Structure of matter, 6

Quantum Flavor Dynamics (QFD), I

That each generation of the quark and lepton periodic tables (see SM 1, p.1) has two that can be flipped into one another by emission or absorption of W bosons is reminiscent of how the angular momentum spin-1/2 component of a charged particle can be flipped between "up" and "down" orientations along some direction (z) in space by emission or absorption of photons. This analogy is made more graphic by labeling flavor rows by a new kind of "spin" (completely unrelated to angular momentum), namely, *weak isospin component*, I_z. The neutrino flavors are assigned a weak isospin component value of +1/2 ("up"), and the negatively charged lepton flavors, –1/2 ("down"). Similarly, the positively charged uct quarks are assigned +1/2 weak isospin components, and the negatively charged dsb quarks, –1/2.

The local gauge theory story for the origin of the electromagnetic force of QED emerges from the indifference of nature to the exact value of the complex phase of the Dirac field describing a charged particle. Similarly, the local gauge theory story for the color force of QCD emerges from the indifference of nature to the exact color of the Dirac field describing a quark. That is, the "world" appears to be symmetric under continuous phase and color transformations. Requiring these transformations to be (local) functions of position and time leads to the necessity of having electromagnetic and color potential fields. The particles associated with these fields are spin-1 bosons that respectively "carry" the electromagnetic and color interactions. In other words, electromagnetic and color forces can be understood as arising from symmetry. It is natural to wonder if the world is similarly symmetric under (local) weak isospin–flavor changing–transformations. Of course, it can't *exactly* be because changing a muon into a muon neutrino or a d quark into a u quark produces change in mass. But, to start, let's ignore this potentially embarrassing fact.

What would a local gauge theory of flavor changing interactions look like? To start, leptons and quarks both have *angular momentum* spin-1/2. Their fields, ψ, therefore obey the Dirac Equation. In analogy with angular momentum spin, each part of the new ψ has two *additional* components corresponding to "up" and "down" weak isospin components. Conservation of weak isospin would require that the density of the lepton field, $|\psi|^2$, be invariant under isospin transformations represented by 2x2 matrices, S: $\psi' = S\psi$. (These are called **SU(2) isospin transformations**.) In the spirit of local gauge theory, these transformations can be functions of space and time—i.e., $S = S(\vec{r},t)$. Such local transformations produce unwanted derivatives-of-S in the Dirac Equation. To cancel these terms, additional (potential) fields, $\phi_I, \vec{A}_I$, are required which transform simultaneously with the lepton fields. The components of the potential fields are also 2x2 matrices. Because they obey Maxwell-like equations, their associated particles would be massless and have angular momentum spin = 1, just like photons and gluons.

When discussing the color force, we saw that the force carriers, the gluons, were 3x3 color matrices whose indices could be viewed as a color and an anti-color. In analogy with gluons, the weak isospin changing matrices introduced above can be thought of as having rows that are isospin values and columns that are anti-isospin values. In other words, the quanta of the isospin potential fields, generically called W, can be envisioned as having indices $W_{I,\bar{I}'}$.

Before proceeding, it is useful to consider how angular momentum behaves in quantum mechanics. The *magnitude* of a spin angular momentum vector can only have certain discrete values: $|\vec{S}| = \hbar\sqrt{s(s+1)}$, where s is 1/2 for "spin-1/2" and 1 for "spin-1". The projection along a direction in space, generically designated S_z, can have the values $\frac{-1}{2}\hbar$ ("down" or ↓) or $\frac{+1}{2}\hbar$ ("up" or ↑) for spin-1/2 (each of these has a magnitude $\frac{\sqrt{3}}{2}\hbar$) and $-h$, 0, or $+h$ for spin-1 (where each of these has magnitude $\sqrt{2}\hbar$). If two spin-1/2 vectors are combined in the right way a spin-1 vector can result. There are four possible +1/2 and –1/2 combinations depicted as: $\uparrow\uparrow, \downarrow\downarrow, \downarrow\uparrow, \uparrow\downarrow$. The $+h$ combination corresponds to $\uparrow\uparrow$, $-h$ to $\downarrow\downarrow$. According to quantum mechanics the two up/down pictures can also be combined into $\downarrow\uparrow + \uparrow\downarrow$ and $\downarrow\uparrow - \uparrow\downarrow$ combinations, both with z-component equal to zero. The difference between these two is that when the two arrows in each term are switched, the first combination doesn't change while the second gets a negative sign. Note that when the two arrows in the up/up and down/down are switched they don't look any different. In other words the set $\{\uparrow\uparrow, \downarrow\downarrow, \downarrow\uparrow + \uparrow\downarrow\}$ forms a spin "triplet" of spin=1 possibilities, while $\downarrow\uparrow - \uparrow\downarrow$ is spin=0 "singlet."

