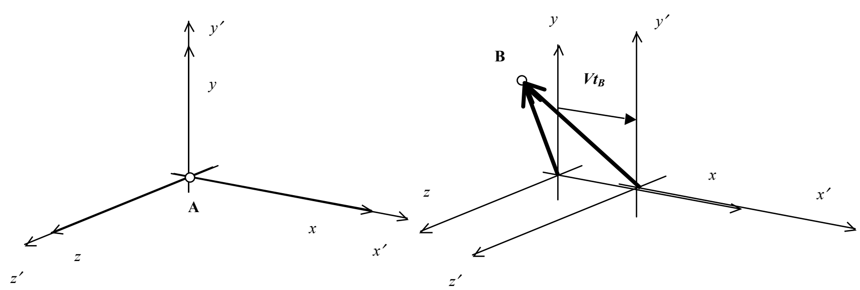
Special relativity, 1

Newton and Maxwell

We have seen that simple $q\vec{v}\times\vec{B}$ magnetic forces are incompatible with Newtonian mechanics. There are far more profound incompatibilities between electromagnetism and Newton. In particular, Maxwell's equations predict that an accelerating charge produces a time changing magnetic field, which, in turn, produces a time changing electric field, which, in turn, produces a time changing magnetic field, which, in turn, produces … . This self-sustaining production of magnetic and electric fields propagates away from the source charge at a finite speed given by $1/\sqrt{\mu_0\varepsilon_0}$ (in empty space), the numerical value of which is approximately 3x10^8 m/s. But, according to which observer? Is there lurking around someplace in Maxwell's formalism an unstated preferred, inertial reference frame (the ether) in which the prediction is true (such as one at rest with respect to the "fixed" stars, for example)?

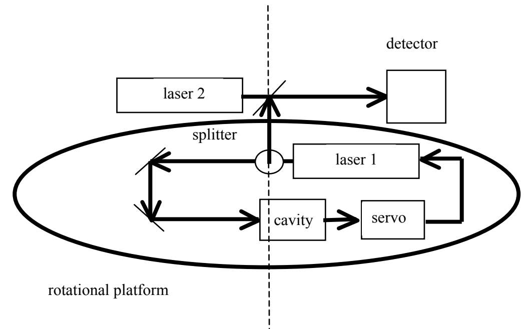
The root of this question is related to how different observers reckon the positions and times of the same events in Newtonian physics. Recall that an "observer" is *not* a person or a single measuring device but rather a collection of perfect sensors and microprocessors with perfectly synchronized clocks capable of recording the occurrence of when and what events happen close to them. This way, events are recorded instantaneously at local points.

Suppose two Newtonian observers O and O′ carry with them spatial coordinate systems aligned as in the figure to the right. O′ moves relative to O with a constant speed V along the $+x$-direction. Event **A** corresponds to the moment ($t'_A = t_A = 0$) the two coordinate origins coincide, while event **B** is some later event. A fundamental assumption in Newtonian physics is that all observers agree on *when* events happen: thus $t'_B = t_B$. They don't agree on *where*, however. For the coordinate systems shown, $y'_B = y_B$, $z'_B = z_B$, *but* $x'_B = x_B - Vt_B$. Letting t_B approach t_A allows us to measure the velocity of a signal propagating from **A** to **B**. O reckons the velocity to have components v_x, v_y, and v_z, while to O′ the components are v'_x, v'_y, and v'_z, with $v'_y = v_y$ and $v'_z = v_z$, but with $v'_x = v_x - V$.



