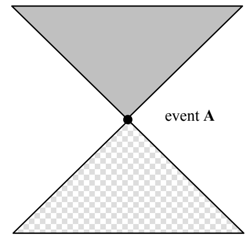
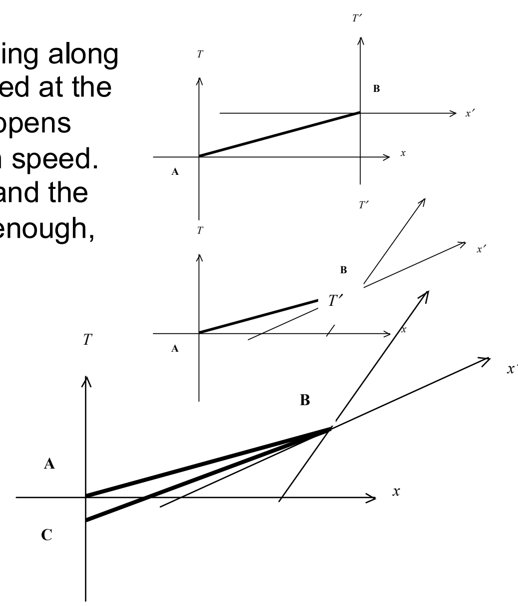
Special relativity, 4

More kinematic consequences of the Lorentz transformations



Light cones: A "light cone" is a set of world lines corresponding to light rays emanating from and/or entering into an event. The figure to the right shows an event (**A**) and four s-t regions in 1+1 dimensions connected to it. The edges of the regions are defined by the world lines of light rays. If a light pulse were emitted from **A** it would spread uniformly in both spatial directions and the *edges* of the uniform gray region above **A** would correspond to all events on that pulse. The proper time intervals between all events on each edge and **A** would be zero. In other words, the edges are collections of events that have a light-like interval relative to **A**. All events, **B**, inside the *uniform* gray area have time-like intervals with respect to **A**, and the associated proper time interval, $\tau_{A \to B}$, is positive, meaning that **A** occurs *before* any of those events. **A** can be *connected to* any event in the uniform region by a signal traveling slower than *c*. The edges of the checkered gray region below **A** similarly have light-like intervals relative to **A**. If a light pulse were emitted from any event on one of those edges it would eventually get to **A**. All of the events, **B**, inside the *checkered* region have time-like intervals with respect to **A** and $\tau_{A \to B}$ is negative, meaning that **A** occurs *after* any of those events. Any event in the checkered region can be *connected to* **A** by a signal traveling slower than *c*. The white regions to the left and right of **A** consist of all events that have space-like intervals relative to **A**. Connecting **A** with such events would require a signal traveling faster than *c*. The proper time between two such events is an *imaginary* number.

Tachyons are tacky! A light cone divides space-time about any event into regions of events that might be connected to the event in question by signals of different speeds. The carriers of signals traveling slower than c are "tardyons." Carriers traveling at speed c are "luxons." And carriers traveling faster than c are "tachyons." You are familiar with signals sent by tardyons (packages sent by FedEx, for example) and by luxons (TV, radio, and so forth). But maybe you are less familiar with those sent by tachyons. That's probably for good reason. It's probably not possible for pieces of ordinary matter (like you and me) to communicate by such signals. Here's why.



To the right is an s-t diagram showing a tachyon (traveling along the bold worldline) emitted from the origin of O at **A** and received at the origin of O′ (at rest relative to O) at **B**. To both O and O′ **B** happens after **A**. Now, suppose that O′ is traveling away from O at high speed. On the s-t diagram of O the T'-axis is tipped over to the right and the x'-axis is tipped upward. If the recession speed of O′ is high enough, this tipping produces the remarkable result that to O′ **A** happens *after* **B**! In other words, the tachyon received at **B** comes out of the future of **B**. See the figure to the right. Events above the x'-axis happen after events on the x'-axis according to O′.

What makes matters worse, is that upon receipt of the tachyon at **B** a second (faster) tachyon signal might be sent back toward the origin of O. Traveling *into* the future of **B** as seen by O′, such a signal would come *out of* the future according to O and, indeed, might be received by O (at **C**) *before* the event (**A**) that supposedly triggered the sequence of events in the first place! Such *acuasal* sequences make one's head to explode: **A** causes **B** according to O, **B** causes **C** according to O′, but **C** causes **A** according to both observers. So, having received the tachyon signal at **C** does O have the option to NOT send the signal at **A**? Does O′ have the option to NOT send the signal to **C**? Such logical impossibilities make us think that if tachyons exist, they can't actually connect ordinary objects, and therefore have no physical reality (to us).

Because we reject tachyonic signals, the events surrounding any event **A** lie in one of three regions: events that could be caused by **A**—**A**'s "future light cone" (the uniform gray region above), events that could have caused **A**—**A**'s "past light cone" (the checkered gray region above), and all events that cannot influence or be influenced by **A**—**A**'s "elsewhere" (the white regions).

Dynamic consequences of the Lorentz transformations

Recall (BK 2) that we previously argued that "Newtonian mechanics is symmetric under Galilean transformations." We now begin to examine in what ways mechanics is modified to be symmetric under Lorentz transformations.

Momentum ain't what you think it is! Consider a simple collision: a mass m, initially traveling with constant (dimensionless) velocity u_0 in the $+x$-direction, collides with and sticks to an identical mass, initially at rest. In Newtonian physics, *momentum* = *mass* x *velocity* is conserved in the collision. Thus, as good Newtonians, we write $mu_0 + 0 = (2m)u_f$ and deduce that the combined mass travels off in the $+x$-direction with a velocity $u_f = u_0 / 2$, after the collision. Now, this story tacitly assumes that the collision is being recorded in an inertial frame (O, say). Suppose we switch to a frame O′ traveling relative to O with a velocity $\beta = u_0$ (i.e., initially attached to the first mass). Because of the Newtonian velocity transformation rule, $u' = u - \beta$, before the collision the first mass is at rest in O′ while the second mass has velocity $-u_0$, and after the collision the two masses stuck together have velocity $-u_0 / 2$. In other words, in O′ momentum before equals momentum after, just as in O: i.e., $0 + m(-u_0) = (2m)(-u_0 / 2)$.

In special relativity, however, the relevant velocity transformation rule is:

$$u'_x = \frac{u_x - \beta}{1 - \beta u_x}$$

(See SR 2.) This means that before the collision according to O′ the first mass has velocity equal to zero and the second velocity equal to $-u_0$ (just as in the Newtonian case), but after the collision the two stuck masses have a velocity equal to $\frac{u_0 / 2 - u_0}{1 - (u_0)(u_0/2)} = \frac{-u_0 / 2}{1 - u_0^2 / 2}$, which is decidedly NOT equal to $-u_0/2$ for u_0 values near 1! Thus, if in the collision momentum is conserved according to O it is not conserved according to O′: i.e., $0 + m(-u_0) \neq (2m)(\frac{-u_0 / 2}{1 - u_0^2/2})$.

If we are to preserve the constancy of momentum, irrespective of observer, for an isolated system (no external forces) we have to define momentum differently. Of course, the new definition has to agree with the old one, *mass* × *velocity*, when velocities are << 1.