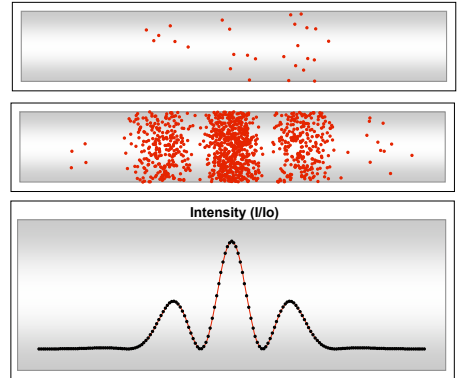


Foundations, 2

The double slit experiment in dim light – photons!

Let's imagine doing the double slit experiment again, but now in *very dim light*. To do so requires putting the laser, plate, and collector in a sealed, light-tight box. Inserting neutral density filters in the beam between the laser and the double slit plate decreases the intensity of the beam striking the plate without changing the color of the beam. In fact, the experiment can be done at such a low intensity that a human eye will not see any light on the CCD collector; but the CCD *can*. Under these conditions, the number of pixels that "light up" during the collection time Δt is small and very rarely do two adjacent pixels light up at the same time no matter how small the pixels are. In other words, the lighting of a pixel seems to be as if it is hit by a tiny particle. The sequence of hits, in dim light, is irregular. An example is shown to the right at the top. Every time the process is repeated we observe a different sequence of hit pixels. It appears to be impossible to predict what the sequence will be before it is generated. On the other hand, if this process is repeated numerous times and the hit pixels are superimposed, a regular accumulated image begins to emerge like that in the middle to the right. When such a time-lapse image is compared with the expected theoretical intensity distribution for bright light (bottom right), it is clear that the pixel hits are gradually filling in the classical interference pattern. Thus, though the hits appear to occur at random, not anything can happen: there are constraints to the randomness. In particular, there are pixels that are more likely to be hit than others and some pixels (almost) never get hit.



Though there are technical barriers to performing the following thought experiment directly, the results have been verified indirectly to great precision. In principle, it is possible to measure the deposited energy per pixel hit. If we do this for a given laser color (i.e., a given wavelength—again, this is a thought experiment: real lasers emit multiple wavelengths) we find that the energies per hit are the same (within experimental uncertainty) no matter how many hits are recorded. In other words, it is *as if* the CCD gets hit again and again by little BBs carrying the same energy. The light falling on the CCD appears to consist of "particles" of light energy. Poetically, **these particles of light are called photons**. Performing the experiment with different light colors shows that the energy-per-hit is determined only by the wavelength:

$$E_{\text{photon}} = \frac{hc}{\lambda}, \quad (1)$$

where h is Planck's constant, c is the speed of light, and λ is the wavelength of the corresponding bright light. This relation has been established in the laboratory with high confidence for all wavelengths of EM radiation, not just visible light. Thus, photons, just like ordinary particles, carry energy. Note that for light $c = \lambda f$, so (1) is equivalent to $E_{\text{photon}} = hf$, where f is the frequency of the corresponding bright light.

Quantum units

In SI units, h has the rather unlovely value of 6.62×10^{-34} J-s. In the microrealm of quantum mechanical things, the joule is vastly too large an energy unit. More useful is the *electron volt* (eV). **One eV is the change in kinetic energy one electron would experience in passing through an electric potential difference of one volt.** The conversion between eV and joules is **1 eV = 1.6×10^{-19} J**. Thus, in eV, $h = 4.14 \times 10^{-15}$ eV-s. This is still not beautiful. It turns out that h is almost always usefully multiplied by c , giving a quantity in eV-m. But (similar to the situation for the joule), the meter is a wildly inappropriate measure of distance on the quantum scale; there, nanometers (nm) are much better. After all is said and done, then, we have $hc = 1240$ eV-nm (well, among friends, roughly 1000). That's a lot prettier. *Memorize it!*
Useful tip: whenever you see h always multiply (and divide) by c and use eV-nm.

