

General relativity, 9

Cosmic development

As discussed in GR 8, the cosmic scale factor a in the FLWR s-t obeys the Friedmann equation $\left(\frac{da}{dT}\right)^2 - \frac{8\pi G}{3c^2} \rho a^2 = -\frac{k}{r_0^2}$, with $\rho = \rho_C \left[\left(\Omega_r/a^4\right) + \left(\Omega_m/a^3\right) + \Omega_v \right]$. Recent data from the WMAP satellite indicates that ($\pm 2\%$) $\Omega_v \approx 0.72, \Omega_m \approx 0.28, \Omega_r \approx 5 \times 10^{-5}$. (See table at the end.) This implies that, if we set $a = 1$ (today), $\rho \approx \rho_C$ and $k \approx 0$. One interpretation of this is that the FLWR s-t is infinite in (spatial) extent.

The full time-course for how a changes can be obtained by integrating the $k = 0$ Friedmann equation:

$$\int_0^a \frac{da'}{a' \sqrt{\frac{\Omega_r}{a'^4} + \frac{\Omega_m}{a'^3} + \Omega_v}} = T \left(\frac{8\pi G \rho_C}{3c^2} \right)^{1/2} = tH_0.$$

It is instructive to evaluate the time between $a = 0$ and $a = 1$, i.e., the “age of the universe,” if, in turn, only radiation, only matter, and only vacuum-energy were ever present. In each case, we set the relevant $\Omega = 1$ and the others to zero. For a radiation-only filled universe, the integral on

the left is just $\int_0^1 \frac{da'}{a' \sqrt{\frac{1}{a'^4}}}$, which results in

$$t_{rad,only} = 1/(2H_0) = 6.99 \times 10^9 \text{ y} \quad (1)$$

about half of the value found in the table at the end of these notes. If matter were the only source

of gravity, the integral would be $\int_0^1 \frac{da'}{a' \sqrt{\frac{1}{a'^3}}}$, yielding a time

$$t_{mat,only} = 2/(3H_0) = 9.31 \times 10^9 \text{ y}, \quad (2)$$

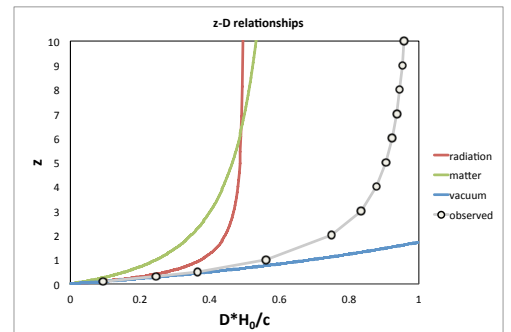
again, far smaller than the value in the table. Finally, in a vacuum-energy only universe, $\int_0^1 \frac{da'}{a'}$,

with

$$t_{vac,only} = -\ln(0)/H_0 = \infty \text{ y}, \quad (3)$$

this time, wildly greater.

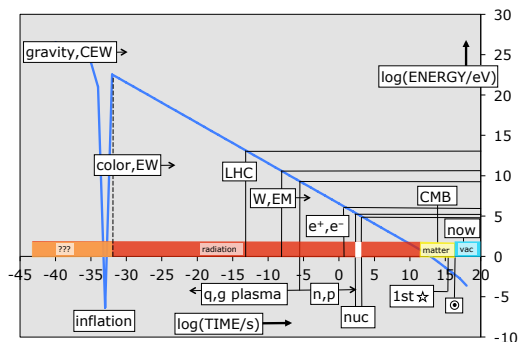
If the integrals above are evaluated from a to 1 instead of from 0 to 1, the times on the right hand sides are the times (for each energy density scenario) before the present that a had its value. Multiplying those times by the speed of light yields the distances light would have to travel to reach us from a source when the universe length scale factor was a . This allows a plot of $z = 1/a - 1$ versus distance to be made for the three different energy density scenarios. Such a plot is shown to the right. On



it, “observational” data from an empirical formula for galactic z versus D is included. (Note that there is currently a slight difference in D values depending on how distance is measured.) Clearly, observed values of galactic z are much smaller at very large distances than can be accounted for by radiation and/or matter only. Moreover, an evaluation of the complete integral above with all three energies contributing with their current values of Ω shows that $t_{\text{now}} \approx 1/H_0 \approx 13.8$ billion years. As the oldest known stars (ones in our own galaxy) are at least 13 billion years old (as dated by isotopic abundances) the contribution of vacuum (“dark”) energy is essential for making sense of this observation.