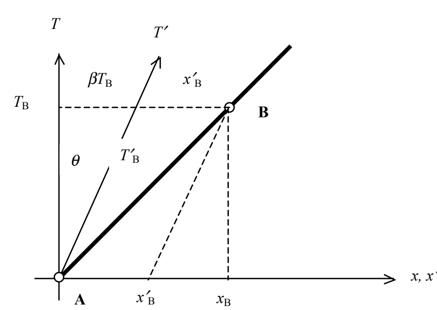
Back to Maxwell. Maxwell says that an accelerating charge generates electromagnetic radiation that travels at speed c in all directions. But if two Newtonian observers are moving relative to one another, then only one of them can observe speed c in all directions. The other must observe a different speed along the direction of their relative motion. So, who gets to observe c? What distinguishes one observer from another? Sound is also a "radiating" wave phenomenon (where a speaker, for example, is the "source charge") and for sound there is a clear answer: sound is carried by a medium, such as the air in a room; observers at rest with respect to the air measure the "book value" of the speed of sound, while observers moving relative to the air measure other values. When Maxwell published his c-result in the 1860s, it was commonly assumed that electromagnetic radiation must also be carried by something—the "ether"—and that the preferred observers were those at rest relative to the ether.

This idea has been tested by several different experiments, first by **Michelson** and **Morley** in the 1880s, and to much higher accuracy, by **Brillet** and **Hall** in 1978. These experiments don't measure c directly but instead attempt to measure differences in c in different directions. They all employ interference, combining two beams of light that, if Newtonian physics were exactly correct would have different light speeds. By altering the beams' directions the resulting interference pattern would change, thus implying which directions were preferred and which not. The Brillet and Hall set-up is depicted in the figure to the right. Laser 1, a beam splitter, two mirrors, a cavity and servomechanism are all mounted on a platform that can be rotated. The servomechanism continually adjusts the frequency of laser 1 so that the same standing wave mode is maintained in the cavity. The splitter deflects a portion of laser 1's beam up the axis of rotation where it is joined with a second beam from laser 2, which is fixed in the lab (with constant frequency). The combined beam then enters a photon detector. Then the platform is rotated to different orientations. If in one orientation the beam from laser 1 is in a frame at rest with respect to the ether (due to Earth's motion), it surely won't be in another orientation (provided Earth is moving relative to the ether). Any change in count rate in the detector as the platform is rotated signals a phase shift between beams 1 and 2, which, if correlated to a frequency adjustment in laser 1, would indicate that light travels at different speeds in different directions of motion relative to the "preferred reference frame." The Brillet-Hall apparatus is so sensitive that frequency shifts of 1 part in 10^{15} are detectable. (Even more recently sensitivity has been raised to 1 part in 10^{17}.) We now know that the Solar system is moving at a speed of about 370 km/s ($1.2\text{x}10^{-3}$ times the speed of light) relative to the cosmic background radiation (the modern version of fixed stars) and we expect that shifts arising from such motion might be on the order of $(V/c)^2$ = $(1.2\text{x}10^{-3})^2$, which is well within detectable limits. No such shift has ever been found. If light speed depended on direction, this experiment would certainly have found it. Thus, **we can confidently conclude that the speed of light is independent of the motion of observers relative to the light source and to each other**. This is a profoundly disturbing result, at least for Newtonian thinking. (The null result described above might be explained by Earth "dragging" a bit of ether with it. A similar experiment has now been done with spacecraft across the solar system, with again a null result. So, the idea of ether dragging is highly dubious.)



Newtonian and Einsteinian relativities

Relativity is the set of rules by which two different observers reconcile their differences as to the positions and times of an event each assigns. As mentioned in BK 2, the study of relativity is often greatly clarified by using **space-time (s-t) diagrams**. As before, an s-t diagram is a 2D graph in which one axis (horizontal) represents a direction in space and the other (vertical) represents time—this is "1+1 spacetime." The space and time coordinates for two different observers can be depicted on the same diagram. This is shown, for

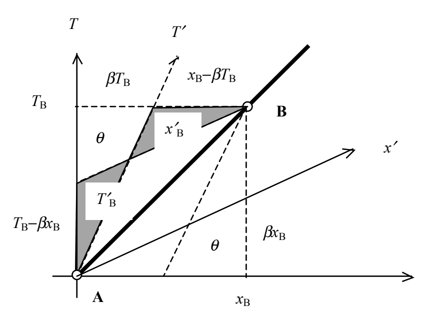


example, in the figure to the right. Recall that the continuous record of events on an s-t diagram for a given object is called the object's *worldline*.

