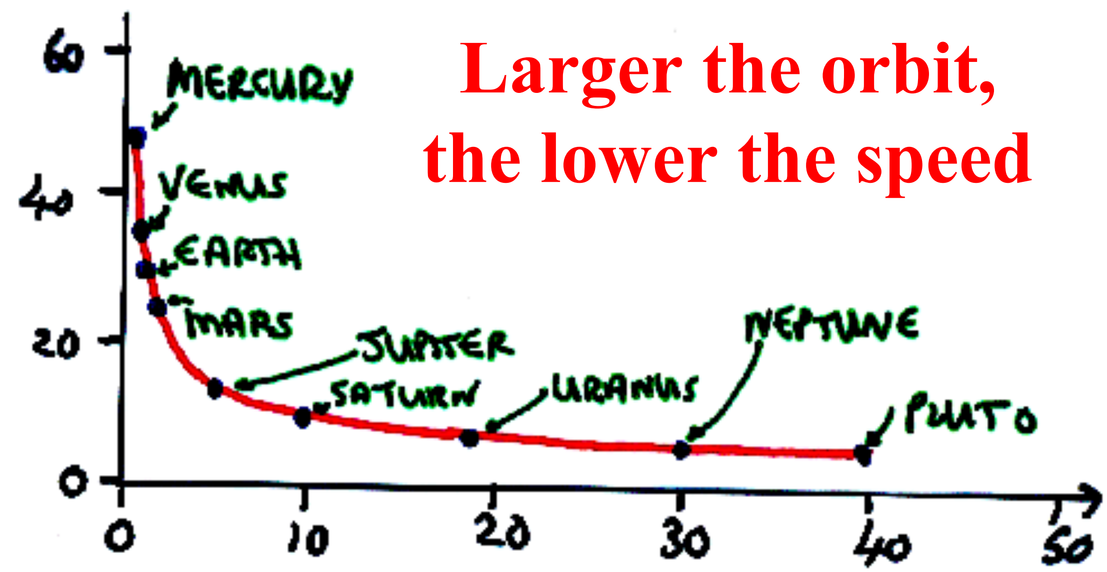
$$= 7.6 \text{ km/s} \qquad (\text{or } 27\times10^{3} \text{ km/hr})$$

