

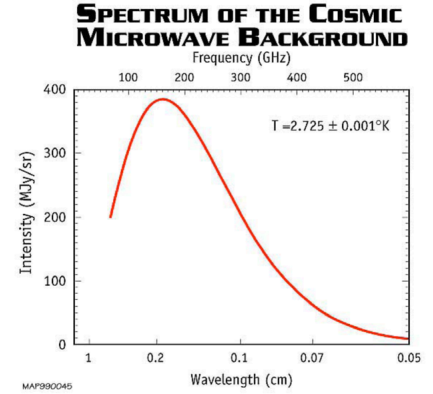
General relativity, 8

The Cosmic Microwave Background (CMB)

As previously noted, the universe is filled with microwave radiation. The frequency spectrum of this ubiquitous radiation follows a blackbody curve, as shown to the right.

(<http://map.gsfc.nasa.gov/media/ContentMedia/990015b.jpg>)

Note that photon energy (proportional to $1/\text{wavelength}$) increases to the right. You might think the curve shown is the plot of a theoretical equation, but what is shown is actual measured data taken during the flight of the COBE (Cosmic Microwave Explorer) satellite/microwave observatory in 1990. The uncertainties in the measurements are about the thickness of the curve plotted. When compared with a theoretical blackbody curve the disagreement is less than one part in 2000. The CMB isn't LIKE a blackbody spectrum—it IS a blackbody spectrum! Note that the maximum in the CMB spectrum is at a wavelength of about 2 mm. The energy of a photon of that wavelength is $E = hc/\lambda \approx 6.6 \times 10^{-4}$ eV.



Where does a blackbody spectrum come from? A blackbody spectrum results from a mish-mash of processes—primarily atomic and molecular transitions and Doppler shifts. Though transitions between bound states are associated with discrete colors, Doppler shifts and transitions involving unbound states can produce a broad blurring of such “lines,” with the end product being an energy density-frequency relation,

$$\rho(f, \Theta) = \frac{8\pi hf^3}{c^3} \frac{1}{\exp(hf/k_B\Theta) - 1}, \quad (1)$$

that is characterized by a single parameter—temperature, Θ (we reserve the symbol T to refer to time) measured in kelvins (K)—and is independent of the material from which the radiation emerges. The quantity k_B is Boltzmann’s constant, 8.617×10^{-5} eV/K. The temperature parameter fixes both the energy density of the radiation and the shape of the frequency distribution. The maximum in the distribution occurs at the photon energy $hf_{\max} = 2.82k_B\Theta$. As the measured value of hf_{\max} is about 6.6×10^{-4} eV, Θ must be about 2.7 K. Very careful fitting of (1) to the measured values shown in the figure above leads to $\Theta = 2.725 \pm 0.001$ K. If (1) is integrated over all frequencies we get the energy per unit volume, ρ_r , associated with blackbody radiation at temperature Θ :

$\rho_r(\Theta) = (8\pi^5/15) [k_B^4/(hc)^3] \Theta^4 = 4.725 \times 10^3 \Theta^4$ (eV/m³). Dividing (1) by hf gives the number of photons per unit volume per unit frequency,

$$n_\gamma(f, \Theta) = \frac{8\pi f^2}{c^3} \frac{1}{\exp(hf/k_B\Theta) - 1}, \quad (2)$$

so the integral of that over all frequency produces the density of photons, n_γ :

$n_\gamma(\Theta) = (19.23\pi)(k_B/hc)^3 \Theta^3 = 2.028 \times 10^7 \Theta^3$ (ph/m³). For $\Theta = 2.725$ K, $\rho_r \approx 0.26$ MeV/m³ (the energy density for luminous matter is about 0.24 GeV/m³, about 1000 times greater) and $n_\gamma \approx 4.1 \times 10^8$ photons/m³ (the number density for luminous matter is about 0.26 proton/m³—i.e., about 1.6×10^9 photons per proton). The energy of the *most likely* photon (where (2) has its maximum) is $1.59k_B\Theta$, which is 3.7×10^{-4} eV at 2.725 K.

