Chapter 2

Problem 2.5

(a) $A = \frac{1}{\sqrt{2}}$

(b) $\Psi(x,t) = \frac{1}{\sqrt{2}}\left(e^{-i\omega t}\psi_1(x) + e^{-4i\omega t}\psi_2(x)\right)$. $|\Psi(x,t)|^2 = \frac{1}{2}(\psi_1^2 + \psi_2^2) + \cos(3\omega t)\psi_1\psi_2$.

(c) $\langle x\rangle = \frac{a}{2} - \frac{16a}{9\pi^2}\cos(3\omega t)$. The amplitude of the oscillation is therefore $\frac{16}{9\pi^2}a$.

(d) $\langle p\rangle = \frac{16}{3\pi^2}m\omega a\sin(3\omega t)$.

(e) The energies which will be measured are E_1 and E_2, each with probability $\frac{1}{2}$.

$$\langle H\rangle = \frac{5\pi^2\hbar^2}{4ma^2} = \frac{1}{2}E_1 + \frac{1}{2}E_2.$$

Problem 2.7

(a) $A = \frac{2\sqrt{3}}{a^{3/2}}e^{i\alpha}$

(b) $\Psi(x,t) = \sum_{n=1}^{\infty}\frac{4\sqrt{6}}{n^2\pi^2}\sin(\frac{n\pi}{2})e^{-iE_nt/\hbar}$. Note that only odd values of n contribute to the sum.

(c) $96/\pi^4$

(d)

$$\langle H\rangle = \sum_{\substack{n=1\\ n\text{ odd}}}^{\infty}\frac{48}{n^2\pi^2}\frac{\hbar^2}{ma^2} = \frac{6\hbar^2}{ma^2}$$

Problem 2.8

(a) $\Psi(x,0) = \begin{cases}\sqrt{\frac{2}{a}}, & 0 < x < a/2\\ 0, & \text{otherwise}\end{cases}$

(b) $\left(\frac{2}{\pi}\right)^2$

Problem 2.13

(a) $A = 1/5$

(b) $\Psi(x,t) = \frac{1}{5}\left(3\psi_0(x)e^{-i\omega t/2} + 4\psi_1(x)e^{-3i\omega t/2}\right)$, $|\Psi|^2 = \frac{1}{25}\left(9\psi_0^2 + 16\psi_1^2 + 24\psi_0\psi_1\cos(\omega t)\right)$.

(c) $\langle x\rangle = \frac{24}{25}\sqrt{\frac{\hbar}{2m\omega}}\cos(\omega t)$, $\langle p\rangle = -\frac{24}{25}\sqrt{\frac{m\omega\hbar}{2}}\sin(\omega t)$.

(d) Energy $\frac{1}{2}\hbar\omega$ with probability 9/25; energy $\frac{3}{2}\hbar\omega$ with probability 16/25.

Problem 2.15

0.157

Problem 2.19

$J = \frac{\hbar k}{m}|A|^2$.

Problem 2.34

(a) $R = 1$.

(b) $R = \frac{(\sqrt{(E)} - \sqrt{E - V_0})^4}{V_0^2}$

(c) $T = 0$ for $E < V_0$.

(d)

$$T = \frac{4\sqrt{E}\sqrt{E - V_0}(\sqrt{E} - \sqrt{E - V_0})^2}{V_0^2}$$