Chapter 4

Problem 4.2

(a)

$$\psi(x,y,z) = \left(\frac{2}{a}\right)^{3/2} \sin(\frac{n_x \pi}{a} x) \sin(\frac{n_y \pi}{a} y) \sin(\frac{n_z \pi}{a} z), \quad E = \frac{\hbar^2 \pi^2}{2ma^2}(n_x^2 + n_y^2 + n_z^2).$$

(b) The energies (E) and degeneracies (d) are

$$E_1 = 3\epsilon,\ d_1 = 1, \quad E_2 = 6\epsilon,\ d_2 = 3, \quad E_3 = 9\epsilon,\ d_3 = 3,$$

$$E_4 = 11\epsilon,\ d_4 = 3, \quad E_5 = 12\epsilon,\ d_5 = 1, \quad E_6 = 14\epsilon,\ d_6 = 6,$$

where

$$\epsilon = \frac{\hbar^2 \pi^2}{2ma^2}.$$

(c) E_{14} can be obtained by $n_x = n_y = n_z = 3$ *or* $n_x = 5,\ n_y = 1 = n_z$, *etc.* . Thus the degeneracy is 4, which is a purely numerical coincidence, not easily viewed in terms of physics.

Problem 4.9

The ground state wave function is

$$\psi = \begin{cases} A\sin(kr), & r \le a \\ Be^{-\kappa r}, & r \ge a, \end{cases}$$

where

$$k = \sqrt{2m(E+V_0)}/\hbar, \quad \kappa = \sqrt{-2mE}/\hbar,$$

$$B = A\sin(ka)e^{\kappa a},$$

and E is the lowest solution to

$$\cot(ka) = -\kappa/k.$$

The normalization constant A is determined from

$$1 = 4\pi \int_0^\infty dr\, r^2 \psi^2(r).$$

Problem 4.13

(a) $\langle r \rangle = 3/2a$, $\langle r^2 \rangle = 3a^2$.

(b) $\langle x\rangle 0$, $\langle x^2\rangle = a^2$.

(c) $\langle x^2\rangle = 12a^2$.

Problem 4.14

a

Problem 4.15

(a)

$$\Psi(\mathbf{r},t) = -\frac{i}{\sqrt{2\pi a}4a^2} re^{-r/2a}\sin\theta e^{-iE_2t/\hbar}.$$

(b) $E = \frac{1}{2}E_1 = -6.8$ eV.

Problem 4.17

(a) $V(r) = -\frac{GMm}{r}$.

(b) $a = \frac{\hbar^2}{GMm^2} = 2.34\times 10^{-138}$ m.

(c) $n = 2.53\times 10^{74}$.

(d) $\Delta E = 2.09\times 10^{-41} J$. $\lambda = 1$ light year.

Problem 4.24

(b) The normalized eigenfunctions are just the spherical harmonics, Y_n^m. The degeneracy is $2n+1$.

Problem 4.27

(a) $A = \frac{1}{5}$.

(b) $\langle S_x\rangle = 0$, $\langle S_y\rangle = -\frac{12}{25}\hbar$, $\langle S_z\rangle = -\frac{7}{50}\hbar$.

(c) $\sigma_{S_x} = \frac{\hbar}{2}$, $\sigma_{S_y} = \frac{7}{50}\hbar$, $\sigma_{S_z} = \frac{12}{25}\hbar$.

Problem 4.29

(a) The eigenvalues and eigenspinors are

$$\pm\frac{\hbar}{2}, \quad \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ \pm i\end{pmatrix}.$$

(b) $\pm\frac{\hbar}{2}$ with probabilities $\frac{1}{2}|a\mp ib|^2$.

(c) $\frac{\hbar^2}{4}$ with probability 1.

Problem 4.32

(a) $\frac{1}{2}[1 + \sin\alpha \sin(\gamma B_0 t)]$

(b) $\frac{1}{2}[1 - \sin\alpha \sin(\gamma B_0 t)]$

(c) $\cos^2(\frac{\alpha}{2})$

Problem 4.33

(a) $H = -\gamma \mathbf{B} \cdot \mathbf{S} = -\frac{\gamma B_0 \hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

(b) $\chi(t) = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{i(\gamma B_0/2\omega)\sin(\omega t)} \\ e^{-i(\gamma B_0/2\omega)\sin(\omega t)} \end{pmatrix}$.

(c) $\sin^2\left(\frac{\gamma B_0}{2\omega} \sin(\omega t)\right)$.

(d) $\frac{\pi\omega}{\gamma}$