Example: As stated in Fn1, the wavelengths of visible light lie in the range 400 nm (violet) to 700 nm (red). A **violet photon** carries an energy of $1240 \text{ eV-nm} / 400 \text{ nm} = \mathbf{3.1 \text{ eV}}$, while a **red photon** carries $1240 \text{ eV-nm} / 700 \text{ nm} = \mathbf{1.8 \text{ eV}}$. That's the range of photon energies humans can see—values also **well worth memorizing**.

Example: An FM radio station has a broadcast power of 1000 W (= $1000 \text{ J/s} / 1.6 \times 10^{-19} \text{ J/eV} = 6.25 \times 10^{21} \text{ eV/s}$). Suppose the carrier frequency of the station is 100 MHz. This corresponds to a wavelength of $3 \times 10^{17} \text{ nm/s}$ (the speed of light in nm) divided by $100 \times 10^6 \text{ Hz}$, or $3 \times 10^9 \text{ nm}$ (i.e., 3 m). The energy of such a radio photon is $1240 \text{ eV-nm} / 3 \times 10^9 \text{ nm} = 4.1 \times 10^{-7} \text{ eV}$ (very small). The number of photons released per second by the station is $6.25 \times 10^{21} \text{ eV/s} / 4.1 \times 10^{-7} \text{ eV/photon} = 1.5 \times 10^{28} \text{ photons/s}$ (very large). Your radio antenna is bombarded with so many photons per unit time that the granular nature of EM radiation in radio broadcasts (and almost all other macroscopic sources of EM radiation) is not observable.

Photons are massless!

All Newtonian particles have mass. To determine the mass of the photon we borrow a general result from Maxwell and a second general result from *special relativity*. The result from Maxwell has to do with the fact that EM radiation carries both energy and momentum. Momentum transport in EM waves results in the phenomenon of radiation pressure. Maxwell shows that in EM radiation *energy-density* = $c \times$ *momentum-density*. If the Maxwell result carries over to photons (we now know that not *all* of Maxwell's predictions are exactly correct at the quantum level, but this one is) then

$$E_{\text{photon}} = cp_{\text{photon}} \quad (2)$$

In (2) p_{photon} is the *magnitude* of the photon momentum (remember, in Newtonian physics, energy is a scalar but momentum is a vector). The second result, borrowed from special relativity, is that the total energy and momentum of a freely moving (i.e., no external forces) particle are related by

$$E = \sqrt{(pc)^2 + (mc^2)^2} \quad (3)$$

Of course, if the particle is at rest ($p = 0$), (3) becomes

$$E = mc^2 \quad (4)$$

(which is Einstein's famous formula). This is called the particle's *rest energy*.

Example: The mass of an electron is 9.1×10^{-31} kg. What is its rest energy in eV?

Solution: $mc^2 = 9.1 \times 10^{-31} \text{ kg} \cdot (3 \times 10^8 \text{ m/s})^2 = 8.2 \times 10^{-14} \text{ J} / 1.6 \times 10^{-19} \text{ J/eV} = 5.1 \times 10^5 \text{ eV} \approx 0.5 \text{ MeV}$ (among friends). **Memorize this.**

Useful tip: whenever you see a mass, always multiply (and divide) by c^2 and use eV. Often you can use the tips for h and for m at the same time so that the dividing occurs automatically.