Now back to the W fields with two isospin indices. We can think of the W fields as having *four* possibilities $W_1 = W_{\uparrow\uparrow}$, $W_2 = W_{\downarrow\downarrow}$, $W_3 = W_{\uparrow\downarrow+\downarrow\uparrow}$, and $W_4 = W_{\uparrow\downarrow-\downarrow\uparrow}$. What do these fields do when they interact with a lepton? In analogy with the gluon fields and their color/anti-color indices, the one "index" is an isospin and the other "index" is an "anti-isospin." If both indices are the same as the isospin carried by the lepton then there is no interaction. If one is different, however, then the lepton's isospin is annihilated and replaced by the $W's$ other index.

Gell-Mann's early organization of the lightest baryons and mesons (SM 3, p.2) employed two hypothetical properties called the z-component of strong isospin (I_{Sz}) and strong hypercharge (Y_S), which were connected to electric charge, Q (in units of electron charge), by the equally hypothetical Gell-Mann-Nishijima formula: $Q = I_{Sz} + \frac{1}{2}Y_S$. The triumph of the quark model has rendered these hypothetical properties obsolete, but if weak isospin is a valid property of matter, perhaps so also is *weak hypercharge*, Y, and perhaps also $Q = I_z + \frac{1}{2}Y$. Thus, to get the neutral electric charge using this relation of the neutrinos and the negative electric charge of the other leptons requires they carry weak hypercharge = –1. In addition, the quarks must carry weak hypercharge = +1/3 (do you see why?). For now, let's assume all of the seemingly unmotivated ideas for the weak interactions are true.

Assuming the Gell-Mann-Nishijima formula in weak form applies to the W fields, and that *their hypercharge is zero*, the particles of W_1 should have electric charge = +1 and the quanta of W_2 should have electric charge = –1; that sounds a lot like the putative $W^{\pm}$ bosons discussed in SM 5 (p.2-3). So maybe this crazy idea of deriving the weak interaction from isospin switching symmetry has some merit after all. And if it does, the prediction is that *there should be electrically neutral weak exchange bosons* as well.

From a more sophisticated version of the model outlined above, Sheldon Glashow, Steven Weinberg, and Abdus Salam in the 1960s predicted a neutral weak-force carrying boson, Z^0 (in addition to the $W^\pm$ bosons) and *indirect* evidence for it was obtained in 1973. GWS were awarded the 1979 Nobel Prize for their prediction, even though *direct* experimental evidence for the $W^\pm$ and Z^0 didn't come until four years later. These particles were inferred from the analysis of collisions involving beams of protons and anti-protons at CERN in Switzerland, in 1983. In high-energy $p,\bar{p}$ collisions many things can happen, among them $p+\bar{p}\to W^\pm + e + \nu_e$, $p+\bar{p}\to Z^0 \to e^- + e^+$. In the first process the W^+ is accompanied by an electron and an electron anti-neutrino, while the W^- is accompanied by a positron and an electron neutrino. To infer the existence of the $W^\pm$ requires looking for events in which high-energy electrons or positrons emerge and not much else. The second process can be confused with the much more likely $p+\bar{p}\to\gamma\to e^- + e^+$. The probability of the latter is calculable using QED and this has to be subtracted from the measured yield to look for residual events not explainable by electrodynamics. In any case, billions of events were examined and about *ten* corresponding to each process were found. That was enough to convince the Nobel committee to award their Prize to CERN's Carlo Rubbia and Simon van der Meer. The masses of the $W^\pm$ are both about 80 MeV (the two particles are antiparticles of each other), while that of the Z^0 is about 90 GeV. As a consequence, if these particles are responsible for carrying the weak force, the associated range would have to be about 10^{-9} nm–about 1/1000 times the size of a nucleus. The weak force is not weak because its intrinsic strength (i.e., α_W) is small (it's actually, about four times stronger than the electromagnetic strength, α_E), but because particles have to be so close to interact via $W^\pm$ or Z^0 exchange. (Incidentally, the detection of a neutrino by the Cherenkov radiation produced when the neutrino kicks an electron out of an atom (see SM 5) is due to Z^0 exchange.)

Weak interactions violate parity symmetry

Before worrying about how mass screws up the flavor-changing symmetry, another important aspect of the weak interactions has to be reckoned with. Until 1956, it was common wisdom that the laws of physics worked equally well in the real world or in a mirror reflection of the real world. More precisely, it was believed that physical processes would be identical under the position-vector reflection, or "parity," transformation, $\vec{r}\to-\vec{r}$. Such a transformation has several implications: velocity switches direction, $\vec{v}\to-\vec{v}$; acceleration switches direction, $\vec{a}\to-\vec{a}$; because of Newton's Second Law and the fact that mass is independent of $\vec{r}$, force switches direction, $\vec{F}\to-\vec{F}$. Not all of the objects we traditionally call vectors transform this way. For example, angular momentum, $\vec{L}=\vec{r}\times m\vec{v}$, does not switch direction under parity transformation because $-\vec{r}\times m(-\vec{v})=\vec{r}\times m\vec{v}$. Magnetic field, $\vec{B}$, is another example; the magnetic force, $\vec{F}=q\vec{v}\times\vec{B}$, changes sign when $\vec{r}\to-\vec{r}$ and so does $\vec{v}$, thus, $\vec{B}$ does not. (Objects like angular momentum and magnetic field are more properly called *pseudovectors.*) So what does "physical processes are identical under parity transformation" mean? It means that if something is conserved in a process for $\vec{r}$, it will also be conserved in that process for $-\vec{r}$.

