Phys 3750 Midterm Exam 3 Name____________________

You may use a pen/pencil and a 3"×5", handwritten note card on the exam. Answer all questions as completely as possible; show all of your work. If you run out of space on a problem, continue on the back of that sheet of paper. Good Luck!

Q1. (5 points each) Short and Simple.

(a) What kind of field is $\nabla \cdot (f \nabla g)$?

(b) What is a Hankel function?

(c) For a solution to the wave equation in cylindrical coordinates, write down the two Bessel functions that can be involved in a cylindrically symmetric solution.

(d) What is true about the functions $Q_l(s)$ as $s \to 0$? As $s \to \infty$?

(e) What is the energy current density associated with the wave $q(x,t) = q_0 \cos(kx - kct)$?

(f) What is the r dependence of the radiative component of the energy current density for a spherically symmetric traveling wave?

Q2. (10 points) If the Fourier transform of $f(x)$ is $h(k)$, find the Fourier transform of $x^2 f(x)$.

Q3. (10 points) Calculate the divergence of $\mathbf{V}(\mathbf{r}) = \frac{y}{x^2 + y^2}\hat{\mathbf{x}} + \frac{x}{x^2 + y^2}\hat{\mathbf{y}}$

Q4. Consider the partial differential equation $\frac{\partial q(x,t)}{\partial x} = c\, q(x,t) \frac{\partial q(x,t)}{\partial t}$ where c is a constant.

(a) (20 points) Use separation of variables to find two related ordinary differential equations that are equivalent to this equation.

(b) (10 points) Find solutions to the two ordinary differential equations that you found in (a).

Q5. (10 points) The fourth Legendre polynomial is $P_3(s)=\frac{1}{2}(5s^3-3s)$. Find the normalized version of this function.

Q6. (10 points) The second spherical Bessel function is $j_1(s)=\frac{\sin(s)}{s^2}-\frac{\cos(s)}{s}$. Show that this function vanishes as $s\to 0$.