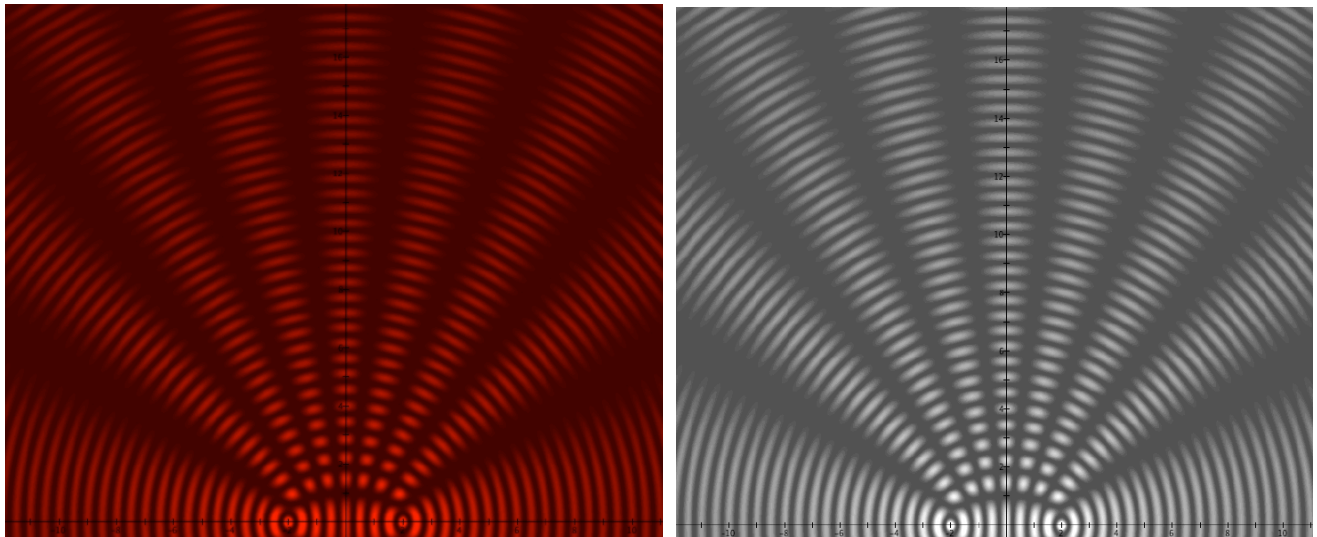
The m that appears in these relations is known as the *rest mass*—the mass you would measure by catching up to the particle and weighing it. It’s an intrinsic property of the particle. If we combine (2) and (3) we discover the remarkable result that $m_{\text{photon}} c^2 = 0$: **despite the fact that they carry energy and momentum, photons are massless.** Note that if we insert for the energy of the photon hc/λ in (2) we find the important consequence

$$p_{\text{photon}} = \frac{h}{\lambda}. \quad (5)$$

Just because a photon is massless does not mean it doesn’t transport momentum! Momentum is *not* equal to mass times velocity for particles with speeds near or at the speed of light. In particular, the momentum of the photon is *not* $m_{\text{photon}} c$; it’s what (5) says it is.

Wave-particle duality

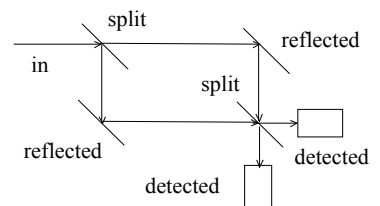
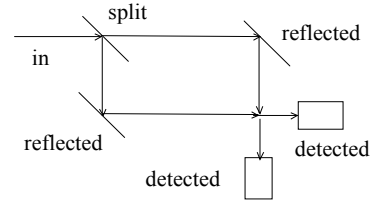
That the momentum and energy of a photon refer to a wavelength means that there is something about photons that is “waving” as well. The images below give a hint.



The red image on the left depicts the instantaneous EM energy density (u) distribution for a bright light. This pattern propagates away from the two “plugs” at the bottom of the image heading at speed c toward the top. At a maximum intensity point on the collector (at the top edge of image, say) energy rises and falls rapidly as the blobs of energy hit one after another. The CCD collects energy over a duration that is long compared with the blob rise and fall time. The continuous dark lines are “lines of nodes,” places where there is never any energy. Where they intersect the collector the interference pattern is dark. The grayscale image to the right depicts what occurs in very dim light. It is identical to the red image except now there is essentially no energy in the blobs. These blobs correspond to regions of high probability of

finding a photon—even if the experiment is done one photon at a time. (See the Appendix, below.) Thus, **what “waves” with a photon is the probability of finding it.** The energy distribution on the left and the probability distribution on the right have the same wavelength and both fly from bottom to top at speed c .