## Planetary Orbital Velocity:

$$v_{or} \propto \sqrt{\frac{\mathbf{M}}{\mathbf{r}}}$$

M = mass of the Sun

km/s



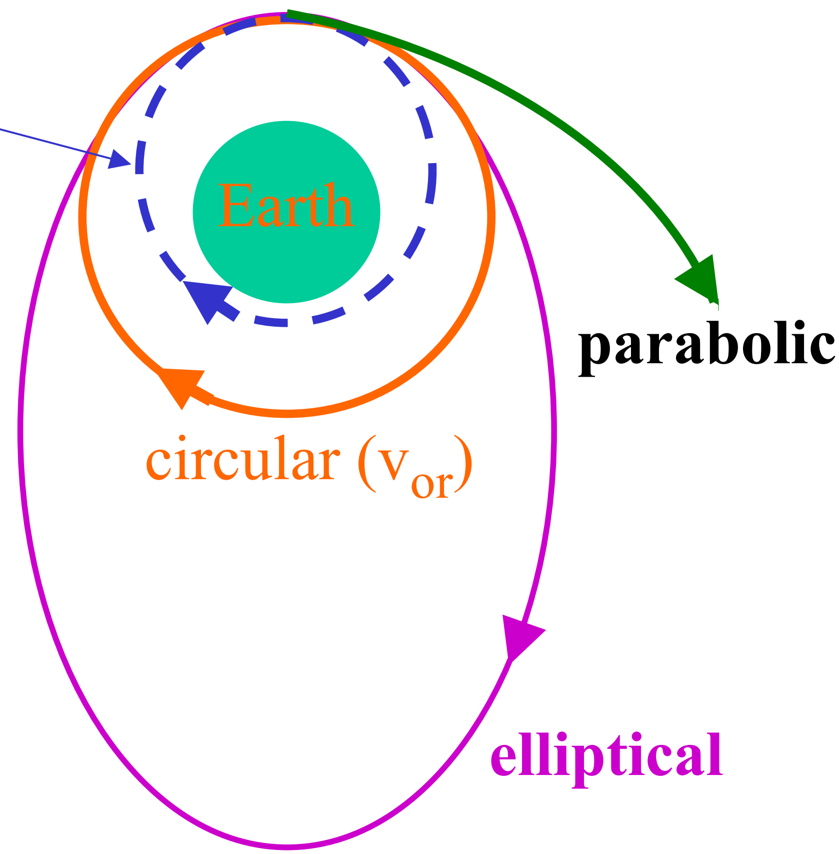

Mean Distance (Astronomical Units)

**Larger the orbit, the lower the speed**

# Qu: How to achieve orbit?

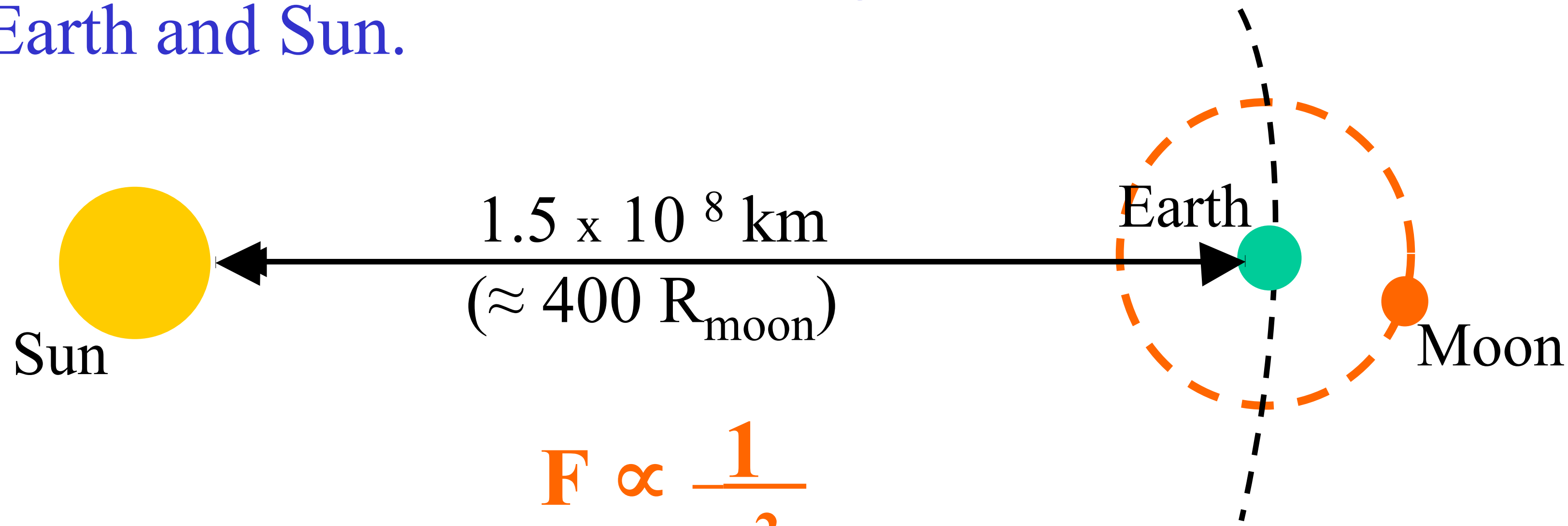
- Launch vehicle rises initially vertical (minimum air drag).
- Gradually rolls over and on separation of payload is moving tangentially at speed $v_{or}$ produces circular orbit.

