Problem 1.31

We are to show that conservation of momentum for all closed systems of particles implies Newton's third law. Following the *Hint* in the text, we can restrict attention to a single pair of particles. We assume that $\vec{p}_1 + \vec{p}_2$ is conserved, which means for all motions of the particles

$$\frac{d}{dt}(\vec{p}_1 + \vec{p}_2) = \frac{d\vec{p}_1}{dt} + \frac{d\vec{p}_2}{dt} = 0.$$

From Newton's second law and the assumption that the net external force is zero on all particles we have that

$$\frac{d\vec{p}_1}{dt} = \vec{F}_{21}, \quad \frac{d\vec{p}_2}{dt} = \vec{F}_{12},$$

where $\vec{F}_{ij}$ is the force of the i^{th} particle on the j^{th} particle. Therefore, for all configurations of the particles

$$\vec{F}_{12} + \vec{F}_{21} = 0 \quad \Longleftrightarrow \quad \vec{F}_{21} = -\vec{F}_{12},$$

which is Newton's third law.