

# Task-performing Dynamics in Irregular, Biomimetic Networks

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*Understanding self-organized collective dynamics—especially in sparsely connected, noisy, and imperfect networks—has important implications for designing and optimizing task-performing technological systems as well as for deciphering biological structures and functions. We note that stomatal arrays on plant leaves might provide an ideal example of task-performance in this context. Guided by observations of stomatal networks, we examined a simple model of task-performing, collective dynamics that included state noise, spatial rule heterogeneity, dynamic modules, and network rewiring. Our results indicate that task-performance in such networks can actually be enhanced by various kinds of spatial and temporal irregularity. © 2007 Wiley Periodicals, Inc. Complexity 12: 14–21, 2007*

**Key Words:** network dynamics; spatiotemporal noise; cellular automata; majority task

## 1. INTRODUCTION

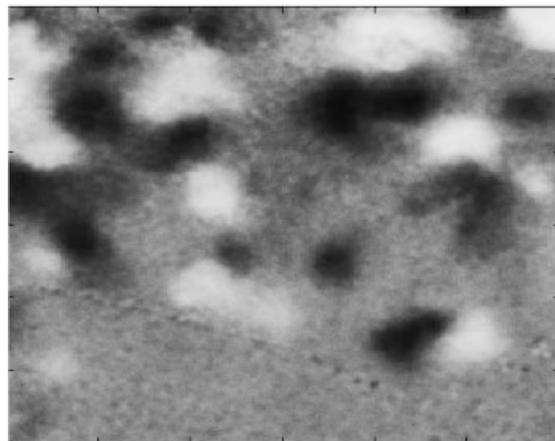
**D**ynamical processes on networks often exhibit self-organized collective behavior in which the activities of large numbers of nodes spontaneously synchronize without the intervention of a central processing unit (CPU). It is sometimes possible to interpret these self-organized collective configurations in terms of *performing a task*. This particular kind of task performance has been achieved in carefully engineered hardware systems [1–3] and has been conjectured to underlie some aspects of how living organisms function [4–6]. One difference between technological and biological systems is their response to irregularity. In

technological systems, specialized equipment or procedures are frequently required to detect and correct errors, whereas biological systems often manage successfully in highly variable environments without any obvious error correction capability. It is of great interest, therefore, to identify how biology's apparent innate robustness to imperfections might be mimicked in technological systems manufactured—or forced to operate—in complex conditions. In this article, we propose that *stomata*—microscopic, variable aperture pores—on a leaf constitute an ideal reference system for developing at least some types of noise-resistant, task-performing networks. The argument goes as follows.

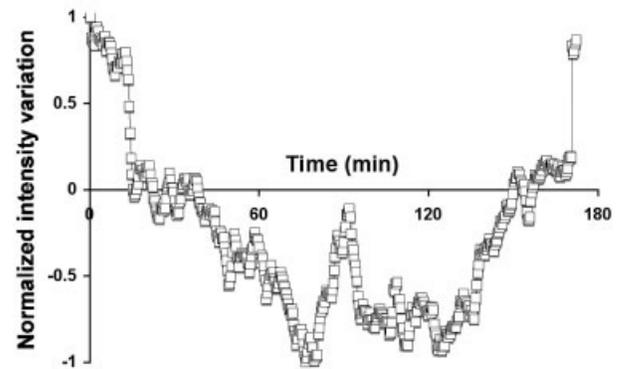
A central hypothesis in plant biology is that through the “correct” adjustment of their stomata plants solve a constrained optimization problem involving maximizing, over

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**FIGURE 1**



(a)



(b)

(a) Chlorophyll fluorescence during a patchy stomatal episode. The image corresponds to a 6.25-cm<sup>2</sup> region on a leaf of the plant *Xanthium strumarium* L. and indicates whether, over a period of 3 min, the fluorescence intensity of a pixel is increasing (white), decreasing (black), or remaining constant (gray). Increased fluorescence implies stomatal aperture closure and vice versa. The density of pixels is roughly the same as the density of stomata and dark or bright patches include hundreds to thousands of stomata all opening and closing (respectively) synchronously. (b) Chlorophyll fluorescence intensity variation during a patchy stomatal episode. Shown is a typical pixel intensity change time series over the course of 3 h. The variations shown are about  $\pm 10\%$  of the absolute intensity at that pixel. Measurement noise is Gaussian, with  $\pm 1\%$  variation.