- If speed less than $v_{or}$, craft will descend to Earth in an (decaying) elliptical orbits.
- If speed greater than $v_{or}$ it will ascend into a large elliptical orbit.
- If speed greater than $\sqrt{2v_{or}}$ it will escape earths gravity on parabolic orbit!



# Lunar Orbit

- Not a simple ellipse… due to gravitational force of Earth and Sun.



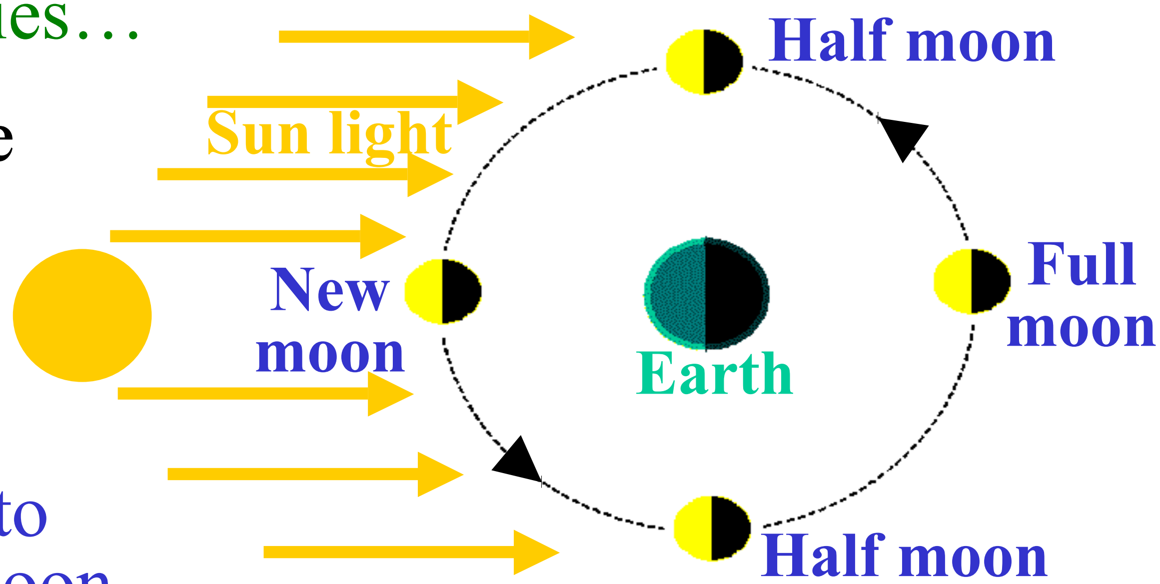

$$F \propto \frac{1}{r^2}$$

- Gravitational attraction between Earth and Moon provides **centripetal acceleration** for orbit.

- **Sun's** gravitation **distorts** lunar **orbital ellipse**. (Orbit oscillates about true elliptical path.)

# Moon

- The only satellite Newton could study and played a key role in his discoveries…
- Phases known since prehistoric times…
- Moonlight is reflected sunlight.
- Phases simply due to geometry of sun, moon and Earth.




Phases repeat every 27.3 days (lunar orbit).

**Full moon:**

- Moon on opposite side of Earth from Sun.
- Fully illuminated disk.
- Rises at sunset and sets at sunrise (due to Earth’s 24 hr rotation).
- Lunar eclipse only during full moon.

- At other times during moon's orbit of Earth, we see only a part of illuminated disk.

**New moon:**

- Moon on same side of Earth as the Sun.
- Essentially invisible (crescent moons seen either side of New moon).
- Rises at sunrise and sets at sunset (i.e. up all day).
- Solar eclipse condition.

- When moon is in-between full and new phase, it can often be **seen during daylight** too.
- Example: half moon rises at noon and sets at midnight (and vice versa).
- Under good observing conditions (at sunset or sunrise) you can see dark parts of moon illuminated by Earthshine!