Since we are going to be interested in light, and because c is so large, we rescale time so that events on the s-t diagram involving light that are a few meters apart won't all be crammed onto the space axes. Thus, in the figure, T is time measured in meters, that is, $T = ct$, where t is ordinary time in seconds. (Note: one meter of time = 1 m/3x10^8 m/s = 3.3 ns.) A pulse of light covers 1 m of displacement in 1 m of time, so *in these units light has speed* = 1. When time is measured in meters, speed = distance/time is dimensionless. Other speeds in these units are ratios of speeds in ordinary units divided by c: thus, dimensionless speed is V/c. So, what we see in the figure is a pulse of light emitted at event **A** and detected at event **B** somewhere on the mutual x, x' axes according to *two Newtonian inertial observers*, O and O′. More precisely, according to O, **A** has coordinates $(0,0,0,0)$ and **B** has coordinates $(x_B,0,0,T_B)$, where the middle two 0s in each case are the y and z coordinates, while according to O′ the coordinates are $(0,0,0,0)$ and $(x'_B,0,0,T'_B)$

O′ moves to the right according to O with a constant (dimensionless) speed β. The time axis (T) of O is the world line of the spatial origin of O; it is a vertical line. On the other hand, according to O the spatial origin of O′ moves to the right at a constant speed. So, on this s-t diagram the world line of the spatial origin of O′—the T'-axis—is tipped over at an angle, θ, where $\tan(\theta) = \beta$. To find the position of an event according to O, draw a line from the event parallel to the T-axis and determine where it intersects the x-axis; similarly, drawing a line parallel to the x-axis and determining where it intersects the T-axis finds the time of the event. Two such (dotted) lines are shown in the figure emerging from **B**. The x' and T' values of an event are found by drawing similar lines, but in this case the line parallel to the T'-axis is tilted (as shown). We want T and T' to be equal for any event (these are Newtonian observers), so we have to space the hash marks on the T'-axis a bit farther apart than on the T-axis.

Consequently, the (dimensionless) velocity of the light pulse according to O is $u_{AB} = x_B/T_B$, while according to O′ it is $u'_{AB} = x'_B/T'_B = (x_B - \beta T_B)/T_B = u_{AB} - \beta$. In other words, if O records the speed of the pulse as 1 (i.e., on a world line with slope equal 1 in the figure—like the bold one) then O′ will record it as $1 - \beta$. As noted above, these days we can measure such differences, even if they might be very small, to very high accuracy and we don't observe them. We have excellent reason to believe that u_{AB} and u'_{AB} are both equal to 1.



Of course, as the s-t diagram is drawn above, that can't be. Something has to change. The problem is that $T' = T$ in Newtonian physics. Since the distance between **A** and **B** is less for O′ than for O, the time between those events will have to be shorter as well, so that the speeds connecting them are the same. One way of doing that is to redraw the s-t diagram as in the figure to the right. In this figure the x'-axis is tilted up by the same angle that the T'-axis is tilted over. One triangle (shaded) in the figure has T'_B and $T_B - \beta x_B$ on adjacent sides, while a second (shaded) has x'_B and $x_B - \beta T_B$ on the corresponding adjacent sides. The angle between these adjacent sides is θ in both cases, so the triangles are similar. Thus, $\frac{T'_B}{T_B - \beta x_B} = \frac{x'_B}{x_B - \beta T_B}$. Now, we can write $u'_{AB} = \frac{x'_B}{T'_B} = \frac{x_B - \beta T_B}{T_B - \beta x_B} = \frac{u_{AB} - \beta}{1 - \beta u_{AB}}$. If $u_{AB} = 1$, then $u'_{AB} = \frac{1-\beta}{1-\beta \cdot 1} = 1$, also. This is just the right result!

Tilting the x'-axis is something completely new: it is due to **Einstein** who first proposed essentially doing this in 1905. Einstein's idea—now known as *special relativity*—has many observable consequences beyond getting the speed of light to be the same for all inertial observers, and we turn next to examining some of them.