The radiation epoch

The radiation temperature varies as $1/a$. Because a is smaller earlier, the temperature is also higher earlier. Indeed as $a \rightarrow 0$ going backward in time the temperature initially must have been exceedingly high—so high that electrons could not have been bound to nuclei in neutral atoms, nor could protons and neutrons have been bound in stable nuclei. In the earliest moments, the state of matter in the universe must have been very different from what we observe about us now. It can be argued that the radiation epoch is where all the cosmic action is. This is again emphasized by the logarithmic history graph first found in BK1 (see right). In it, the radiation epoch starts at the Planck time, 10^{-43} s, and continues until about 10^{12} s, where transitions into the epoch of ordinary and dark matter begin. In logarithmic time the radiation epoch spans the vast majority of cosmic history. Trying to understand aspects of the structure and evolution of the earliest phase of the universe is the primary goal of the last portion of this course.

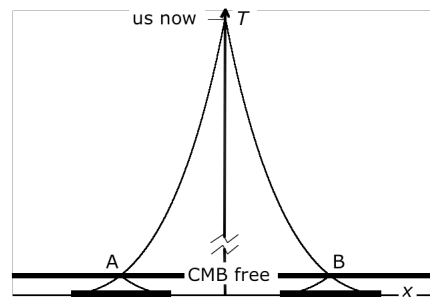


We know a few important things about the universe soon after it emerged from the radiation epoch: matter consisted of protons and neutrons bound in a small number of different light nuclei (mostly hydrogen and helium), along with electrons bound to the nuclei in neutral atoms; blackbody photons—the CMB—permeated the universe. There must also have been dark matter and vacuum energy, the former being immediately important for the evolution of a , the latter only important much later. Eventually, gravity (enhanced by dark matter) organized clouds of hydrogen into stars and planets and galaxies, and ultimately chemistry made complex molecules and life. Despite the apparent simplicity of the post-radiation epoch, a number of questions persist. If the initial state of matter was pure energy (extremely high energy “photons”), it is quantum mechanically mandatory that particle-antiparticle pairs would have followed. Electrons, protons, and neutrons are “matter” not “antimatter.” So where did all the antimatter go? Why are the only electromagnetically active particles electrons, protons, and neutrons? What is dark matter? What is vacuum energy?

(As noted in GR8, other potentially very informative signals should have leaked out of the dense plasma of the radiation epoch earlier than the origin of the CMB if their carriers did not interact with the photons, electrons, and nuclei then present. A prime candidate for such a signal would be carried by neutrinos—particles whose primary interaction with matter is via the weak force. The last significant scattering of neutrinos would have occurred at a much higher temperature than the recombination temperature, 3×10^3 K, when they and other particles would have been much more energetic. Traveling more-or-less unhindered to us near the speed of light such neutrinos would form a “past neutrino-cone” that would define a larger and earlier “neutrino-

visible” universe. Unfortunately, we don’t yet know how to detect these evasive particles. Like the CMB photons, such neutrinos will be very red-shifted and not very energetic—so not good candidates for initiating nuclear reactions, which is one way they are detected at present.)

And there is a mystery regarding the CMB. Consider the s-t diagram to the right. What is depicted is the detection of CMB photons at Earth (at $x = 0$) now coming from two points, A and B, that are 180° opposite one another in the sky. The coordinate x orders events in space. Currently, x -hash marks are a certain physical distance apart; earlier, that distance is less by a factor of a . Light travels at speed = 1 = (physical distance)/(time).



Earlier, light traveling toward us would have crossed more x -hash marks per unit time than now. That’s why the past light cone for us now has curved sides. Events A and B happened just when the CMB became free of the primordial plasma. The past light cones for events A and B are depicted assuming that a varies earlier due only to radiation energy dominance. The thick bars along the x -axis represent all of the events that might have caused A and B in this scenario. The bars don’t overlap. That means that in this scenario A and B share no common ancestral event: they are causally independent.

