Recap

Centripetal acceleration:

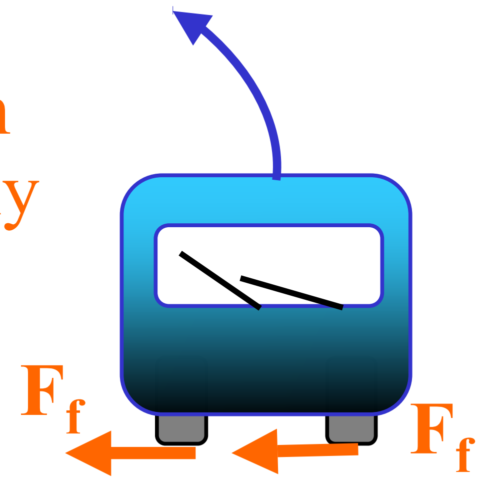
$$a_c = \frac{v^2}{r} \quad m/s^2 \quad \text{(towards center of curvature)}$$

- **A centripetal force F_c is required to keep a body in circular motion:**
- **This force produces centripetal acceleration that continuously changes the body's velocity vector.**

$$F_c = m a_c = \frac{m v^2}{r} \quad \text{(Newtons)}$$

- **Thus for a given mass the needed force:**
  - **increases with velocity ^{2}**
  - **increases as radius reduces.**

Example: The centripetal force needed for a car to round a bend is provided by **friction**.



- If **total** (static) frictional force is **greater** than required centripetal force, car will successfully round the bend.

- The higher the velocity and the sharper the bend, the more friction needed!

$$F_s > \frac{mv^2}{r}$$

- As $F_s = \mu_s N$ - the friction depends on surface type (μ_s).

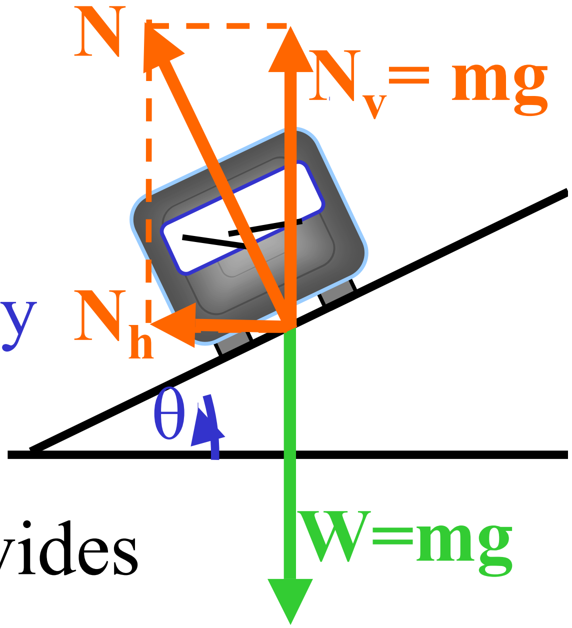
- Eg. If you hit ice, μ becomes small and you fail to go around the bend.
- Note: If you start to skid (locked brakes) μ_s changes to its kinetic value (which is lower) and the skid gets worse!

Moral: Don't speed around tight bends! (especially in winter)

Motion on a Banked Curve

- **The normal force N depends on weight of the car W and angle of the bank θ.**

- **There is a horizontal component (N_h) acting towards center of curvature.**



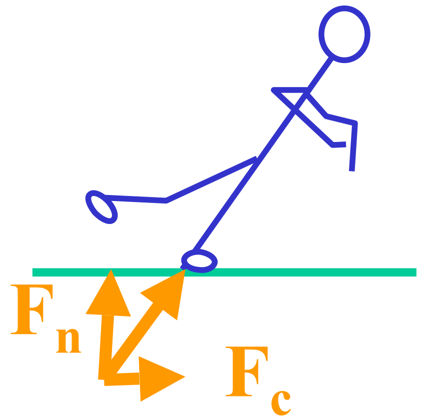
- This extra centripetal force can significantly reduce amount of friction needed…

- If $\tan\theta = \frac{v^2}{rg}$ then the horizontal N_h provides **all** the centripetal force needed!