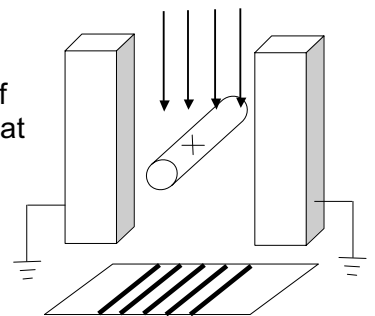
In order to produce interference, the photon probability wave has to pass through both slits at the same time. But, that’s curious because when a photon is detected it deposits its energy at a “point” (well, in a really small pixel)—as a particle would. So, does the photon go through both slits, or just one? To check this, a slight variation of the double slit experiment has been done. The apparatus is depicted in the diagram to the right. A beam of photons comes “in” from the left and is split 50% to the right and 50% down by a “beam splitter.” Thereafter both split beams are fully reflected, winding up in two detectors as shown. When the beam intensity is reduced to one photon at a time the detectors “click” unpredictably and independently, with equal number of clicks *eventually* being recorded in each. That is, in this set-up the photons truly travel over independent paths and act just like BBs. Now, one thing is added to the experiment—a second beam splitter just before the detectors (see right). The second beam splitter causes confusion over which path the photon might have taken. Depending on the difference in the lengths of the two paths, what is now observed is *unequal* detector clicking. In fact, as the path lengths are changed a little (for example, by moving and tilting one of the fully reflecting mirrors) the two clicking rates also change. Indeed, sometimes one detector receives nothing and the other all of the clicks and vice versa. (This is analogous to changing the angle θ in the double slit intensity equation.) Clearly, this behavior indicates that there is interference between the two paths, just as if the photon were a wave spread out over the whole apparatus. In other words, if we can tell which path the photon travels over—that is, if we do a particle-type experiment—then we get a particle-like result; if we can’t tell—that is, if we do a wave-type experiment—then we get a wave-like result (until it’s detected, of course, at which point the wave “collapses into a particle” and is destroyed).



Now, here’s a really cool and wildly weird thing. It is possible to insert the second beam splitter *after* the photon has entered the apparatus and *before* it has reached a detector. This wrinkle is referred to as a **delayed choice experiment**. When the splitter is out, the detectors count 50-50. When it’s in, the detectors count unequal amounts, meaning there is interference! Thus, it is as if the photon carries both particleness and waviness at the same time but it only makes up its mind at the last instant which property it will reveal. The fact that photons carry the potential to show particle properties and wave properties depending on what kind of measurement is done is called **wave-particle duality**.

Double slit experiment done with electrons

The figure to the right is a sketch of an experiment in which a beam of electrons (the arrows) passes through a “double slit” apparatus. Electrons that hit the positively charged wire or the grounded side plates are lost, but those that pass through the gaps between the plates and the wire are deflected by their attraction to the central wire and finally collide with a CCD detector (at the bottom of the figure). The electrons are accelerated from “rest” through a known potential difference ΔV , so each has kinetic energy equal to $e\Delta V$ (where e is the magnitude of the electron charge). When the current is



large, the collecting plate shows a double slit interference pattern! Measuring the slit separation and the distance between adjacent maxima allows the effective “wavelength” of the electron beam to be calculated. When the accelerating voltage is changed the maxima get closer or farther apart, indicating that the wavelength depends on voltage. The experimentally determined relation is $\lambda = \text{constant}/\sqrt{\Delta V}$. Though it doesn’t immediately look like it, we have actually seen this relation before. When the electron is accelerated from rest through ΔV its kinetic energy increases from zero to $e\Delta V$. (This is the work-energy theorem.) The Newtonian form of kinetic energy is $K = mv^2/2$, which, because $p = mv$, is also $K = p^2/2m$. In other words, for the accelerated electron $\sqrt{\Delta V} = p/\sqrt{2me}$. Plugging into the empirical relation between wavelength and accelerating voltage yields $\lambda = (\text{constant} \cdot \sqrt{2me})/p$. The *measured* value of the numerator turns out to be just Planck’s constant, h . In other words,

$$p_{\text{electron}} = \frac{h}{\lambda}, \quad (6)$$

which is the same relationship as for photons. Louis de Broglie hypothesized that massive particles might have a wavelength given by Equation (6) in 1924, three years before its experimental confirmation by **Clinton Davisson** and **Lester Germer**. For this reason, the λ in (6) is known as the *de Broglie wavelength*.