In the earliest fractions of a second after $a = 0$ there must surely have been wild quantum/thermal fluctuations. Thus, it is reasonable to expect that the thermal spectrum of CMB photons in the A part of the sky should in general be quite different from that in the B part. But, as mentioned in GR8, the CMB is extraordinarily smooth all over the sky. This puzzle is often referred to as the “horizon problem.” Something must have happened early on (before the freeing of the CMB) to erase the wild variability of the universe’s “birth.”

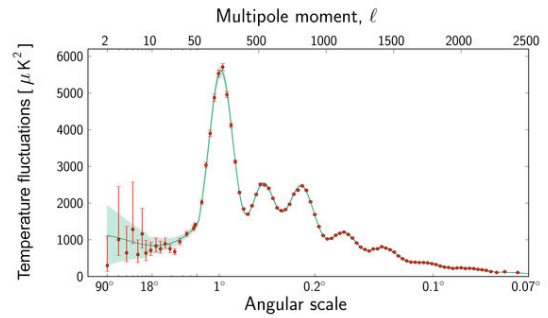
Inflation

The leading candidate for this erasure is “inflation.” One “explanation” for inflation is that the vacuum energy, Ω_V , actually consists of two parts: $\Omega_V = \Omega_{V, \text{today}} + \Omega_{V, \text{early}}$. The “early” part, in this story, was much larger than the “today” part, but, at some cutoff time, t_C , dropped to a smaller value. As a increased, in this scenario, Ω_r / a^4 fell below $\Omega_{V, \text{early}}$ at a time, t_I ($t_{\text{plank}} < t_I < t_C$),

producing a solution to the Friedmann equation similar to $a(t) = a_I \exp\left[\sqrt{\Omega_{V, \text{early}}} H_0 (t - t_I)\right]$; t_I is the onset of inflation (at about 10^{-35} s on the history graph). In other words, during this epoch a increased exponentially rapidly, from $a(t_I) = a_I$ to $a(t_C) = a_I \exp\left[\sqrt{\Omega_{V, \text{early}}} H_0 (t_C - t_I)\right]$. Now,

depending on what the values of t_I , t_C , and $\Omega_{V, \text{early}}$ were, $a(t_C) / a_I$ might have been 10^{25} (or greater). Such a rapid stretching of length scale would have allowed the past light cones of events A and B above to overlap. It would have had the effect of pulling apart regions of unusually high density—much like what happens to dots of ink on the surface of a rapidly inflating balloon. It would also produce a plummeting of the radiation temperature. In the usual inflation scenario, the “stuff” supplying the $\Omega_{V, \text{early}}$ (the so-called “*inflaton* field”) might, at sufficiently low temperature, be radically “supercooled,” and as a consequence might have undergone a phase transition (similar to a supercooled vapor that becomes a liquid). The “latent heat” released in the phase transition could then have “reheated” the universe to a high temperature, resulting in many highly energetic particles, especially high-energy photons. In the inflation hypothesis, shortly after t_C (about 10^{-33} s on the history graph), radiation would have again ruled, but with much tamer fluctuations. In fact, inflation predicts that large spatial scale fluctuations should now be more

prevalent than small ones and that prediction is exactly borne out by WMAP and Planck data (the latter shown to the right; the dots are data, the solid curve is the prediction; note that the spatial scale, as measured by “angular size,” gets smaller to the right). As mentioned previously, the distribution of CMB fluctuations provides an additional, independent corroboration of the spatial flatness of the universe. The large bump at a size of about 1° corresponds to patches of similar temperature that are equal to how far light could have traveled between $a \approx 10^{-35}$ (inflation) and $a_{CMB} \approx 10^{-3}$ (CMB released) in a radiation dominated universe with $k = 0$. No patches larger than this would be expected because that would require a correlating signal traveling faster than light.