It seems so obvious that physics should be invariant under parity transformation that it's a wonder anybody would have suggested otherwise. In a 1956 paper, however, **C.N. Yang** (see *Appendix* below) and T.D. Lee pointed out that though electromagnetic and strong forces had been experimentally demonstrated to be insensitive to parity transformation, no similar experiments had yet been done involving weak interactions. They suggested several possible experiments to test this, including measuring the rate, in different directions, of decay products when spin-polarized ^{60}Co nuclei undergo beta decay: $^{60}Co \rightarrow {}^{60}Ni + e^- + \bar{v}_e$. In this process, a neutron in the cobalt nucleus becomes a proton in the nickel nucleus while the electron and antineutrino come out. The electrons are easily detected. The experiment consists of placing a sample of ^{60}Co in a strong magnetic field (to orient the nuclear spin) and measuring electron rates along the magnetic field direction and opposite it. The nuclear spin of ^{60}Co is $5\hbar$ "up" (i.e., in the direction of the external magnetic field) and the nuclear spin of the (excited state of) ^{60}Ni it decays into is $4\hbar$ (up). Thus, the electron and antineutrino have to emerge from the decay with opposite velocities to conserve momentum and with their $\hbar/2$ spins *both up* to conserve angular momentum. For a free fermion velocity and spin are either parallel ("right-handed") or antiparallel ("left-handed"). So when a ^{60}Co nucleus beta decays in this experiment the emitted electron emerges either in the direction of the nuclear spin as right-handed or opposite to it as left-handed.

To test parity invariance for this process, simply count the number of electrons emerging in the up direction versus the number emerging down. In a parity-transformed world, magnetic field does not change direction, nuclear spin (an angular momentum) does not change direction, but velocity does change direction. For parity transformation symmetry there must be equal numbers of right-handed electrons going up as left-handed ones going down. There's a technical problem to doing this experiment, though. To keep the nuclear spins aligned requires low temperature—about 0.003K! In a triumph of experimental design and execution "Madame" (see *Appendix* below) **C.S. Wu** accomplished this on December 27, 1956, and found that, amazingly, the decay rates were different in different directions. In fact, we now (with better experimental resolution) know that electrons *only* come out in the down direction. Beta decay is *not* symmetric under parity transformation. (This astonishing result yielded Yang and Lee the Nobel Prize. Unfortunately, like several of our previous stories of great work going unrewarded [i.e., Leavitt, Hubble, Alpher, Bell, Rubin], Madame Wu—who was the genius behind making the beta decay experiment work—did *not* share the glory.) Today, parity asymmetry has been established in *all* weak processes that involve neutrinos.

Since electrons only come out the south pole of the decaying nucleus in this experiment, anti-neutrinos must only come out the north pole. The electrons are always left-handed with spin up and momentum down. The anti-neutrinos must always be right-handed (spin up and momentum up). A similar experiment can be performed with ^{58}Co, which decays by "inverse beta decay" (involving a positron and a real neutrino). The directions of the emerging particle and antiparticle are reversed in this experiment indicating that only left-handed neutrinos and right-handed positrons are involved.

Appendix

C.N. Yang shared the Nobel Prize with his colleague T.D. Lee for proposing that the weak interactions might not obey parity symmetry. He is even more famous, perhaps, for inventing the first example of a local gauge theory for the description of forces other than electromagnetism. Though different from his original idea, local gauge theory (now often referred to as "Yang-Mills theory") is *the* theoretical structure underlying all of the Standard Model of Particle Physics.

C.S. Wu, widely known as "the First Lady of Physics," despite having a PhD and holding a full professorship at Columbia University preferred to called Madame Wu. Though she did not share the 1957 Nobel Prize with Yang and Lee (go figure!), she won numerous awards in her career including the Comstock Prize of the National Academy of Science, the Bonner Prize of the American Physical Society, the Wetherill Prize of the Franklin Institute, the U.S. National Medal of Science, and the (inaugural) Wolf Prize (often considered as prestigious as the Nobel). In 1995 (two years before she died) Yang and Lee established the Wu Chien-Shung Education Foundation in Taiwan in her honor to identify and encourage young scientists.