In GR7 we noted that $\rho_r \propto 1/a^4$; but, for a blackbody $\rho_r \propto \Theta^4$, so it must be that the blackbody temperature varies as $1/a$. In GR7, we found that the energy density in relativistic particles would equal the energy density in matter when (at a time T_R) the scale of the universe was $1/3200$ ($=3.1 \times 10^{-4}$) of what it is presently. At that time the temperature of the CMB would have been about 8.7×10^3 K. The energy of the photons most likely to be encountered in such a blackbody field is about 1.2 eV. Photons only interact with luminous matter and today luminous matter is mostly neutral atomic hydrogen (over 90% of the atoms in the universe). The minimum energy required to excite hydrogen from the ground state is 10.2 eV and the minimum energy required to ionize is 13.6 eV. So it might seem like radiation at T_R is too cool to interact much with matter. But, as shown above, the blackbody spectrum has a long high-energy tail. Equation (2) can be used to calculate the density of photons with energy above 10.2 eV at any temperature, and for $\Theta = 8.7 \times 10^3$ K the density of such photons would be about 10^5 times greater than the density of atoms. Indeed, at T_R all of the atoms in the universe must have been ionized. The mean free path for photons in such an ionized cloud would be much smaller than the size of the then “visible” universe. Photons would be trapped in this hot, randomly fluctuating “soup”; such a medium is a perfect environment for creating a blackbody spectrum. Thus, it is likely that the origin of today’s CMB spectrum occurred at a time in the development of the universe when radiation was hot enough to ionize hydrogen.

But the CMB photons we detect today do not come directly from the time T_R . To see why, consider the ionization/recombination reaction $H + \gamma \leftrightarrow p^+ + e^-$. Here, H stands for “neutral hydrogen atom,” γ for “photon,” p^+ for “proton,” and e^- for “electron.” The number density of photons changes in time in two ways: by reaction and by expansion. Thus, $\dot{n}_\gamma = (\dot{n}_\gamma)_R + (\dot{n}_\gamma)_a$, where the last term is due to the increase of the universe’s scale factor ($\dot{n} = dn/dt$). Today, the last term dominates; the number of photons is constant, but the volume they occupy is increasing. At T_R , however, the reaction term overwhelmed expansion, and the reaction must have been in equilibrium. The photons observed in the CMB come from a time, T_{CMB} , when reaction became less important than expansion. Best estimates suggest that this crossover would have occurred for a about 10^{-3} of the present value, and at a temperature of about 3×10^3 K. At that point, the cosmic photons became “frozen in” (number fixed) with a spectrum corresponding to a blackbody.

Note that, for such an a ($=1/(1+z)$) the redshift factor z is 1000. Recall that the larger is z the older is the light, and that the largest observed galactic z is about 10. In other words, the CMB spectrum observed today can be used to infer the state of matter in the universe at a much earlier time than when galaxies or even stars were present; it is a precious vestigial treasure telling us about the ancient days of the cosmos. (Though not the earliest. There should be other such treasures—such as relic neutrino and gravitational wave backgrounds—but so far our extremely limited ability to detect such things means that we are unable to take advantage of their existence. Should that situation change in the future we will then have an incredibly more detailed picture of the state of the even earlier universe.)

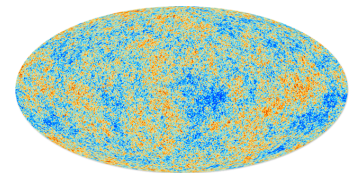
The CMB spectrum is an exceedingly well-confirmed observational fact; from it important inferences about an earlier cosmic epoch can be extracted. This perspective, however, is the reverse of the historical order of things: the blackbody CMB was actually predicted theoretically many years before it was first observed. In 1948, **Ralph Alpher**, then a youngish post-doc at Johns Hopkins, published a theoretical argument that a blackbody CMB should exist at the present time if

the universe “started” in an extremely hot state of pure radiation—the so-called “Big Bang.” In his PhD dissertation, presented earlier the same year, Alpher calculated that in a cooling Big Bang universe a handful of light nuclear species—predominantly hydrogen and helium—could be synthesized in the first few minutes. His calculated values agreed well with the observed cosmic (not stellar) abundances of those elements. In extending his calculations beyond the epoch of cosmic nucleosynthesis, Alpher realized that there would be a time when radiation would not be able to interact with matter much and that the state of radiation at that moment would be preserved as a kind of “relic” of the “hot, old days.” He showed that the present temperature of this relic radiation is determined by the energy levels in hydrogen and by the present number density of nucleons (i.e., luminous matter). In 1948, the density of luminous matter was not as well-known as it is today, but based on the then accepted value, Alpher predicted that the relic blackbody radiation should have a temperature of about 5 K. Though he tried hard to persuade experimentalists to look for this radiation, none, at the time, thought that it would be intense enough to measure.

For 17 years, the issue of the CMB lay dormant—forgotten, in fact, by the scientific community. Then, in 1965 Arno Penzias and Robert Wilson, while working on a new satellite communication system for Bell Labs in New Jersey, observed that despite heroic efforts to get rid of it there was always an irritating hiss in their radio telescope. Moreover, the hiss did not seem to depend on where their receiver was pointing, indicating possibly that it was coming from the receiver itself. They cooled their apparatus and fussed over it—even climbed inside to sweep out pigeon droppings that had accumulated in the “horn.” Nothing helped. They found that the noise in their system had a blackbody spectrum (though with their apparatus they could only measure part of the spectrum) with a temperature of about 3 K. Unwittingly, Penzias and Wilson had stumbled on Alpher’s CMB. Despite the blind serendipity of their discovery, they were justifiably awarded the 1978 Nobel Prize in Physics. (It is a historical curiosity that two of the most profound discoveries of twentieth century physics were discovered by accident at Bell Labs—the unwitting CMB discovery and, 40 years earlier, the totally fortuitous observation by Davisson and Germer that electrons are both particles and waves.)