Example: What is the de Broglie wavelength of an electron accelerated from rest through a potential difference of 100 V?

Solution: Inserting $p = h/\lambda$ into $K = p^2/2m$ yields $K = h^2/2m\lambda^2$ or $\lambda = \sqrt{h^2/2mK}$. **As stated above, the rules in doing quantum mechanical calculations are (a) multiply h times c and m by c^2 whenever they appear and (b) use eV and nm (never J and m).** This leads to

$$\lambda = \sqrt{(hc)^2/2mc^2K} = \sqrt{(1240 \text{ eV}\cdot\text{nm})^2/(2\cdot 5\times 10^5 \text{ eV}\cdot 100 \text{ eV})} = 0.124 \text{ nm},$$

about the size of an atom. Be sure you understand all of the steps in the above equation string.

(Historical comment: It is a wonderful bit of historical irony that two of the most important discoveries of 20th century physics occurred at Bell Labs in New Jersey—both by accident! In both cases the discoverers were faithfully engaged in their employer’s main mission: improving electronic communication. The first of these accidents, involving *vacuum tubes*, demonstrated that electrons are actually not little BBs. The second, 40 years later and dealing with *communication satellites*, showed that the universe is filled with roughly 14-billion year-old electromagnetic radiation—the Cosmic Microwave Background—produced shortly after the so-called “Big Bang.” In any case, in April 1925, Bell scientists Clinton Davisson and Lester Germer were investigating how a beam of electrons bounced off a plate of nickel in a vacuum tube, when they inadvertently brought the hot tube in contact with cold (“liquid air”) vapor. The glass cracked and the nickel plate rapidly oxidized. Upon reheating the plate to remove the oxide layer on its surface, they observed a remarkable change in the reflection pattern (produced presumably because reheating the plate caused it to become more like a perfect single crystal rather than the highly polycrystalline form it was originally in): instead of a more-or-less uniform distribution of reflected electrons, now there were special directions relative to the incident beam in which many electrons reflected and other directions where almost none reflected—much like an interference pattern. At the time of their accidental finding, Davisson and Germer had no idea that a year earlier de Broglie in his PhD dissertation had suggested that electrons might have a wave character and that, if so, the associated wavelength should be $\lambda = h/p$. After Davisson learned of

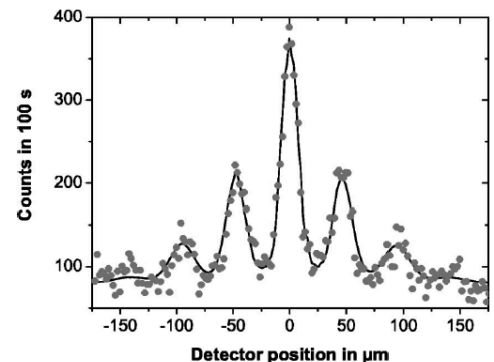
de Broglie's conjecture at a conference in Europe in 1927 he returned to Bell and he and Germer performed a series of systematic studies of the interference pattern as a function of electron momentum and verified that de Broglie was correct. Largely because of Davisson and Germer's work, de Broglie was awarded the 1929 Nobel Prize in Physics. Later (1937), Davisson also received the Nobel Prize; Germer never got anything—other than gracious acknowledgement by Davisson in his Nobel Lecture. Go figure.)

Just like photons, the electron double slit experiment can be done one electron at a time, and just like photons, the interference pattern builds up electron after electron. Thus, electrons have an associated probability wave. Also, just like photons, if a wave measurement is performed on an electron it reveals wave-like properties. In the above apparatus that means not detecting which side of the wire that the electron passes through and observing an interference pattern. On the other hand, if which side of the wire *is* detected (by placing small conducting loops just below the wire, for example) then the interference pattern converts into the two blobs you would expect if the electrons were particles. Electrons, like photons, have *wave-particle duality*.