- **In this case no friction is necessary and you can safely round even an icy bend at speed…**

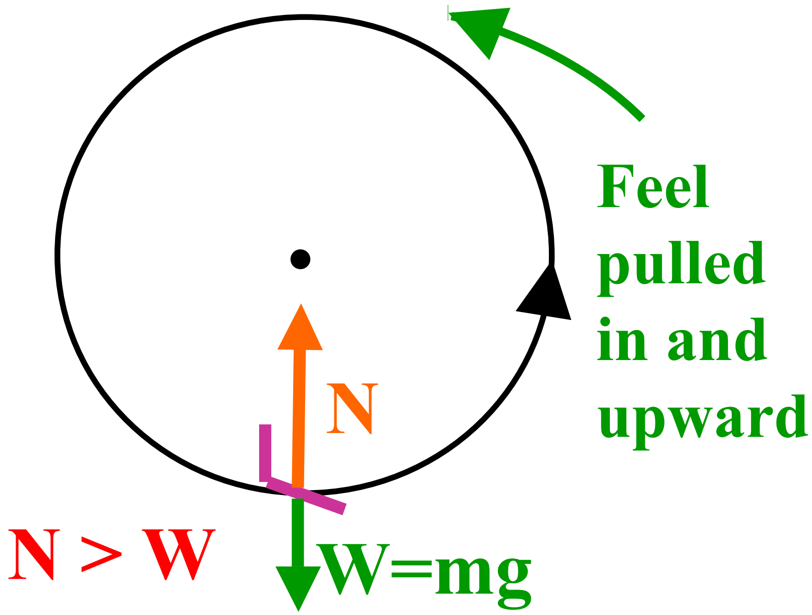
Ice Skating and Bike Racing

- Ice skaters can’t tilt ice so they lean over to get a helping component of **reaction force** to round sharp bends.

