Problem 3.9

Consider three different approximations to the derivative $f'(x)$ of a function $f(x)$ in terms of a small parameter $h << 1$:

(1) Forward difference: $\Delta_1 f = \frac{f(x+h)-f(x)}{h}$

(2) Backward difference: $\Delta_2 f = \frac{f(x)-f(x-h)}{h}$

(3) Central difference: $\Delta_3 f = \frac{f(x+\frac{1}{2}h)-f(x-\frac{1}{2}h)}{h}$

Use Taylor's theorem to show that the error in the derivative introduced by not taking the limit as $h \to 0$ is of order h for $\Delta_1 f$ and $\Delta_2 f$, but is of order h^2 for $\Delta_3 f$, and hence $\Delta_3 f$ is the more accurate approximation of the derivative.