<http://sci.esa.int/planck/51555-planck-power-spectrum-of-temperature-fluctuations-in-the-cosmic-microwave-background/>

Primordial nucleosynthesis

Before about $a = 10^{-8}$, the temperature of the radiation field would have been so high that nuclei could not be bound. The constituent neutrons and protons would be pulled apart by collisions with other relativistic particles (including photons, of course). A short time earlier the universe would be filled with photons, neutrinos, electrons, neutrons, and protons. This state of matter is the primordial goo that George Gamow (Ralph Alpher’s PhD mentor) called “Ylem,” supposedly descendent from the ancient Greek word, *hule*, meaning “matter.” The point at which the Ylem particles are in thermal equilibrium occurs just a few 10s of seconds after $t = 0$.

Fusing neutrons and protons together forms heavier nuclei. The fundamental building block for all nucleosynthesis (the sequential formation of heavy nuclei) models is the deuteron, ${}^2\text{H}$, the isotope of hydrogen consisting of 1 proton and 1 neutron, formed by $n + {}^1\text{H} \leftrightarrow {}^2\text{H}$ (where ${}^1\text{H}$ designates the proton in “nuclear physics speak”). Fusion occurs only when the nucleons approach within about 10^{-13} m of one another. To obtain any significant rate of deuteron formation requires high density of the reactants. The fusion reaction competes with a second, dissociation reaction, namely, $\gamma + {}^2\text{H} \leftrightarrow n + {}^1\text{H}$. The binding energy of the deuteron is only about 0.2 MeV, so if the radiation field contains a significant density of photons with energy above 0.2 MeV, ${}^2\text{H}$ won’t exist long enough for additional fusion to take place. As described in GR8, the photons in the CMB become frozen in when the rate of change of photon density due to reactions with atoms falls below the rate of change of photon density due to cosmic expansion. A similar argument can be applied to the density of ${}^2\text{H}$. It changes due to the reactions above and also due to expansion. When a is small (less than about 5×10^{-8}), photons are hot and the dissociation reaction (photon plus deuteron) dominates. When a is too small very little ${}^2\text{H}$ can form. As a gets bigger, the neutron-proton reaction begins to dominate and deuterium accumulates. At bigger a still, expansion becomes dominant; at some point nuclear material gets too dilute and the fusion stops. The window of opportunity for nucleosynthesis lasts for only a few minutes. What emerges from this window is very sensitive to the ratio of photons to the sum of neutrons and protons at the start. Too many photons and almost no heavy nuclei emerge; too few photons and the chemical composition of the universe would be drastically different from what it is now.

In his 1948 PhD dissertation, Alpher did the first quantitative calculation for Big Bang nucleosynthesis. He assumed that after $n + {}^1\text{H} \leftrightarrow {}^2\text{H}$, a sequence of neutron capture processes led to $n + {}^2\text{H} \leftrightarrow {}^3\text{H} \rightarrow {}^3\text{He} + e^- + \bar{\nu}_e$, $n + {}^3\text{He} \leftrightarrow {}^4\text{He}$, and so forth. The putative next capture reaction, $n + {}^4\text{He} \leftrightarrow {}^5\text{He}$, does “not” happen because ${}^5\text{He}$ decays in about 10^{-21} s, far too fast to serve as a step in building higher mass nuclei. So the neutron-capture scenario dead-ends at ${}^4\text{He}$. Nevertheless, Alpher was able to show that such a narrow window of fusion could account for the observed cosmic abundances of ${}^1\text{H}$, ${}^2\text{H}$, ${}^3\text{He}$, and ${}^4\text{He}$. His work demonstrated that the curious observed disproportion between hydrogen and helium could be explained naturally assuming a hot early universe—a monumentally important contribution to modern cosmology.