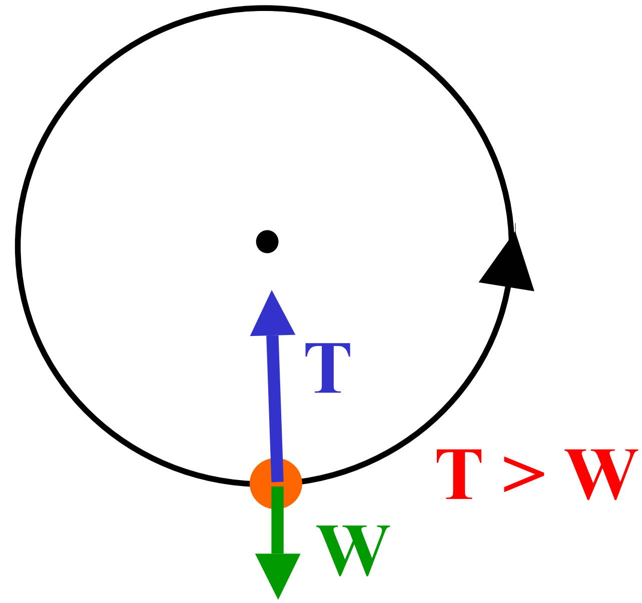


Vertical Circular Motion

Ferris Wheel



Ball on String



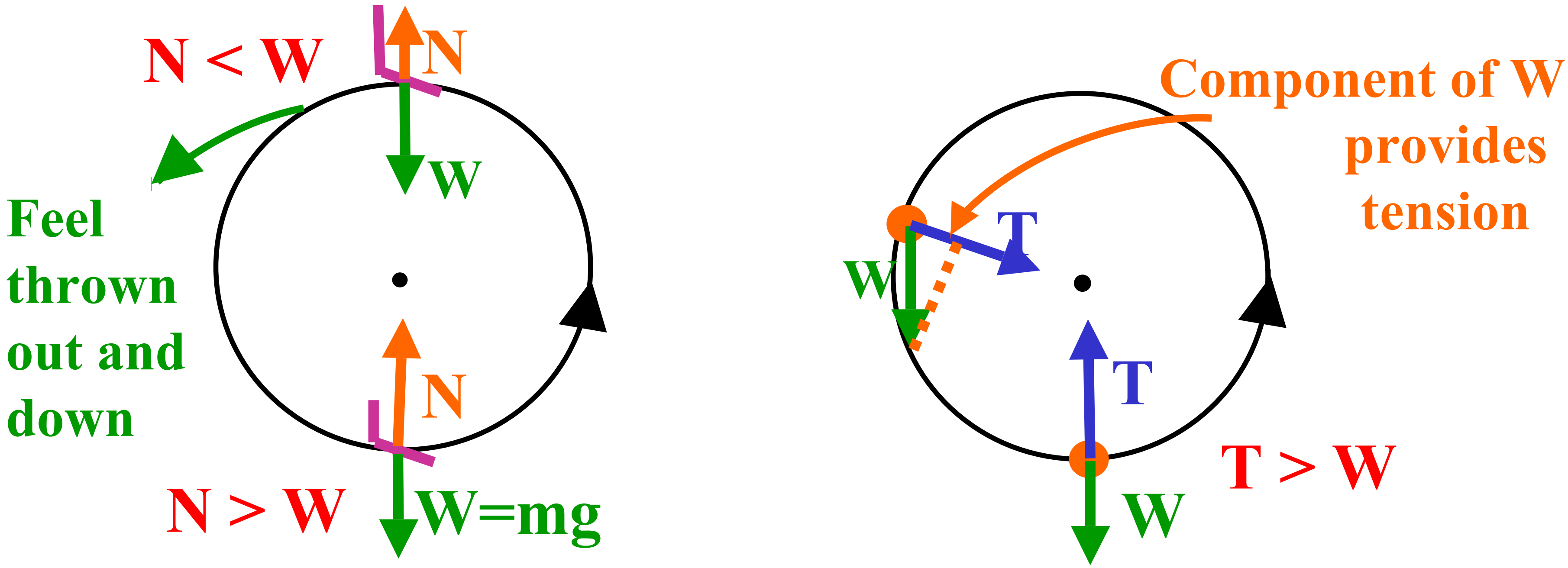
Bottom of circle:

- **Centripetal acceleration is directed upwards.**
- Net force is thus directed upwards:

$\mathbf{F_{net} = N - W = ma_c}$ N=apparent weight (like in elevator)

Thus: $\mathbf{N = W + ma_c}$ **i.e. feel heavier/larger tension**

Top of the Circle



Weight is only force for centripetal acceleration downward

$N = W - m\,a_c$ i.e. **lighter / less tension**

Special condition if $W = m\,a_c \rightarrow$ feel weightless (T=0)

then $a_c = g = \frac{v^2}{r}$ or $v = \sqrt{r\,g}$ **(larger r, higher v)**

Newton's Law of Universal Gravitation

Questions:

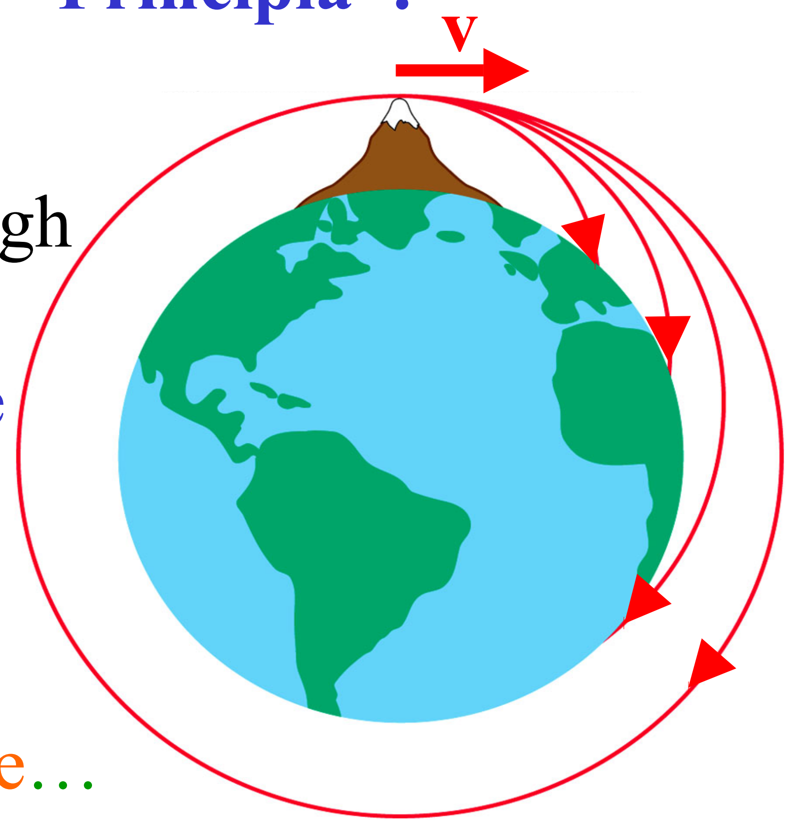
- What role does **centripetal acceleration** play in the motions of heavenly bodies?
- What **forces** are acting to cause their motion?
- We know the planets are moving in **curved paths** (orbits) around the Sun.
- What force is **ever present** to cause the necessary centripetal acceleration?

Answer: It must be **gravity** … but how?

❖ **Newton's earth shattering breakthrough!**

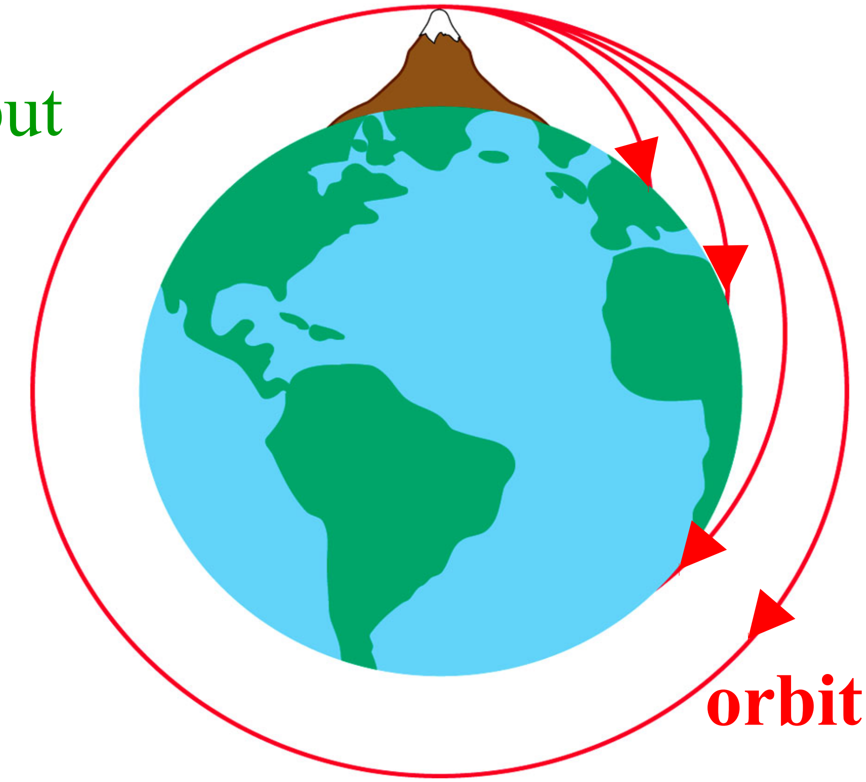
- Newton realized that the motion of a projectile launched near the Earth's surface and the moon's orbit around the Earth are similar!
- He realized that the moon is also under the influence of gravity and is actually continuously falling towards Earth.

❖ **Famous sketch from Newton's "Principia":**



- Imagine a projectile launched horizontally from an incredibly high mountain. (Olympus Mons)
- The larger the initial velocity, the further it will travel.
- At very high velocities, the curvature of the Earth becomes important in determining the range…

- In fact, if velocity is high enough it will never land…
- It will keep falling (free-fall), but the Earth's surface (curvature) keeps dropping away at the same rate!



