Physics 3710 – Problem Set #3 *Relativistic kinematics, I*

1. GPS satellites orbit the Earth with a speed of about 4 km/s.

(a) It takes a time Δt_E for the satellite to make one orbit according to the Earth . How much time, Δt_S, does it take on board the satellite? Express your answer in terms of Δt_E and γ.

(b) Express the difference, $diff = \Delta T_E - \Delta T_S$, in terms of Δt_E and γ.

(c) Using the binomial expansion, show that, for small β, $1 - \frac{1}{\gamma} \approx \frac{1}{2}\beta^2$.

(d) Relative to an observer fixed to Earth, what is β for the satellite?

(e) Suppose Δt_E = 12 h exactly. What is the value of $diff$ in seconds?

(f) The smallest error in position that an uncorrected GPS system would produce in one orbit is $c \cdot diff$. How many meters is $c \cdot diff$? If the target error is less than 10 m, does the GPS system have to take special relativity into account?

2. As seen from Earth, a particle created at an altitude of 25 km (event A) decays just as it reaches Earth's surface (event B), 100 μs after creation.

(a) What is the lifetime in the Earth frame in units of km?

(b) What is the dimensionless speed of the particle re Earth?

(c) What is γ for the particle's frame re Earth?

(d) What is the lifetime of the particle in its own rest frame in km?

(e) How far is Earth's surface from the particle in the particle's rest frame at the moment of creation?