Corroborated now by many high quality space-based measurements (especially those from COBE, WMAP [Wilkinson Microwave Anisotropy Probe] (2003), and, most recently, Planck (2009)), Alpher’s prediction of the CMB arguably stands as one of the giant contributions to modern science. Though over the last 60 years many have tried to produce one, there is no compelling explanation for the CMB other than that the universe must have had a hot beginning. Long ignored, Alpher’s seminal work is beginning to gain ever-greater mention in books and historical papers. Unfortunately, Alpher died in August 2007 (two weeks after being awarded the National Medal of Science) without receiving the much-deserved recognition of a Nobel Prize. The 2006 Nobel Prize was awarded to John Mather and George Smoot, principal investigators on the COBE mission, “for their discovery of the blackbody form and anisotropy of the cosmic microwave background radiation.” While the COBE results undoubtedly deserved recognition, the Nobel committee could have rectified its slight of Alpher by including him this one last time—but chose not to.

A last point about the observed CMB: COBE, WMAP, and Planck show that the CMB is *extraordinarily smooth*. The false color image to the right shows data from Planck recording variations of the CMB temperature from the overall average value of 2.725 K; the difference between the “warmest” and “coolest” regions of the CMB are on the order of 10^{-4} K. This is an amazing result because it says vastly separated portions of the universe are strongly correlated with one another. Exactly how this correlation arose is still a matter of debate. A frequently invoked, but still *highly* debated, explanation is “inflation,” the hypothesis that during a very brief period early in the history of the universe the length scale (a , in the FLWR



solution of Einstein's equations) expanded exponentially rapidly, thus wiping out any strong variations.

FLWR dynamics

We have seen that the form of the FLWR proper time contains several important qualitative features: (1) the possibility of describing a nonstationary universe whose expansion or contraction is encoded in a time-varying cosmic scale factor a , which, coupled with Hubble's galactic redshift measurements, appears at this time to be increasing; (2) a simple explanation for the Hubble law relating the apparent recessional velocity of galaxies to their distance of separation; and (3) a plausible explanation for why the universe is bathed in a more-or-less uniform sea of blackbody radiation. To more convincingly demonstrate the validity of the FLWR s-t picture, we need to know more about how a depends quantitatively on what the universe contains.

To determine a completely requires using Einstein's full general relativistic field equations. It is sufficient for our purposes to simply note the result: if the energy density in the universe is a known function of length scale only, a satisfies the single ordinary differential equation

$$\left(\frac{da}{dT}\right)^2 - \frac{8\pi G}{3c^2}\rho a^2 = -\frac{k}{r_0^2}. \quad (3)$$

Equation (3) is often referred to as the *Friedman Equation*. In (3), a and ρ (the total mass-energy density in the universe) are functions of time. The right hand side of (3) is the spatial curvature that appears in the FLWR proper time. Usually, of course, one would take the square root and write da/dT by itself on one side of the equation, but expressing (3) as shown allows us to make contact with Newtonian gravity (sort of). In particular, suppose we, located at $r=0$, are inside a uniform sphere of mass of radius $X=ar_0$ (the physical radius of the universe when the FLWR length scale is a) looking at another galaxy not too far from us (so that its recessional speed is much less than the speed of light) at a physical distance $x=ar$, where r is the constant coordinate distance from us. Multiply (3) by the constant $\frac{1}{2}mr^2$, where m is the mass of the galaxy we are watching, and switch to time t in s instead of T in m. Equation (3) then becomes

$$\frac{1}{2}m\left(\frac{dx}{dt}\right)^2 - \frac{GM(x)m}{x} = -\frac{1}{2}mc^2k\frac{r^2}{r_0^2},$$

where $M(x)=4\pi\rho x^3/3$ is the mass-energy inside a radius x surrounding us. The first term on the left hand side of this equation is recognizable as the Newtonian kinetic energy of the galaxy; the second term looks like the gravitational potential energy, but now due to the interaction of the galaxy with all of the mass *and energy* within a sphere of radius x . The interpretation of the right hand side is plausibly the galaxy's total mechanical energy. The right side is negative if $k=+1$, corresponding to a universe that is closed with positive spatial curvature. This is analogous to the case for closed Newtonian planetary orbits with negative total energy (i.e., with speeds slower than the escape speed). The right side is positive if $k=-1$, similar to open (escape) hyperbolic Newtonian orbits. The case, $k=0$, is analogous to the "just-barely-open, escape-energy" parabolic Newtonian orbits.

