Circular Motion (Chapter 5)

So far we have focused on **linear motion** or motion **under gravity** (free-fall).

Question: What happens when a ball is twirled around on a string at **constant speed**?

Ans: Its **velocity continuously changes in direction**.

This implies:

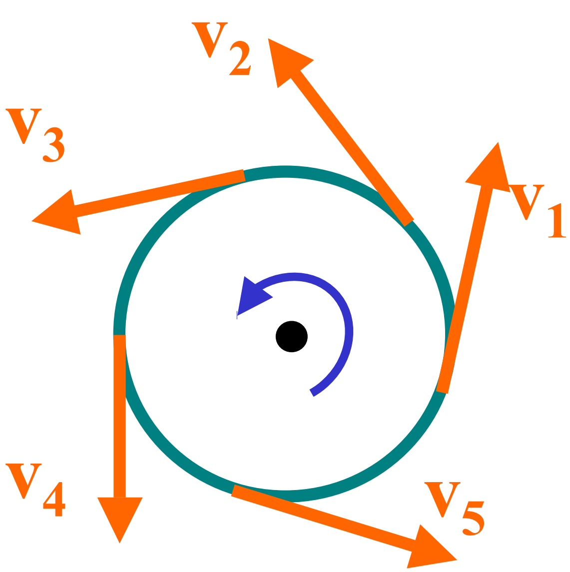
- The velocity change is caused by an **acceleration**.
- By Newton's 2^{nd} law an acceleration requires a **force**!

Big questions:

- What is the **nature** of this force / acceleration?
- What is the relationship between the **acceleration** and the **velocity** of the ball and the **radius** of curvature?

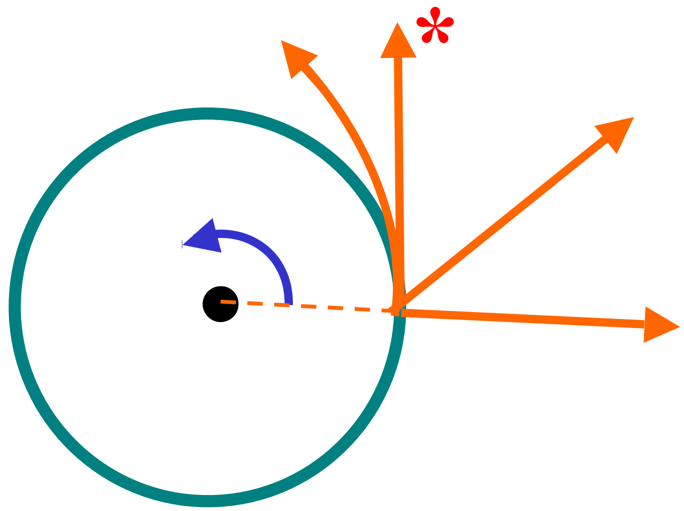
Ball on a String

- In the absence of gravity, **tension** provides the **only force** action on the ball.
- This **tension** causes ball to **change direction of velocity**.



Instantaneous velocity vector changing in direction but its magnitude stays constant.

