Physics 3710 – Problem Set #1 *Power law and logarithmic thinking*

Problems 1-5 refer to the first figure in BK1.

1. For the $\log_{10}$(time/s) scale in the figure some interval starts at *x* and ends at *y*. Assume both *x* and *y* are <u>integers</u> (positive, negative, or zero). *Approximately* how long, in seconds, is the interval *y*–*x*?

2. On the "history plot" in BK1, "inflation" corresponds to a speculated rapid cooling of the universe followed by a rapid reheating. The latter can be thought of as the "Big Bang," the epoch after which all the observed forces and particles emerge. Approximately how many seconds after $t = 0$ does the Big Bang end?

3. On a log-log plot a power law function $y = Ax^p$ is a straight line with slope *p*. (That is, $\log(y) = p\log(x) + \log(A)$. It doesn't matter what the base of the logarithm is.) The figure shows that $\log_{10}$(energy/eV) is approximately linear in $\log_{10}$(time/s) for both the matter and radiation dominated epochs, but with slightly different slopes. If you draw straight lines through the matter and radiation curves on the figure in BK1, you get roughly the following sets of points on the lines: for matter (12, 0.5) and (15, -1.5); for radiation (–28, 20) and (10, 1). Determine *p* in each case assuming it is a simple rational fraction.

4. The smallest "physical" time mentioned in BK1 is the so-called *Planck time*, t_P. The Planck time is a combination of gravity, relativity, and quantum mechanics, of the form $t_P = G^x c^y \hbar^z$ where *x*, *y*, and *z* are dimensionless constants. If *M*, *L*, and *T* are the fundamental dimensions "mass," "length," and "time," respectively, the fundamental dimensions of *G* are $M^{-1}L^3T^{-2}$, of *c* are L^1T^{-1}, and of $\hbar$ are $M^1L^2T^{-1}$. Show that $x = z = 1/2$ and $y = -5/2$. (This an exercise in dimensional analysis, a useful technique well worth learning.)

5. Using the results of Problem 4, insert the values of *G*, *c*, and $\hbar$ in SI units and show that for t_P $\log_{10}$(time/s) ≈ -43.