some period of time, the uptake of CO<sub>2</sub> while avoiding desiccation due to evaporative water loss [7]. Plants lack neural tissue that might serve to coordinate stomatal activities over widely separated parts of the organism, so it might seem reasonable to assume that each stoma solves this problem autonomously. On the other hand, it has recently been established that stomata interact [8], and that they therefore cannot be completely independent—i.e., stomata are effectively “wired” together in a network. A dramatic demonstration of stomatal network dynamics is *stomatal patchiness*, a phenomenon in which patches of hundreds to thousands of contiguous stomata open and close in synchrony [9], with adjacent patches often having quite different average apertures [see Figure 1(a)]. Stomatal patchiness has been observed in numerous species in the laboratory and in the wild. Its commonplace nature suggests the possibility of an important adaptive function, though what that might be is as yet unsettled.

In laboratory experiments, patchiness is induced by an applied perturbation—for example, a sudden, uniform reduction in ambient humidity. Despite all efforts to perform such experiments under identical conditions it is never possible to predict when patches will appear, if they will be dynamic or static, or what their duration will be. Typically, patchiness—when it occurs—does not become observable until tens of minutes after the perturbation is applied. In the interim, stomata undergo aperture adjustment, but with no evidence of collective organization. Sometimes when

patches do subsequently form, they persist for hours and are richly dynamic [see Figure 1(b)]—growing and shrinking in size, changing shape, and seemingly moving across a leaf’s surface as coherent entities. As discussed in detail in Peak et al. [10], patchiness, during such long-lived dynamic episodes, has the same persistent, power-law correlations in space and time as are found in model systems that exhibit self-organized, collective behavior [11]—including some designed to perform a task.

The last point leads us to propose the following interpretation. In experiments in which a uniform step-perturbation is introduced, both the initial and final aperture states are solutions to an optimal gas-exchange problem for the respective environmental conditions. The final uniform aperture state is achieved in two stages. First, during the transient period immediately after the perturbation, fast physical and chemical processes lead to each stoma attempting to adopt the new correct aperture value more-or-less independently. These fast processes also produce “errors,” however. These errors consist of small islands of stomata with stable, incorrect apertures, presumably arising from a competition between multiple (possibly suboptimal) attractors of the stomatal dynamics. In the second, more leisurely period that follows—the one in which coherently moving patches are sometimes observed—all apertures adjust collectively to be closer to the correct value. The dynamics in this latter epoch apparently serves to destabilize the locally incorrect apertures and facilitate their adjust-

ment to more optimal values. In other words, we speculate that the task of the stomatal network is to identify the correct aperture among those present and to reset erroneous apertures to the correct value.

Finally, it is important to note that stomatal systems exhibit a number of irregularities that are not found in perfect task-performing networks. These include the following: (i) State noise: During episodes of dynamical patchiness, there is irreducible temporal variability in stomatal aperture [Figure 1(b)]. (ii) Heterogeneous interactions: Stomata of most leaves vary in size, orientation, and spacing. Preliminary evidence (Sharples, Peak, and Mott, unpublished data) suggests that this variation affects local interactions among individual stomata. (iii) Modularity: During stomatal patchiness, synchronization of stomata spreads across regions of the leaf that are separated by small veins. Thus, these regions appear to function like dynamical modules, with neighboring modules capable of influencing one another.

Irrespective of whether our conjecture about the task performed by stomatal networks is literally true, the more general task of identifying and correcting the nodal states in networks lacking central control is likely to have broad technological applicability. We therefore explore here the question, to what extent can networks perform the “identify and correct” task in the face of spatiotemporal variability without an error management capability? In the next section we define a set of simulations, designed to mimic characteristics of stomatal networks, that probe these issues. We summarize our empirical findings in the succeeding section and conclude with a discussion of their implications.