- **Circular orbit around the Earth---Wow!**

- So the same force that controls the motion of objects near Earth's surface (as described by $d = ½\, a.t^2$, and $v = a\, t$) also acts to keep the moon in orbit!

Question: What is the nature of this force?

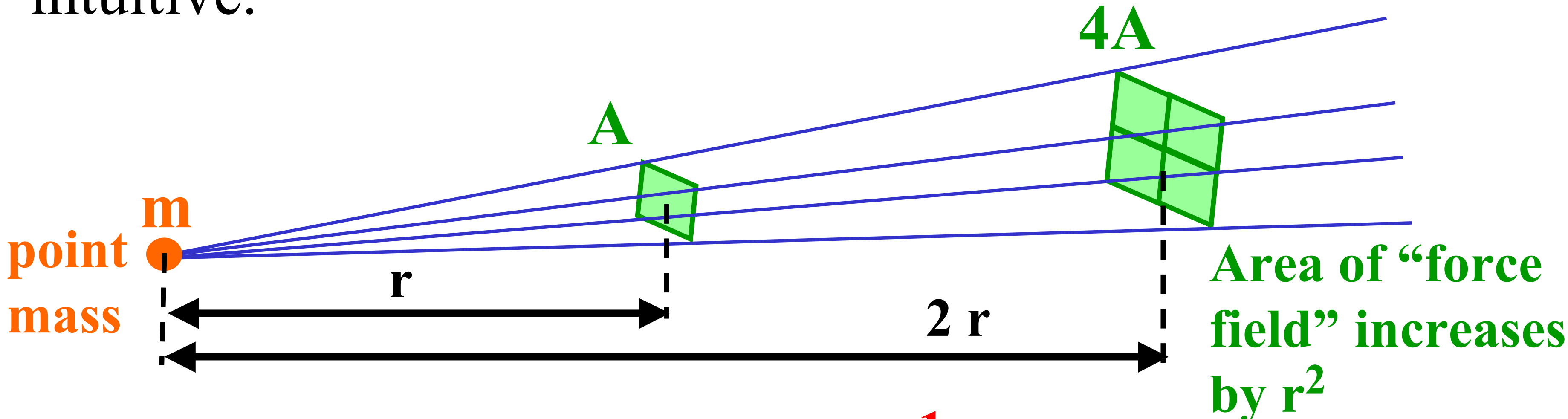
Nature of Universal Gravitational Law

- Newton's 2nd law applied to free-falling object:

$$F = m\,g \quad \text{(weight force)}$$

❖ Thus: mass is key to the general description of gravity – intuitive.

- But how does gravitational force vary with distance?
- Expect force to decrease in strength as distance increases – intuitive.



Actual: Force $\propto \frac{1}{r^2}$

- Many forces in nature exhibit a $\frac{1}{r^2}$ relationship…

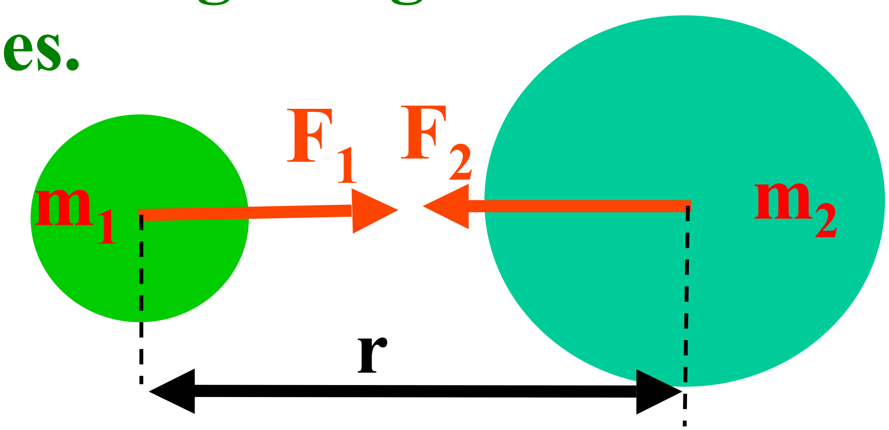
Newton's Gravitational Law

The gravitational force between two objects is proportional to their masses and inversely proportional to the square of the distance between their centers.

