

1. A test mass attracted to Sun is dropped from rest at a distance of one Earth-Sun radius ($r_0 = 1.5 \times 10^{11}$ m). Suppose the Sun were a black hole. How long would it take for the test mass to reach the Schwarzschild radius according to an observer riding on the test mass? Use the proper time expression on page 2 of GR5. Give your answer in years.
2. How much time (measured in meters) does it take for a photon to travel from $1.0001r_s$ to $10r_s$ when there is no gravity (i.e., as in special relativity) according to an observer outside the photon? (That is, express T in terms of r_s .)
3. Repeat Problem 2, now for spacetime outside a blackhole described by Schwarzschild gravity. (Hint: start with $dr/dT = (1 - r_s/r)$ for a photon [page 1 GR5] and integrate to find T in terms of r_s .) Is T the same, smaller, or larger than in problem 2?
4. There is (probably) a supermassive black hole with mass equal to 4×10^6 solar masses at the center of the Milky Way galaxy.
 - (a) What is r_s for this black hole?
 - (b) Assuming Earth were orbiting the black hole at the same distance as it is from Sun (1.5×10^8 km—is that outside r_s ?) how long would a “year” be? (Take the orbital period to be the classical value $T = 2\pi\sqrt{\frac{r^3}{GM}}$.)