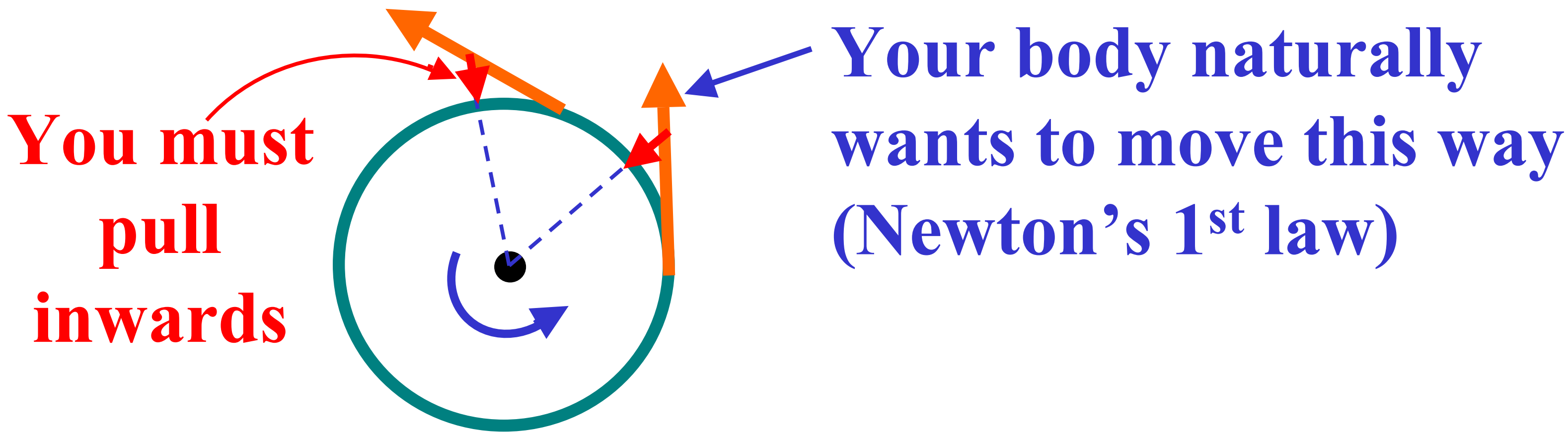
Question: What happens if you let go of the string?



Answer: Ball travels in direction of last **instantaneous vector**. (Newton's 1st law)

Let's imagine you are on a kid's "roundabout"…

Question: Why do we feel an outward force if it's not really there?



- However, to keep you in circular motion you must apply a force inwards to change your direction.
- Your pulling inwards creates the sensation that the roundabout is pushing you outwards!

Centripetal Acceleration

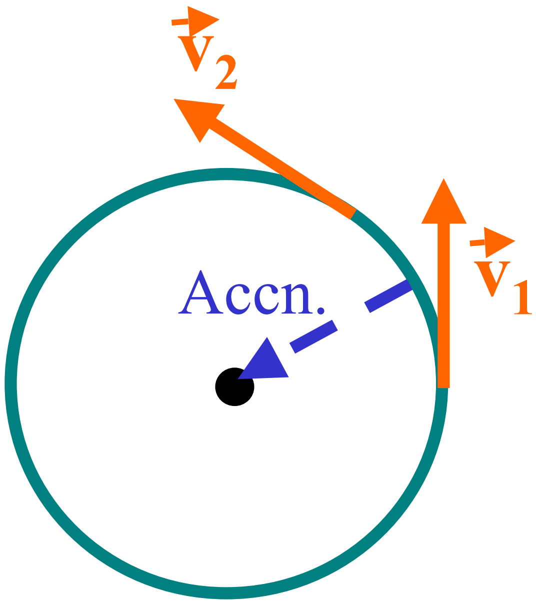
- The force (tension) causes an acceleration that is **directed inwards** towards center of curvature.
- ie. The string is continuously **pulling** on the ball towards the center of curvature causing its **velocity** to constantly **change**.
- This is called **centripetal acceleration (a_c):**

❖ **Centripetal acceleration is the rate of change in velocity of an object due to a change in its direction only.**

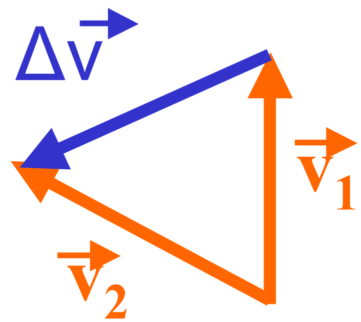
❖ **It is always perpendicular to the velocity vector and directed towards the center of curvature.**

- There is **NO** such thing as **centrifugal** (ie outward) force.