$$F = \frac{G\, m_1\, m_2}{r^2} \quad \text{(Newtons)}$$

- **F is an attractive force vector acting along line joining the two centers of masses.**
- **G = Universal Gravitational Constant**

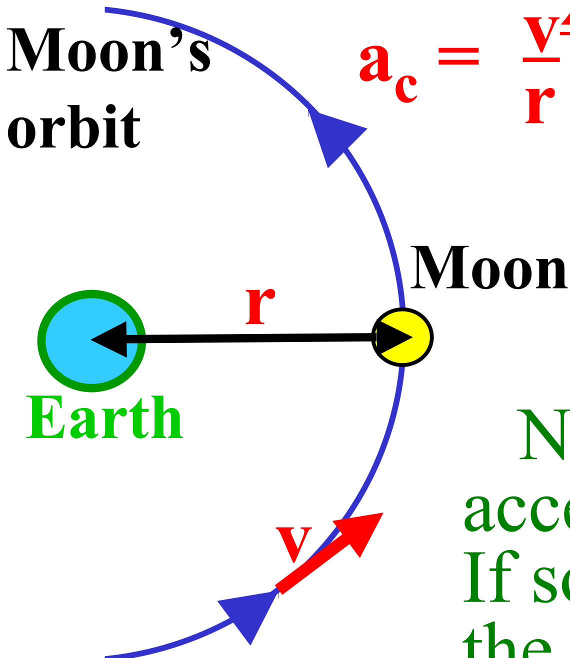
$G = 6.67 \times 10^{-11}\ N.m^2/kg^2$
(very small)



($F_1 = -F_2$)

Note: G was not measured until > 100 years after Newton! - by Henry Cavendish (18th cen.)

- Newton proved this $1/r^2$ dependence using Kepler's laws (next lecture) and he applied his knowledge of centripetal acceleration and his ideas on gravity to the moon…
- Centripetal acceleration of moon for circular motion:



$$a_c = \frac{v^2}{r}$$

r = 60.3 Earth radii, or 3.84×10^8 m

(i.e. ≈ 0.4 million km)

v = 1.02 km/s

$a_c = 0.0027\ \text{m/s}^2 = 2.7 \times 10^{-3}\ \text{m/s}^2$

Newton argued that Earth's gravitational acceleration (i.e. force) decreases with $1/r^2$ If so, then the acceleration due to gravity at the moon's orbital distance ($g_{\bullet}$) is:

$$g_{\bullet} = \frac{9.81}{r^2} \approx \frac{9.81}{60^2} = 2.7 \times 10^{-3}\ \text{m/s}^2$$

❖ **Moon's centripetal acceleration is provided by Earth's gravitational acceleration at lunar orbit.**

Gravitational Attractive Force

- **As 'G' is very small the gravitational attraction between the two every-day objects is extremely small.**
- Example: 2 people of mass 150 kg and 200 kg separated by 0.1 m

$$F = \frac{G\, m_1 m_2}{r^2} = \frac{6.67 \times 10^{-11} \times 150 \times 100}{0.1 \times 0.1} \text{ Newtons}$$

$$= 2 \times 10^{-4}\ \text{N} \quad (\text{i.e. } 0.0002\ \text{N})$$

However, as masses of planets and in particular stars and even galaxies are **HUGE**, then the gravitational attraction can also be enormous!

