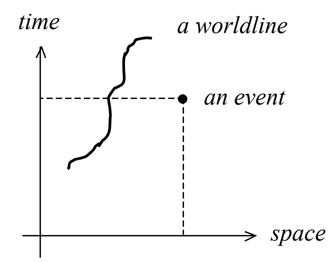
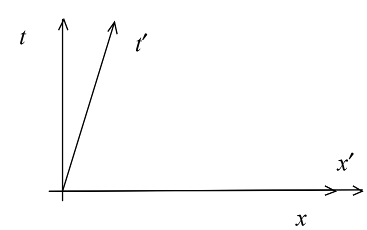
Background, 2

Ordinary, everyday, Galilean/Newtonian relativity

An "event" is something that happens at a point in space, at an instant in time. In physics, ***relativity* means the rules by which two observers can compare and make sense of measurements each makes of the positions and times of the same events.** In physics, *an observer is not a person or an individual measuring device*. Such isolated "detectors" are plagued by experimental issues of parallax, delay times, and so forth. For our purposes, **an *observer* will always mean *an infinite collection of rigidly attached, perfect sensors and microprocessors whose internal clocks are perfectly synchronized***. The sensor and microprocessor "closest" to an event gets to record that event's position and time. The sensors and microprocessors that make up each observer are magic: they can pass through one another without crashing and breaking. Thus, two perfect observers can be in motion relative to one another and each can record (infinitely rapidly and without error) positions and times of events.



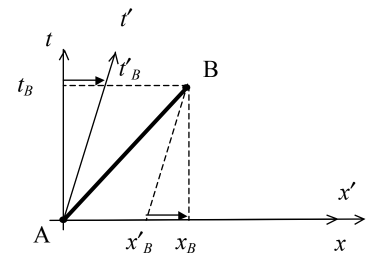
The collection of all possible positions and times is *spacetime*. Spacetime is the arena in which events occur. The study of relativity is greatly aided by drawing *spacetime* (*s-t*) *diagrams*. Because physical space is (at least) 3-dimensional, s-t diagrams suppress one or more directions of space. A 1+1 *s-t diagram* (1 dimension of space + 1 dimension of time) looks like the figure to the right. A dot on an s-t diagram is an *event*. A sequence of events whose space and time coordinates change continuously is a *worldline*. An s-t diagram can be made into a quantitative tool by laying off hash marks along the respective axes indicating units of space and units of time. If that's done, the x and t coordinates of an event can be found by, respectively, drawing a line parallel to the t-axis until it hits the x-axis (that's the x coordinate) and vice versa for t. Note that the t-axis on an s-t diagram is the worldline of the spatial origin. The x-axis *at any instant*, t, is the collection of all events that have that same time, t, that is all events that happen *simultaneously* at t. (That might sound funny, at first, but all points on the x-axis get older just like the origin does.)



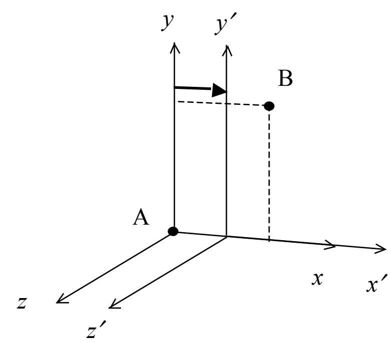
Now, we can superpose the s-t diagrams for two observers to draw inferences about what each records. Suppose observer O′ moves with constant speed in the $+x$-direction, according to O. Suppose the x- and x'-axes are parallel (as are y and y', and z and z') and that the two spatial origins coincide at $t' = t = 0$. The s-t diagram of O also showing the space and time axes of O′ looks like the figure to the right. The t'-axis is tipped to the right because it is the worldline of the origin of O′ and that origin is traveling to the right according to O. (If O′ moves in the $-x$-direction the t'-axis is tipped to the left.) The x- and x'-axes are parallel because ***in Newtonian physics time is universal***: all Newtonian observers reckon that an event happens at the same time. Since the x-axis at time t is the set of all events at t and the x'-axis at t' is the set of all events at t', *and* $t' = t$, it must be that x- and x' are parallel.

