Questions 1-3 refer to: The distance from Sun to Earth is defined as 1 astronomical unit (1 AU).

1. The speed of Earth in its (approximately circular) orbit about Sun is
   (a) 1 AU/yr
   (b) $2\pi$ AU/yr
   (c) $2\pi$ AU/hr
   (d) $2\pi$ AU/s

2. The centripetal acceleration of Earth in its (approximately circular) orbit about Sun is
   (a) $4\pi^2$ AU/yr$^2$
   (b) $4\pi^2$ AU/day$^2$
   (c) $4\pi^2$ AU/hr$^2$
   (d) $4\pi^2$ AU/s$^2$

3. The magnitude of Sun’s gravitational field at Earth is
   (a) $4\pi^2$ AU/yr$^2$
   (b) $4\pi^2$ AU/day$^2$
   (c) $4\pi^2$ AU/hr$^2$
   (d) $4\pi^2$ AU/s$^2$

4. Suppose the maximum height of an ocean tide at some place on Earth due to Moon (only) is 1 m. Assuming the height of the tide is proportional to the gravitational tidal field, the maximum height due to Sun (only) is about
   (a) 1 m
   (b) 2 m
   (c) 200 m
   (d) 0.5 m

Questions 5-6 refer to: A small diameter shaft is drilled from north to south poles in a large, uniform spherical planet. The planet has the same mass and radius as Earth and doesn’t rotate.

5. At a distance in the shaft halfway from the center of the planet to its surface the magnitude of the gravitational field is about
   (a) 0 N/kg
   (b) 1 N/kg
   (c) 5 N/kg
   (d) 10 N/kg

6. The time for a small object to fall once from the north pole to the south pole is about
   (a) 24 hr
   (b) 90 min
   (c) 45 min
   (d) 5 min
Questions 7-10 refer to: In general, the total mechanical energy, $E$, of Earth in its orbit around Sun can be expressed as

$$E = \frac{L^2}{2m} \left( \frac{dx}{d\phi} \right)^2 + \frac{L^2}{2m} x^2 - GMmx$$

where $x = 1/r$, $m$ is the mass of Earth, $M$ is the mass of Sun, and $L$ is Earth’s angular momentum relative to Sun.

7. Which term is the gravitational potential energy of Earth due to Sun?
   (a) $E$
   (b) $\left( \frac{L^2}{2m} \right) \left( \frac{dx}{d\phi} \right)^2$
   (c) $\left( \frac{L^2}{2m} \right) x^2$
   (d) $-GMmx$

8. Which term is Earth’s orbital kinetic energy?
   (a) $E$
   (b) $\left( \frac{L^2}{2m} \right) \left( \frac{dx}{d\phi} \right)^2$
   (c) $\left( \frac{L^2}{2m} \right) x^2$
   (d) $-GMmx$

9. Assuming Earth’s orbit is circular which (one) term is zero?
   (a) $E$
   (b) $\left( \frac{L^2}{2m} \right) \left( \frac{dx}{d\phi} \right)^2$
   (c) $\left( \frac{L^2}{2m} \right) x^2$
   (d) $-GMmx$

10. Assuming Earth’s orbit is circular the total energy $E$ equals
    (a) $GMmx$
    (b) $\left( \frac{L^2}{2m} \right) \left( \frac{dx}{d\phi} \right)^2$
    (c) $\left( \frac{L^2}{2m} \right) x^2$
    (d) $-\left( \frac{L^2}{2m} \right) x^2$

11. A photon traveling vertically a distance $L$ near the surface of Earth has a fractional change in wavelength given by $z = (\lambda_J - \lambda_r) / \lambda_r = \pm G_m L / c^2$, where the sign depends on the direction the photon is traveling. For a photon traveling downward 1000 m the fractional shift is about
    (a) $+10^{-13}$
    (b) $+10^{-7}$
    (c) $-10^{-7}$
    (d) $-10^{-13}$
Questions 12-14 refer to: A stationary black hole with Schwarzschild radius $r_s$ is located at $r = 0$. A photon travels radially from event A at $r = 4r_s$ to event B at $r = r_s$. Assume that the photon's motion is described by the Schwarzschild spacetime and that time is measured in meters.