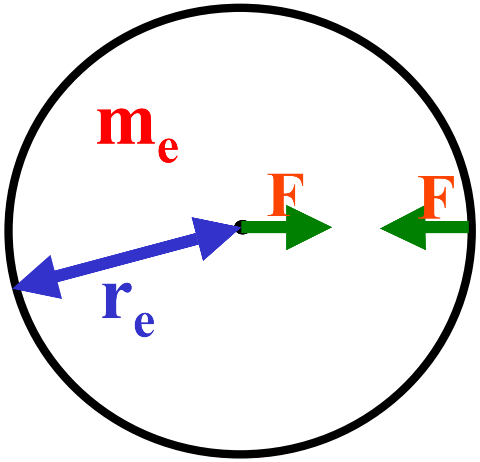
Example: Force of attraction between Earth and Moon.

mass of Earth = 5.98×10^{24} kg
mass of Moon = 7.35×10^{22} kg
r = 384×10^{3} km

$F \approx 2 \times 10^{20}$ N !

(i.e. 200,000,000,000,000,000,000 N)

How is Weight Related to Gravitation?



r_e = radius of Earth = 6370 km

m = mass of an object

m_e= mass of Earth = 5.98×10^{24} kg

Gravitational force of attraction:

$$F = \frac{G\, m_1\, m_2}{r^2} \quad \text{(Newtons)}$$

if m = 150 kg, **F = 1472 N** (or ~ 330 lbs wt)

But this force creates the object's weight:

By Newton's 2nd law (F=ma) we can also calculate weight:

W = m g = 9.81 x 150 = **1472 N**

By equating these expressions for gravitational force:

$$m\, g = \frac{G\, m_e\, m}{r_e^2} \qquad \text{or at surface:} \quad g = \frac{G\, m_e}{r_e^2}$$

Result: 'g' is independent of mass of object !!

Acceleration due to gravity 'g' is:

1. Constant for a given planet and depends on planets mass and radius.
2. Independent of the mass of the accelerating object! (Galileo's discovery).

- However, the gravitational force 'F' is dependent on object mass.
- In general, the gravitational acceleration (g) of a planet of mass (M) and radius (R) is:

$$g = \frac{G\,M}{R^2}$$

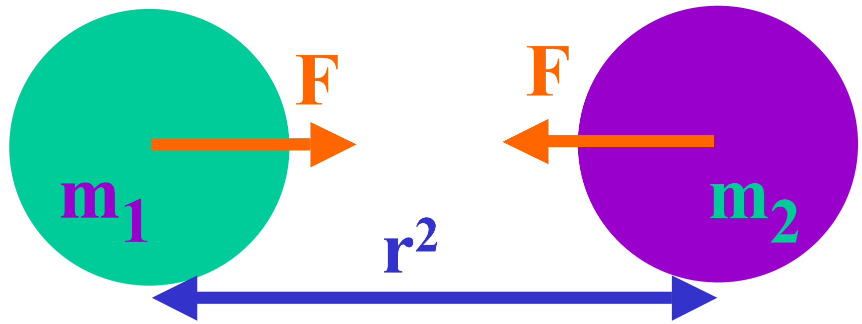
This equation also shows that 'g' will decrease with altitude:

e.g. At 100 km height $g = 9.53\ m/s^2$

At moon's orbit $g = 2.7 \times 10^{-3}\ m/s^2$

Planet	'g' m/s^2
Mercury	3.7
Venus	8.9
Earth	9.8
Moon	1.6
Mars	3.7
Jupiter	26
Saturn	12
Uranus	11
Neptune	12
Pluto	2

Jupiter, Saturn, Uranus, Neptune: no solid surface



- **Newton's 3rd law: Each body feels same force $F = \frac{G m_1 m_2}{r^2}$ acting on it (but in opposite directions)**
- **Thus each body experiences an acceleration!**

Example: Boy 40 kg jumps off a box:

Force on boy: $F = m\,g = 40 \times 9.81 = 392$ N

Force on Earth: $F = m_e\,a = 392$ N

or $a = \frac{392}{5.98 \times 10^{24}} = 6.56 \times 10^{-23}$ m/s^2 ie. almost zero!

Example: 3 billion people jumping off boxes all at same time (mass 100 kg each)

$$a = \frac{3 \times 10^{9} \times 100 \times 9.81}{5.98 \times 10^{24}} = 5 \times 10^{-13} \text{ m/s}^2$$

Conclusion: The Earth is so massive, we have essentially no effect on its motion!