Let's see what the rules are for converting from unprimed space and time coordinates to primed coordinates (and vice versa). It's convenient to do so with specific examples that will be instructive when we consider Einsteinian (special) relativity.

Example: Suppose event A is the emission of a sound pulse from $x_A = x'_A = 0$ at $t_A = t'_A = 0$. Suppose the pulse travels in the $+x$-direction, according to O, with a constant speed equal to $v_x > V$, where V is the constant speed of O′ relative to O. Note that *worldlines with steeper (alternatively, flatter) slopes on an s-t diagram represent slower (alternatively, faster) motions*. Suppose the pulse is detected (event B) at position x_B at time t_B. The s-t diagram to the right shows how to determine x_B and x'_B as well as $t_B = t'_B$ by drawing the appropriate (dotted) lines. In order for $t_B = t'_B$ the hash marks on the two axes have to be spaced differently. The length of each of the two arrows is Vt_B. As a result, $x'_B = x_B - Vt_B$. The velocity of the pulse according to O′ is $v'_x = x'_B / t'_B = (x_B - Vt_B)/t_B = v_x - V$: the two observers disagree about where B occurs and about the pulse speed. But they do agree on how to reconcile their differences. That's Newtonian relativity for you.



Example: In the previous example, we only considered the x and x' coordinates of B. Suppose B occurs at (x_B, y_B) in O and (x'_B, y'_B) in O′. The figure to the right shows the situation. The figure is *not* an s-t diagram; it shows the three-dimensional spatial coordinate systems of O and O′, as viewed by O, at the instant event B occurs. Event A happened at the origin of O t_B earlier. As above, the arrow has length Vt_B. The dotted lines determine (x_B, y_B) and (x'_B, y'_B). It's clear that $x'_B = x_B - Vt_B$ and $y'_B = y_B$. If the z coordinates are included we get $z'_B = z_B$. The velocity of the pulse in the directions perpendicular to x are $v'_y = y'_B / t'_B = y_B / t_B = v_y$ and the same for z.

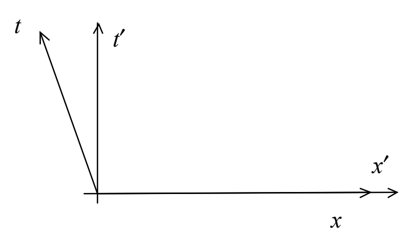


Event B doesn't have to be detection of the sound pulse. It could be any event with coordinates (x, y, z, t) in O and (x', y', z', t') in O′. As long as the spatial origins coincide at $t = t' = 0$ the rules of translation for Newtonian relativity are

$$\begin{aligned} t' &= t \\ x' &= x - Vt \\ y' &= y \\ z' &= z \end{aligned} \quad \text{along with} \quad \begin{aligned} v'_x &= v_x - V \\ v'_y &= v_y \\ v'_z &= v_z \end{aligned} \qquad (1)$$

If you want to find (x, y, z, t) in terms of (x', y', z', t') just *flip the sign of* V:

$$\begin{aligned} t &= t' \\ x &= x' + Vt' \\ y &= y' \\ z &= z' \end{aligned}$$



and so on. Incidentally, the s-t diagram for O′ looks like the figure to the right. Do you see why? The space and time transformations in (1) are historically called *Galilean transformations.*

