Phys 3750 Midterm Exam 2 Name____________________

You may use a pen/pencil and a 3"×5", handwritten note card on the exam. Answer all questions as completely as possible; show all of your work. If you run out of space on a problem, continue on the back of that sheet of paper. Good Luck!

Q1. (5 points each) Short and Simple.

(a) Is the function $e^{x} + e^{-x}/2$ even or odd? Why?

(b) What is the Gibbs phenomenon?

(c) Calculate $\lim_{x\to 0}\frac{\sin(nx)}{x}$.

(d) Consider the two vectors $\mathbf{a} = \begin{pmatrix} 1 \\ i \\ 2 \\ -i \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 1 \\ 2i \\ 2 \\ 1 \end{pmatrix}$ that live in a four dimensional complex vector space.

Calculate the inner product $(\mathbf{a},\mathbf{b})$.

(e) Calculate the integral $\int_{-\infty}^{\infty}(3+2x+x^{2})\delta(x-1)dx$.

Q2. (5 points) Starting with $q(x,t)=\frac{1}{2}\left[a(x+ct)+a(x-ct)+\frac{1}{c}\int_{x-ct}^{x+ct}b(x')dx'\right]$, show that $a(x)$ is the initial displacement.

Q3. (10 points) Consider a vector space of functions defined on the interval $0 \le x \le L$. Find the norm of the function $f(x)=x^2$.

Q4. The function $q(x,t) = Ae^{-(x-ct)^2/a_0^2}$ is a solution to the wave equation.

(a) (10 points) Describe in words, as completely and precisely as possible, the nature of this solution.

(b) (10 points) Find the initial conditions $a(x)$ and $b(x)$ that give rise to this solution.

Q5. Consider the 1D wave equation on the interval $0 \le x \le L$ with boundary conditions $q(x,t) = q(L,t) = 0$. The general solution can be written as $q(x,t) = \sum_{n=1}^{\infty} \sin\left(\frac{n\pi}{L} x\right)\left[b_n \cos(\omega t) + d_n \sin(\omega t)\right]$.

(a) (5 points) What is the relationship between ω and $n\pi/L$?

(b) (15 points) Starting with the equation above, find an expression for d_n in term of the initial conditions $a(x)$ and $b(x)$. Carefully show your work.

Q6. the function $f(x)=\begin{cases} e^{-Ax} & x\geq 0 \\ 0 & x<0 \end{cases}$.

(a) (15 points) Calculate the Fourier transform $h(k)$ of $f(x)$. Express your answer for $h(k)$ in the form $h_R(k)+ih_I(k)$, where $h_R(k)$ and $h_I(k)$ are both real functions.

(b) (5 points) Find $\int_{-\infty}^{\infty} |h(k)|^2\, dk$.