To summarize, photons and electrons are “elementary” in the sense that they do not appear to be made of anything simpler. All photons have two intrinsic (unrelated to their motion) particle properties, *zero* rest mass and *zero* electric charge (they also carry an intrinsic amount of angular momentum, but we will come back to this presently), and two extrinsic (related to their motion) particle properties, momentum magnitude equal to h/λ and the kinetic energy-momentum relation, $KE_{\text{photon}} = cp_{\text{photon}}$ (**which is very different from that for a slowly moving massive particle**). The intrinsic properties of electrons, on the other hand, include rest mass equal to $0.51 \text{ MeV}/c^2$ and electric charge equal to $-e$ (they also carry an intrinsic amount of angular momentum that we will come back to and another kind of charge called “weak charge,” that we won't discuss further in this course). The extrinsic properties of electrons are a wavelength equal to h/p and a kinetic energy-momentum relation, $KE_{\text{electron}} = p_{\text{electron}}^2/2m_{\text{electron}}$ (if they are traveling slowly compared with c).

It's not just photons and electrons

It's not just elementary photons and electrons that behave in such a bizarre fashion. The double slit experiment has been done with composite systems such as neutrons (neutrons are systems of quarks), different species of atoms, and molecules—one of the biggest being C_{60} “buckyballs.” The figure to the right shows real experimental data; the average speed of the molecules was about 117 m/s, corresponding to a de Broglie wavelength is about 0.005 nm (a small fraction of the molecular diameter).



Indeed, we now believe that *all* objects (massive or not) have wave-particle duality. So how come we don't experience interference effects as we pass, for example, through a doorway—unpredictably veering to the right or left? One answer is h is so small. (Is another possibly that we might always be doing a particle measurement on ourselves?) In order to see interference (diffraction), a wavelength has to be smaller—but *not much smaller*—than the size of the hole the wave passes through. If $\lambda = h/p$ applies to us, then we should expect that when $p \rightarrow 0, \lambda \rightarrow \infty$; so, interference should become noticeable for really slowly moving objects. The

trouble is it's difficult for a large object to move slow enough. Even when we try to stand still, for example, our centers-of-mass are wiggling about with a speed of something like 1 mm/s. A person with a mass of 50 kg would have an average momentum magnitude of about 0.05 kg-m/s while "standing still" and a wavelength of about 10^{-32} m. Such a wavelength is vastly smaller than an atom and can produce no noticeable wave-like effects in the macroscopic world of people and doors.

There is a second reason why big things don't exhibit quantum interference effects. In order for two (or more) waves to interfere their phase difference when they were produced has to not change (at least, not very rapidly): *they have to remain coherent*. When a particle interacts with its surroundings, its wave properties "decohere" and interference vanishes. Big, slow objects such as humans or rocks, interact a lot with their surroundings and quickly lose their quantum wave properties. The classical world we are familiar with results from quantum decoherence.

So, what is quantum mechanics?

Quantum mechanics is a theoretical structure that aspires to grapple with the schizophrenia of wave-particle duality. It allows us to calculate the possible outcomes of such particle measurements as energy, momentum, and angular momentum, and what the average values of those outcomes will be in many repeated experiments arising from wavy probability. In essence, quantum mechanics forbids us from saying anything about where a particle is and how it is moving until a particle measurement is made. In this sense, quantum mechanics defines a particle as what you measure when you do a particle measurement. Though the mathematics of quantum mechanics and its implications seem more than a little nutty, they must in some sense be correct because you can make a lot of money with them!

Appendix: Identical photons like one another

Photons with the same momentum have a curious propensity for traveling together. The intensity of an extended source of bright light fluctuates a little because of this clumping propensity. Even in very dim light, multiple pixels of a CCD detector will typically get "hit" simultaneously. To do a genuine single photon experiment requires some subtlety. A method due to Granger, Roger, and Aspect (Europhysics Letters, **1**, 173-179 (1986)) uses single atoms as sources. In their method, an excited atom emits two photons rapidly one after the other traveling in opposite directions. Detection of one photon (the "trigger") allows the detection of the second (the photon of interest) to be uniquely identified with the first if the times from source site to detector sites are precisely measured. A second photon that arrives at the second detector at a wrong time is counted as extraneous and simply ignored.