Nature of 'a_c'



$|v_1| = |v_2|$ = same speed



$$\vec{v}_1 + \Delta\vec{v} = \vec{v}_2$$

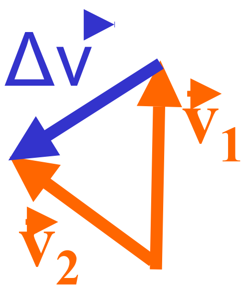
$$a_c = \frac{\Delta\vec{v}}{t}$$

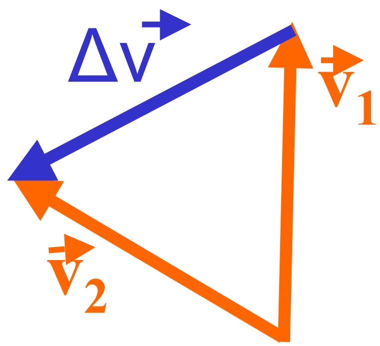
Acceleration is in direction of $\Delta\vec{v}$

(i.e. towards the center)

Dependencies of a_c

1. As speed of object increases the magnitude of velocity vectors increases which makes $\Delta\vec{v}$ larger.

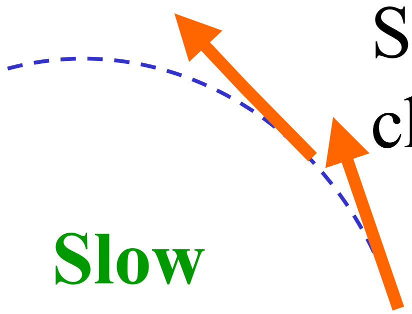




Therefore the acceleration $\left(a_c = \frac{\text{Ä}\vec{v}}{t}\right)$ **increases**.

