Sample Homework Problems

Problem: From Euler's formula, $e^{i\theta} = \cos(\theta) + I\sin(\theta)$, prove that $\cos^2\theta + \sin^2\theta = 1$.

Bad Answer

$$1 = |e^{i\theta}|^2 = |\cos(\theta) + i\sin(\theta)|^2 = \cos^2(\theta) + \sin^2(\theta).$$

Good Answer

This result follows from taking the absolute value square of both sides of Euler's formula. On the right hand side we get

$$|\cos(\theta) + i\sin(\theta)|^2 = [\cos(\theta) + i\sin(\theta)]\,[\cos(\theta) - i\sin(\theta)] = \cos^2(\theta) + \sin^2(\theta).$$

The absolute value square of the left hand side of Euler's formula is

$$|e^{i\theta}|^2 = e^{i\theta}e^{-i\theta} = 1.$$

Therefore we get

$$1 = \cos^2(\theta) + \sin^2(\theta).$$

Problem: The *energy* of a harmonic oscillator can be defined by

$$E = \frac{\gamma}{2}\left\{\left(\frac{dq}{dt}\right)^2 + \omega^2 q^2\right\},$$

where γ is a constant (needed to get the units right). Show that E does not depend upon time if $q = q(t)$ solves the harmonic oscillator equation.

Bad Answer:

$$\frac{dE}{dt} = \gamma\left\{\frac{dq}{dt}\frac{d^2q}{dt^2} + \omega^2 q\frac{dq}{dt}\right\} = 0.$$

Good Answer: For any function $q = q(t)$ the time rate of change of the energy is given by

$$\begin{aligned}\frac{dE}{dt} &= \gamma\left\{\frac{dq}{dt}\frac{d^2q}{dt^2} + \omega^2 q\frac{dq}{dt}\right\}\\ &= \gamma\frac{dq}{dt}\left\{\frac{d^2q}{dt^2} + \omega^2 q\right\}\end{aligned}$$

The quantity in brackets vanishes precisely when the harmonic oscillator equation holds since

$$\frac{d^2q}{dt^2} = -\omega^2 q \quad\Longleftrightarrow\quad \frac{d^2q}{dt^2} + \omega^2 q = 0.$$

So if $q(t)$ solves the harmonic oscillator equation, then $\frac{dE}{dt} = 0$.