Inertial observers

If O′ and O are moving relative to each other with constant speed as above, then $a'_x = dv'_x/dt' = d(v_x - V)/dt = a_x, a'_y = a_y, a'_z = a_z$. That is, if one observer records an object accelerating, the second observer records exactly the same acceleration—provided their relative velocity is constant. If the first observer attributes the object's acceleration to an unbalanced real force (i.e., an acceleration produced by an observed interaction of some kind—as opposed to the observer itself accelerating: this is Newton's [First] Law of Inertia), then the second observer should attribute the acceleration to the same force. That is, for <u>a correctly formulated Newtonian force, $\vec{F}' = \vec{F}$.</u> Such a force cannot depend on *absolute position* or *absolute velocity*, because two relatively moving observers disagree on positions and velocities along the direction of relative motion. In this case, both observers are *inertial observers* and switching from one to the other leaves the Newtonian mechanics *invariant*. We say, "Newtonian mechanics is symmetric under Galilean transformations." This is the first of many examples we will encounter in which **symmetry and dynamics are intimately interconnected**.

<u>Example</u>: Suppose a mass, m, is attached to an ideal spring with force constant, k, that is oriented along the x-axis. According to O the force exerted by the spring on the mass when the mass is at position x is $F_x = -kx$. An observer O′ moving along the $+x$-direction with constant speed V records the position of the mass at the same instant to be $x' = x - Vt$, suggesting that $F'_x = -k(x - Vt)$, i.e., *not* the same force as in O. But, that's wrong. The actual spring force is $-k(x - x_{eq})$ where x is the position of m *relative to* x_{eq}, the position of the mass when the spring exerts no force. In the expression above x_{eq} is apparently O's origin. To O′, O's origin is at $-Vt$. Inserting this into the force expression we get $F'_x = -k[(x - Vt) - (0 - Vt)] = -kx = F_x$. The spring force, properly stated, is a correctly formulated force.

<u>Example</u>: A mass, m, is injected into a container of liquid in the x-direction and thereafter experiences a drag force according to O (at rest with respect to the liquid), $F_x = -\beta v_x$, where β is a drag coefficient. Again, if O′ moves along the $+x$-direction with constant speed V, $v'_x = v_x - V$, so that it would appear that $F'_x = -\beta(v_x - V)$, not the same as F_x. This is again wrong: the correct drag force is $F_x = -\beta(v_x - v_{liquid})$; the drag force is proportional to the velocity *relative to* the liquid. Inserting the velocity transformation from (1) above, $F'_x = -\beta[(v_x - V) - (0 - V)] = -\beta v_x = F_x$. The drag force, properly stated, is a correctly formulated force.

<u>Example</u>: The magnetic force on a particle with electric charge, q, is given by $\vec{F} = q\vec{v} \times \vec{B}$. Suppose q is positive and, to observer O, $\vec{v}$ is in the $+x$-direction at one instant, and $\vec{B}$ is along the y-axis. Then O records an initial acceleration $qv_{x0}B_y/m$ in the $+z$-direction (where m is the particle's mass). In the next instant the particle will begin to move along the $+z$-direction as well as along the $+x$-direction. Now, suppose, as before, that O′ moves along the $+x$-direction with constant speed V. If $V = v_{x0}$ (i.e., O′ catches up with the particle initially) then the acceleration according to O′ is zero initially. According to O′ the particle stays on the x-axis. But that's impossible: the two accelerations are supposed to be the same. In the previous

examples, this dilemma was resolved by noting that the positions and velocities were relative to something, but in the magnetic force law there is no reference to a relative state of motion! Thus, it cannot be that $\vec{F} = q\vec{v} \times \vec{B}$ is a correctly formulated Newtonian force. For awhile in the 19th century, it was thought that Earth was moving through a kind of liquid, called the ether, where electromagnetism was absolutely correct, so that $\vec{F} = q(\vec{v} - \vec{v}_{ether}) \times \vec{B}$ (which would be a good Newtonian force). But very careful attempts to measure motion relative to the ether have failed (with an uncertainty of about 1 part in 10^{17}!), so that can't be the answer. To rectify the situation requires a little help from Einstein's special relativity. We'll get back to this interesting problem presently. For now, try to digest the essential fact that electromagnetic phenomena are not compatible with Newtonian relativity.