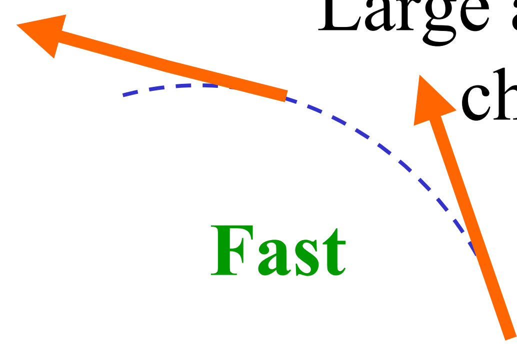
Dependencies of a_c

2. But the greater the speed the more rapidly the direction of velocity vector changes:



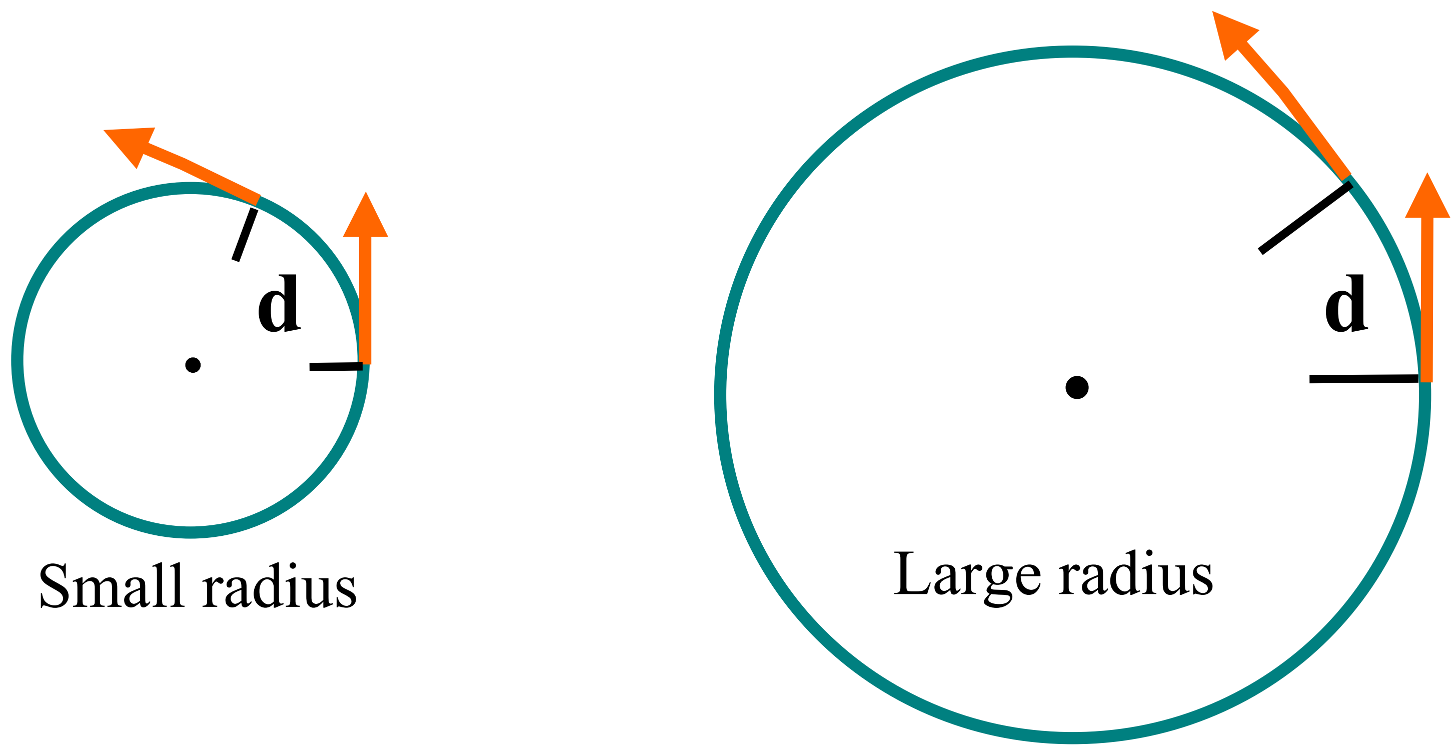
$$\frac{\Delta \vec{v}}{t}$$

increases



Dependencies of a_c

3. As radius decreases the rate of change of velocity increases – as vector direction changes more rapidly.



Same distance (d) moved but larger angle change.

Result: $\frac{\Delta \vec{v}}{t}$ **increases** as radius decreases.

Summary

- Points 1 and 2 indicate that the rate of change of velocity will increase with speed.
- Both points are independent of each other and hence a_c will depend on $(\text{speed})^2$.
- Point 3 shows that a_c is inversely proportional to radius of curvature (i.e. $a_c \propto 1/r$).

Thus: $$a_c = \frac{v^2}{r}$$ m/s^2 (towards center of curvature)

i.e. Centripetal acceleration increases with square of the velocity and decreases with increasing radius.

Example: Ball on a string rotating with a velocity of 2 m/s, mass 0.1 kg, radius = 0.5 m.

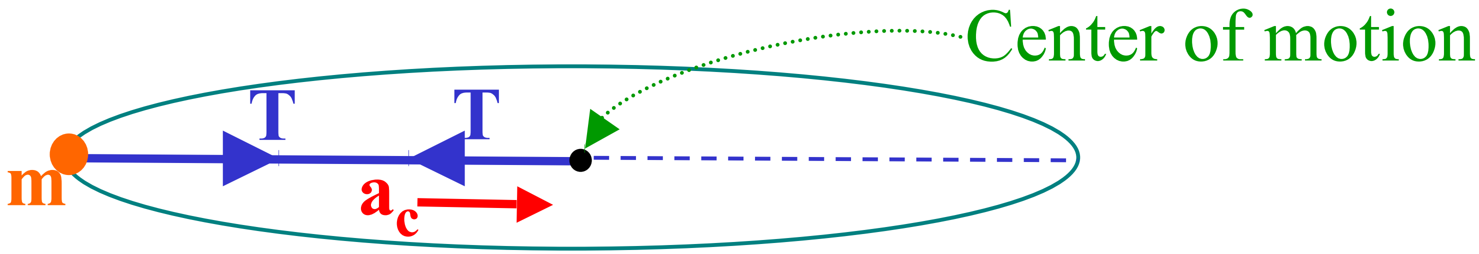
$$a_c = \frac{v^2}{r} = \frac{2 \times 2}{0.5} = 8\ m/s^2$$

What forces can produce this acceleration?

- Tension
- Friction
- Gravitation attraction (planetary motion).
- Nuclear forces
- Electromagnetic forces
- ?