12. “According to the photon” the time between A and B is
(a) zero
(b) $3r_s$
(c) about $9r_s$
(d) infinite

13. “According to the photon” the distance between A and B is
(a) zero
(b) $3r_s$
(c) about $9r_s$
(d) infinite

14. According to an observer far from the black hole and at rest with respect to it, the time between A and B is
(a) zero
(b) $3r_s$
(c) about $9r_s$
(d) infinite

15. The Schwarzschild radius of Sun is 3 km. The Schwarzschild radius of a second star is 6 km. Which one of the following is true?
(a) The mass of the second star is 1 solar mass, and its radius is 0.5 solar radius.
(b) The mass of the second star is 1 solar mass, and its radius is 2 solar radius.
(c) The mass of the second star is 0.5 solar masses, irrespective of its radius.
(d) The mass of the second star is 2 solar masses, irrespective of its radius.

Questions 15-16 refer to: Two galaxies are observed to have redshift factors $z = 1$ and $z = 4$, respectively. The universe scale factor $a = 1$ today.

15. The scale factors when the light from the two galaxies was emitted were, respectively,
(a) 1 and 4
(b) 2 and 5
(c) 1/2 and 1/5
(d) 1 and 1/4

16. The distance light has traveled from the $z = 4$ galaxy is
(a) roughly 4 times the distance from the $z = 1$ galaxy
(b) about the same as the distance from the $z = 1$ galaxy
(c) about 1/2 times the distance from the $z = 1$ galaxy
(d) about 1/4 times the distance from the $z = 1$ galaxy
Questions 17-18 refer to: The energy density of the CMB is roughly a factor of $10^4$ smaller than that of slow massive particles or the “vacuum (”). Assume $a = 1$ now.

17. At roughly what value of $a$ would the densities of the CMB and slow masses be equal?
   (a) $10^1$
   (b) $10^{-1}$
   (c) $10^{-2}$
   (d) $10^{-4}$

18. At roughly what value of $a$ would the densities of the CMB and the vacuum be equal?
   (a) $10^1$
   (b) $10^{-1}$
   (c) $10^{-2}$
   (d) $10^{-4}$

19. Given that the average energy per photon in the CMB is about $10^{-3}$ eV now, at about what value of $a$ would it be 1 GeV ($G = 10^9$)?
   (a) $10$
   (b) $1$
   (c) $10^{-3}$
   (d) $10^{-9}$

20. The temperature of the CMB when $a = 0.5$ was about
   (a) 2 times what it is today
   (b) the same as it is today
   (c) 0.5 times what it is today
   (d) 16 times what it is today

21. The value of $1/H_0$ in years is about
   (a) $10$
   (b) $10^5$
   (c) $10^{10}$
   (d) $10^{15}$

Questions 22-23 refer to: Assume that the universe is spatially flat ($k = 0$) and only ever contained one kind of mass-energy. The Friedmann equation can then be integrated to determine the age of the universe: 
\[ \frac{1}{H_0} \int_{a=0}^{1} \frac{da}{\sqrt{f(a)a^2}} = t_{age}, \]
where $H_0$ is the “Hubble constant” and $f(a)$ is how the energy density varies with $a$.

22. How old would the universe be if it only ever contained slow massive particles?
   (a) $1/(2H_0)$
   (b) $2/(3H_0)$
   (c) $1/H_0$
   (d) infinite
23. Adding the correct amount of vacuum energy to that of slow massive particles is required to make the age of the universe be the observed value
(a) $1/(2H_0)$
(b) $2/(3H_0)$
(c) $1/H_0$
(d) infinite

24. If the ratio of photons to baryons were much greater than its measured value
(a) the ratio of primordial helium to hydrogen would be much smaller
(b) the fluctuations in the CMB would be much smaller
(c) the average energy per photon in the CMB would be much lower
(d) vacuum energy would have become dominant much later

25. An early epoch of exponential growth of $\alpha$ has been proposed in order to explain why
(a) there are so many black holes
(b) the CMB is so smooth
(c) there is so much dark matter
(d) there is so much dark energy