Note that (3) allows for empty universes, i.e., with $\rho=0$, that are spatially curved. Such a universe with $k=-1$ would expand or collapse at a constant rate. As mentioned briefly in GR7, when Einstein discovered this (in 1917), he thought that a time-changing universe was physically (and philosophically) absurd and as a result introduced the now-famous modification to his field equations, a "cosmological constant," Λ . Thus, Einstein's new Friedman Equation is

$(da/dT)^2 = 8\pi G\rho a^2/3c^2 - k/r_0^2 - \Lambda a^2$. A positive cosmological constant makes it possible to have an empty universe ($\rho = 0$) with $k = -1$ that is static ($da/dT = 0$), an esthetically more appealing situation for Einstein. Of course, after Hubble demonstrated the possibility that indeed the universe was expanding, Einstein famously backed off from the cosmological constant idea (his “worst mistake”). It appears now, however, that this term might be a significant player in the fate of the universe after all—though not quite as Einstein had envisioned. Note, incidentally, that the cosmological constant term has the effect of adding to the density of real mass-energy a constant energy density: $\rho_\Lambda = -\Lambda \frac{3c^2}{8\pi G}$. This is energy “from nothing”—a kind of “vacuum energy.” Such an effective constant energy density is *negative* if $\Lambda > 0$. We’ll come back to the cosmological constant later.

To determine a and k you have to insert a functional form for ρ into (3), integrate, then satisfy some initial conditions. Unfortunately, that’s not easy because, as stated previously, we can only make measurements here and now. Our knowledge of what the universe is made of and what its history has been is incomplete, at best. Formally, at least, the density ρ in (3) can be expressed as a sum of three parts: $\rho = \rho_m + \rho_r + \rho_v$, where ρ_m is due to *all* nonrelativistic mass (including dark matter), ρ_r is due to all relativistic particles (photons and neutrinos), and ρ_v is a constant “vacuum” contribution, whatever that might be. (For example, it might be, or least include, ρ_Λ .) There is *at this moment* a critical value of total density that would make the universe spatially flat. To find that value, set $k=0$ in (3), then solve for ρ using the current time: $\rho_c = 3H_0^2 / 8\pi G$, where

$H_0 = \left(\frac{1}{a} \frac{da}{dt} \right)_0$, is the current value (that’s what the subscript “0” means) of the Hubble “constant.”

Inserting the current best estimate of H_0 we obtain $\rho_c \approx 10^{-26}$ kg/m³ (see table of most recent values in GR9). If the total mass/radiation/vacuum energy density is greater than this value, the universe is likely to have *positive spatial curvature and be closed*, and its future is likely to see a halt in expansion and a re-convergence (a “Big Crunch”). If the total density is less than, however, the universe is likely to have negative spatial curvature, be open, and continue to expand forever. At this moment, relativistic particles account for hardly any of the energy content of the universe. Similarly, estimates of the visible matter density indicate that luminous matter only adds up to about 4.6% of the critical density. If dark matter is added in, the total matter density rises to $\rho_m \approx 0.28\rho_c$. Finally, recent measurements of high z objects seem to imply that ρ_v is substantial, with a value around $\rho_v \approx 0.72\rho_c$. In other words, at this moment, it looks like **the total mass/radiation/vacuum energy density in the universe just equals the critical density**. This conclusion is corroborated by independent Planck satellite measurements of the angular size of the largest CMB fluctuations (about 1°), which is in agreement with the FLWR prediction for such fluctuations with $k = 0$. If that is so, **the universe is spatially flat** and its expansion will continue forever—but more slowly than if it had negative spatial curvature.

Finally, it is interesting to note that each of the three parts of ρ depends on a differently. If we let the current value of $a=1$ and $\Omega = \rho/\rho_c$ then $\Omega = (\Omega_r/a^4) + (\Omega_m/a^3) + \Omega_v$, where Ω_r , Ω_m , and Ω_v are the *currently* observed values (5×10^{-5} , 0.28, 0.72). Irrespective of the values of the various Ω s (if they are all non-zero), the universe is dominated by radiation when the cosmic scale factor is very small and by the vacuum when the scale factor is very large. Inserting best current estimates for the various density contributions into (2) and setting $k = 0$, produces a solution

for a that goes to zero a finite time in the past—about 13.8 billion years ago. At that moment, both the density and temperature in the universe would have been infinite. Of course, this whole scenario is based on several assumptions with fairly large uncertainties. In addition, it is clear that GR cannot be trusted at length scales so small. Surely, quantum mechanics must have played a role—perhaps the *defining* role—at the dawn of creation!