Let's consider the ball on a string again…

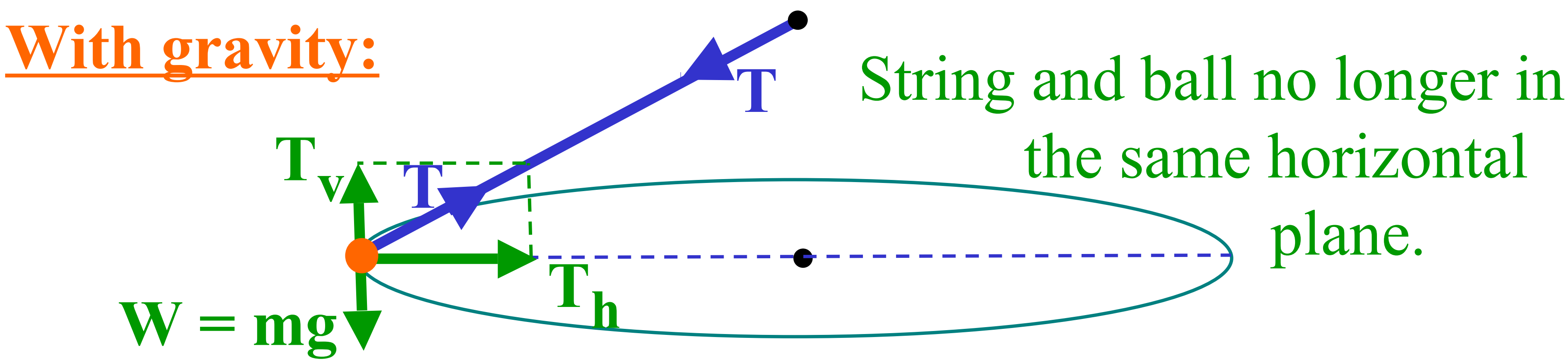
If no gravity:



- Ball rotates in a horizontal plane.

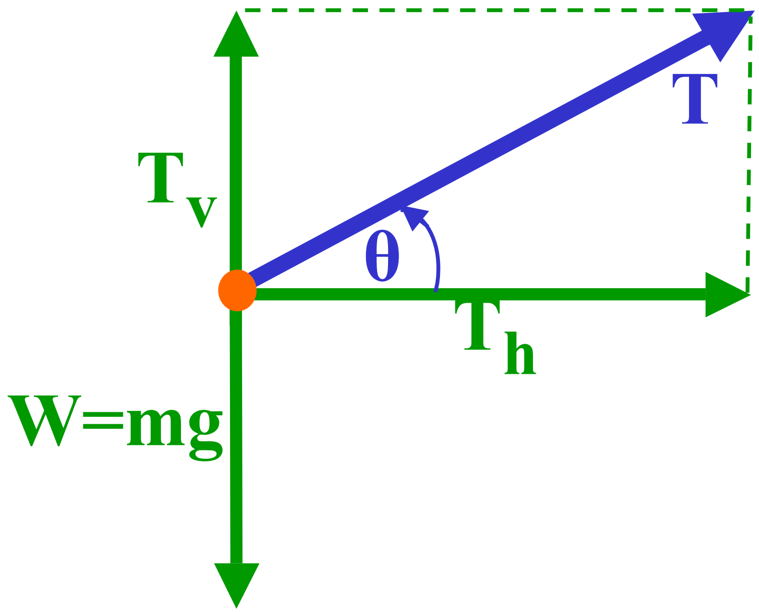
$$T = m\,a_c = \frac{m\,v^2}{r}$$

Let's consider the ball on a string again…



- The **horizontal component** of tension (T_h) provides the necessary **centripetal force**. ($T_h = ma_c$)
- The **vertical component** (T_v) balances the downward **weight** force ($T_v = mg$).