We now know that neutron capture is *not* the correct fusion sequence. After all, there is good evidence that tiny amounts of lithium and beryllium also form in Big Bang nucleosynthesis. Once some ${}^2\text{H}$ is around several additional reactions occur fairly rapidly: ${}^2\text{H} + n \leftrightarrow {}^3\text{H}$, ${}^2\text{H} + {}^1\text{H} \leftrightarrow {}^3\text{He}$, ${}^2\text{H} + {}^2\text{H} \leftrightarrow {}^4\text{He}$, ${}^2\text{H} + {}^2\text{H} \leftrightarrow {}^3\text{He} + n$, and ${}^2\text{H} + {}^3\text{H} \leftrightarrow {}^4\text{He} + n$. Note that some ${}^4\text{He}$ can also be formed by ${}^3\text{H} + {}^1\text{H}$ and ${}^3\text{He} + {}^2\text{H}$. Though the short lifetime of ${}^5\text{He}$ is still a problem, apparently the occasional collision of ${}^3\text{H}$ and ${}^4\text{He}$ can make a little ${}^7\text{Li}$, for example. Other similar reactions are also possible, but time runs out on all of these in short order, and the cosmos is left with only traces of anything heavier than ${}^4\text{He}$. Given our present knowledge of these reactions we can predict the abundance ratios for ${}^2\text{H}/{}^1\text{H}$, ${}^3\text{He}/{}^1\text{H}$, ${}^4\text{He}/{}^1\text{H}$, and ${}^7\text{Li}/{}^1\text{H}$ as functions of the cosmic photon/proton ratio. The measured values of these ratios suggest the latter should be roughly 1.5×10^9 and (as previously mentioned) recent measurements by the WMAP satellite put the value at 1.6×10^9 , a *strong corroboration of the primordial nucleosynthesis hypothesis*.

What at first might have seemed like a cockamamie picture of the universe—that is, the Friedmann-Lemaître-Robertson-Walker space-time picture—in reality has a very impressive set of credentials. These include:

- (a) It explains Hubble’s observed z versus distance rule and, more importantly, why z is observed to be so much larger than the Hubble rule prediction at large distances.
- (b) It resolves the old puzzle of why the night sky is dark even though the universe might be infinite in all directions (i.e., we only see a piece of it, as shown in the light cone figure above).
- (c) It can be extended to predict, as has now been confirmed to great precision, the existence of the Cosmic Microwave Background.
- (d) And, finally, it can be extended to predict the now observed ratios of the light elements provided the photon-to-baryon ratio is as what is observed.

Though alternative theoretical structures for addressing each of these points have been proposed, none has been able to adequately account for all in such an economic and quantitatively accurate way. That the universe had an initial hot, dense phase is almost certainly correct.

Parameters associated with the FLWR cosmological model (as of October 2013)

$$H_0 = \left(\frac{1}{a} \frac{da}{dt} \right)_{\text{present}} = 70.0(\pm 2.2) \text{ km/s/Mpc} = 2.28 \times 10^{-18} \text{ s}^{-1} (\pm 3.1\%)$$

$$1/H_0 = 13.97 \times 10^9 \text{ years, "age"} = 13.77(\pm 0.07) \times 10^9 \text{ years}$$

$$\rho_C = \frac{3H_0^2}{8\pi G} = 9.29(\pm 0.20) \times 10^{-27} \text{ kg/m}^3 = 5.55(\pm 0.12) \text{ protons/m}^3 = 5.21(\pm 0.11) \text{ GeV/m}^3$$

$$\Omega = \rho/\rho_C = \Omega_r + \Omega_m + \Omega_\nu = 1.000(\pm 0.050)$$

$$N_{\text{photons}} / N_{\text{protons}} = 1.616(\pm 0.027) \times 10^9$$

$$\Theta_{\text{CMB}} = 2.725(\pm 0.002) \text{ K}$$

$$E_{\text{CMB photon}} = 6.347(\pm 0.031) \times 10^{-4} \text{ eV}$$

Component	$\Omega = \rho/\rho_C$	kg/m³	protons/m³	GeV/m³
<i>EM radiation</i>	5.04(±0.02)×10 ⁻⁵	4.64×10 ⁻³¹	0.277×10 ⁻³	0.261×10 ⁻³
<i>Luminous matter</i>	0.0463(±0.0016)	4.36×10 ⁻²⁸	0.258	0.241
<i>Dark matter</i>	0.233(±0.015)	2.23×10 ⁻²⁷	1.341	1.257
<i>Dark energy</i>	0.721(±0.017)	6.93×10 ⁻²⁷	4.149	3.892
<i>Neutrinos</i>	less than 0.013			

These data are from WMAP9; data from Planck released in March 2013 are very close to those of WMAP, differing most notably in a slight reappraisal of the age of the universe. The Planck data are still being reexamined for possible systematic errors in one of the satellite's instruments.