Physics 3710 – Problem Set #8 *Relativistic gravity, I*

1. On page 2, GR3, the gravitational shift in frequency as light travels vertically upward a distance L is given by $\lambda_d = (1 + gL/c^2)\lambda_e$ (in conventional units), where g is the surface gravitational field strength, and $L << R_E$. Calculate the fractional change in wavelength, $(\lambda_d - \lambda_e)/\lambda_e$, for light emitted at the base of the Khalifa Tower (in Dubai) and detected at its top, L = 828 m. (And yet this can be measured!)

2. Suppose an electron were a classical *point* mass. It would then have to be a black hole inside its Schwarzschild radius, r_S. What is r_S for an electron?

Problems 3 and 4 refer to: On page 2, GR4, equation (5) describes the shape of free fall orbits in Newtonian gravity. Earth orbits Sun in almost a circular orbit with speed 30 km/s. For Sun, r_S = 3 km.

3. Show that for a circular orbit of radius r and orbital speed v_ϕ, $L/mc = (v_\phi/c)r$. What is the numerical value of L/mc for Earth?

4. Using the latter result, show that for a *circular* orbit equation (5) becomes $2E/mc^2 = (v_\phi/c)^2 - r_S x$. A general result for circular orbits is that the total energy equals the negative of the kinetic energy. Is that numerically correct for Earth? (That is, which term on the right corresponds to kinetic energy?)