Stable Rotating Condition



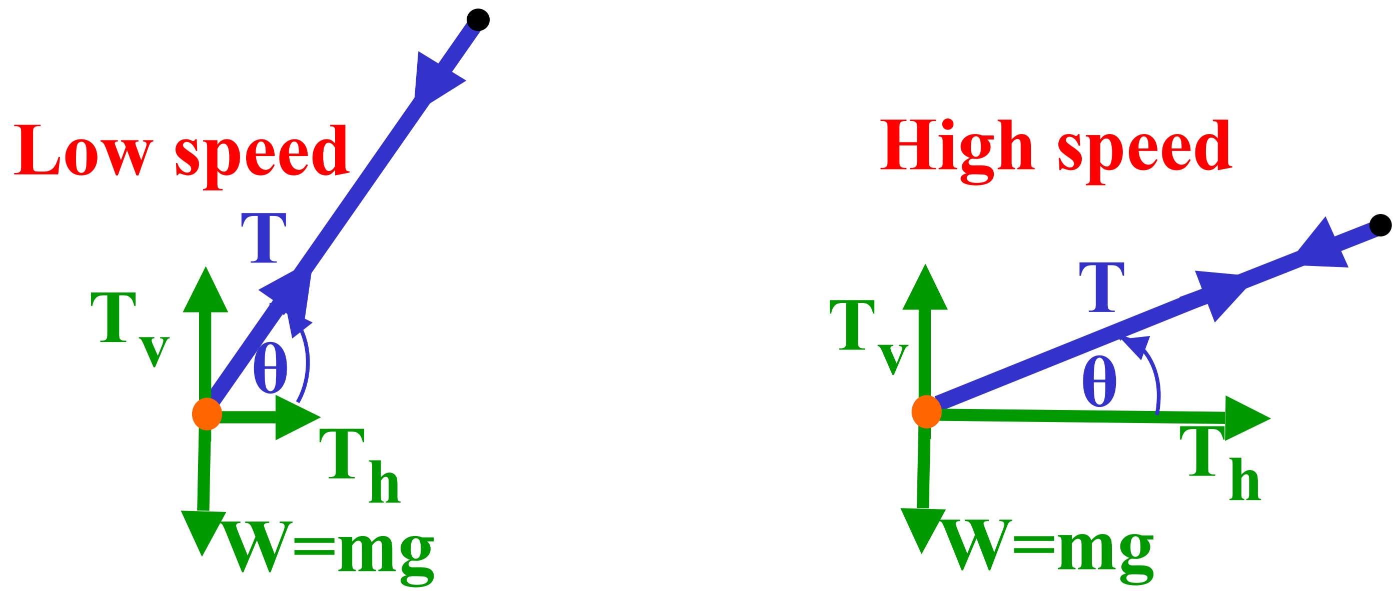
$$T_h = T \cos \theta$$

$$T_v = T \sin \theta = mg$$

$$T_h = \frac{m\,v^2}{r} = T \cos \theta$$

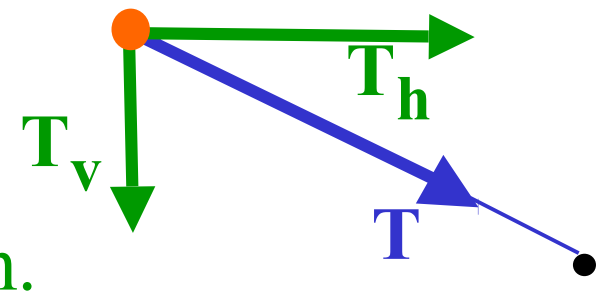
As ball speeds up the horizontal, tension will increase (as v^2) and the angle θ will reduce.

Stable Rotating Condition



Thus, as speed changes T_v remains unaltered (balances weight) but T_h increases rapidly.

Unstable Condition



- T_v no longer balances weight.
- The ball can't stay in this condition.

Ex. again: Ball velocity 2 m/s, mass 0.1 kg, radius=0.5 m.

$$a_c = \frac{v^2}{r} = \frac{2 \times 2}{0.5} = 8\ \text{m/s}^2$$

Centripetal force $F_c = ma_c = 0.1 \times 8 = 0.8$ N

Thus, **horizontal tension (T_h) = 0.8 N.**

Now **double the velocity**…

Centripetal $a_c = \frac{v^2}{r} = \frac{4 \times 4}{0.5} = 32\ \text{m/s}^2$

$F_c = ma_c = 0.1 \times 32 = 3.2$ N

Thus, the **horizontal tension increased 4 times!**

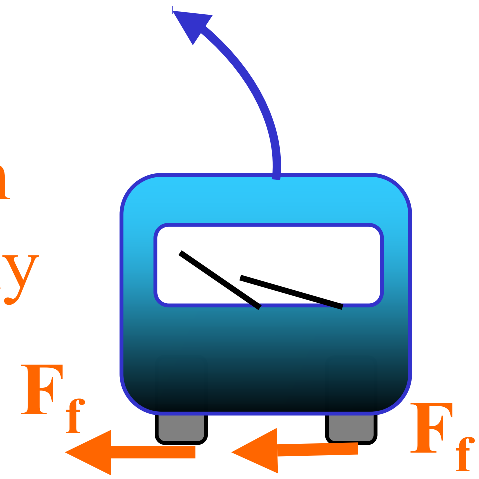
Summary

- **A centripetal force F_c is required to keep a body in circular motion:**
- **This force produces centripetal acceleration that continuously changes the body's velocity vector.**

$$F_c = m\,a_c = \frac{m\,v^2}{r}$$

- **Thus for a given mass the needed force:**
  - **increases with velocity ^{2}**
  - **increases as radius reduces.**

Example: The centripetal force needed for a car to round a bend is provided by **friction**.



- If **total** (static) frictional force is **greater** than required centripetal force, car will successfully round the bend.

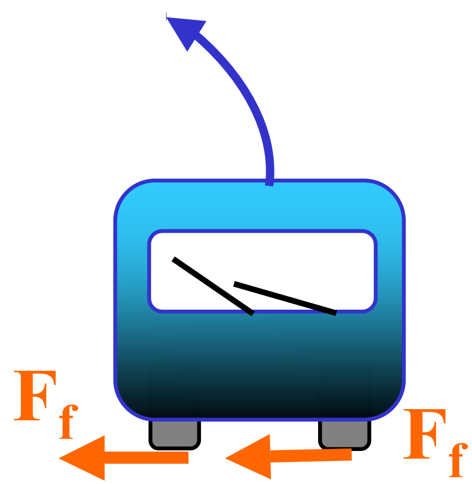
- The higher the velocity and the sharper the bend, the more friction needed!

$$F_s > \frac{mv^2}{r}$$

- As $F_s = \mu_s N$ - the friction depends on surface type (μ_s).

- Eg. If you hit ice, μ becomes small and you fail to go around the bend.
- Note: If you start to skid (locked brakes) μ_s changes to its kinetic value (which is lower) and the skid gets worse!

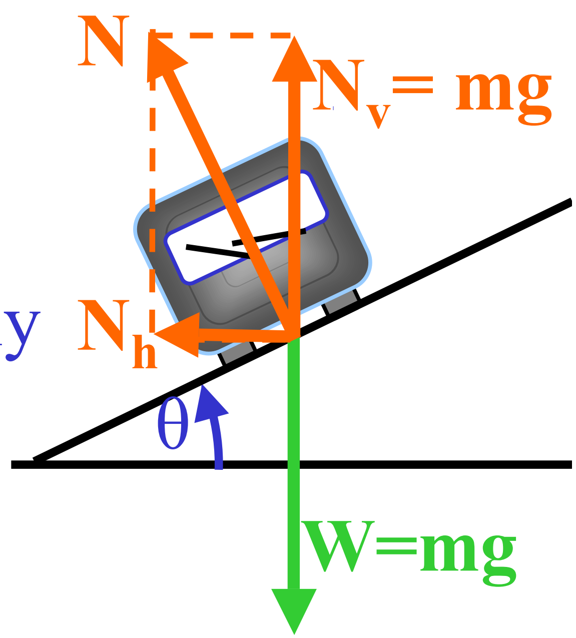
Moral: Don't speed around tight bends! (especially in winter)



Motion on a Banked Curve

- **The normal force N depends on weight of the car W and angle of the bank θ.**

- **There is a horizontal component (N_h) acting towards center of curvature.**

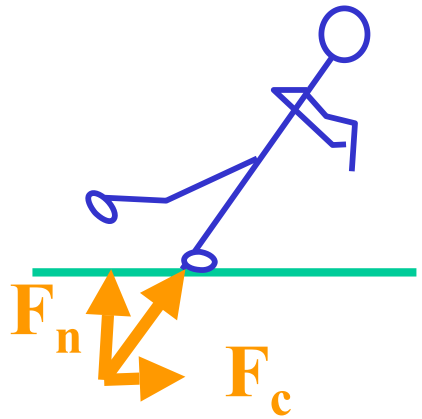


- This extra centripetal force can significantly reduce amount of friction needed…

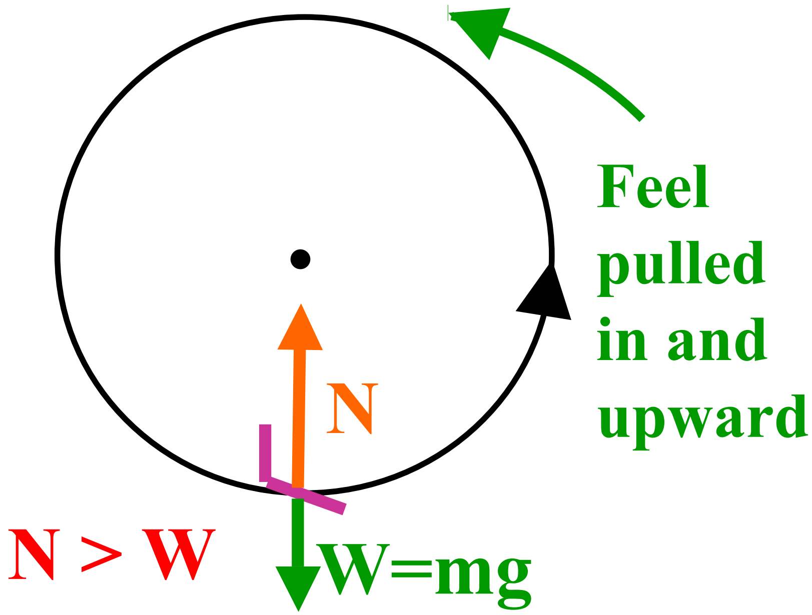
- If $\tan\theta = \frac{v^2}{rg}$ then the horizontal N_h provides **all** the centripetal force needed!

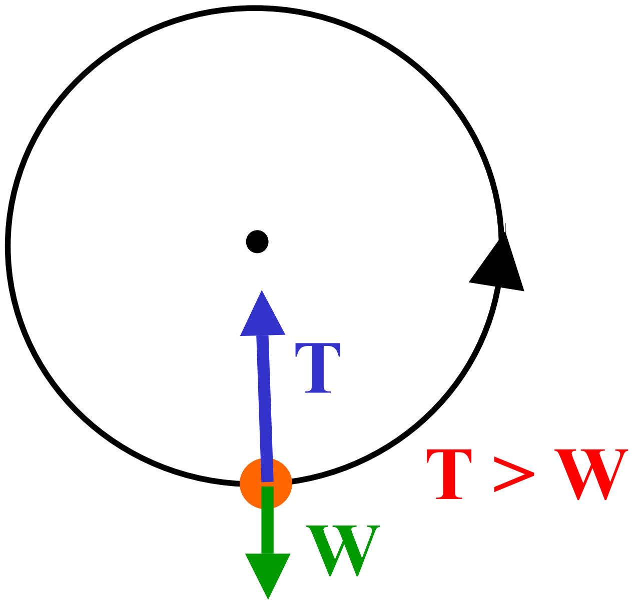
- **In this case no friction is necessary and you can safely round even an icy bend at speed…**

- Ice skaters can’t tilt ice so they lean over to get a helping component of **reaction force** to round sharp bends.



Vertical Circular Motion





Bottom of circle:

- **Centripetal acceleration is directed upwards.**
- Total (net) force is thus directed upwards:

$F_{net} = N - W = ma_c$ N=apparent weight (like in elevator)

Thus: $N = W + ma_c$ i.e. **heavier/larger tension**