## 2. TASK-PERFORMING NETWORKS INSPIRED BY STOMATA

### Network Definitions

We invoke two reasonable simplifications in the remainder of this article. First, we assume that stomatal apertures can be parsed into more-open or less-open than a mid-range value (that might vary from stoma-to-stoma) and that one of these conditions is better for achieving optimal gas-exchange. Such a dichotomous situation is likely to hold in other applications as well, so for simplicity we consider in the following networks whose nodes have only two states, +1 (“open”) and -1 (“closed”), either of which can be the effectively “correct” state. Second, though stomatal apertures adjust continuously in time, we have found—using standard time series analyses [12]—that sampling the stomatal variations at a time scale of about 10 min provides maximal dynamical information. When this interval is used, the continuous stomatal dynamics, in principle, can be recast as a discrete map, and here we adopt that perspective. In short, we take the networks of interest to be equiv-

alent to binary cellular automata (CAs). The “identify and correct” task of interest is therefore equivalent to the much-studied CA *majority* (or “density classification”) *task* [13, 14]—the goal of which is to identify the (presumably correct) state initially in the majority by having the nodes with the (presumably incorrect) minority state change collectively.

We start with perfectly wired networks with each node located on a square lattice and receiving input from itself and its four nearest neighbors only (i.e., 2D CAs with five-node *neighborhoods*). In the noise-free versions of the networks, each nodal unit has perfect input. It unerringly executes a simple state-update rule consisting of 32 “if ... then ...” clauses, where each “if” identifies one of the five-node neighborhood configurations and the “then” corresponds to an action setting the state of the central node in the next time step. At first, we assume all nodes update synchronously. We ultimately relax each of these restrictions to more realistically reflect stomatal characteristics.

For the purposes of accurately mimicking biology, we should equip each node with an update rule extracted from the fundamental physics and physiology of stomata. Unfortunately, the necessary ingredients for constructing such a rule are not yet fully available. Instead, we examine here a variety of majority task-performing rules, searching for commonalities. In our simulations, we examined the performance of the majority task by networks in which: (a) each node executed the “local majority” rule (LM networks); (b) each node executed a “2DGKL” rule (2DGKL networks); (c) different nodes executed different rules drawn from sets of rules found by Sipper ([15] and private communication) using a genetic algorithm (GA) search [14]; we examined four such heterogeneous networks and refer to their average performance as SipHET; and (d) each node executed the predominant rule from the four sets in (c); the average performance of these is referred to as SipHOM.

The LM rule is equivalent to setting the node’s state at time  $t + 1$  to its majority input state at time  $t$ . In other words, an LM network essentially attempts to perform the majority task by having each of its nodes individually do it. The 2DGKL rule is a variant of the LM rule. Suppose the five input ports of each nodal unit are designated  $C$  (center or self),  $N$  (north),  $E$  (east),  $S$  (south), and  $W$  (west); then the 2DGKL rule is: if at time  $t$  the state of a node (i.e., input  $C$ ) is  $-1$  then set its state at  $t + 1$  to the majority of the  $C$ ,  $N$ , and  $E$  inputs; otherwise, set its state at  $t + 1$  to the majority of the  $C$ ,  $S$ , and  $W$  inputs. [In examining different inputs depending on the current node state, this rule is akin to the Gacs-Kurdyumov-Levin (GKL) rule [16] for the majority task in 1D. It differs substantially, however, from the so-called “2DGKL” rule discussed in Jimenez-Morales et al. [17]. Because of interchangeability of state values, as well as  $NE/NW$  and  $SW/SE$  inputs, there are actually four identically performing 2DGKL rules.] Sipper’s four genetically found rule

sets are defined for networks of  $15 \times 15$  nodes with strictly nearest-neighbor architecture and periodic boundary conditions; in each case, there are 40–60 different rules distributed over the 225 nodes, with the predominant (i.e., most frequent) rule appearing at 30–60 sites. Sipper’s heterogeneous sets have strong spatial correlations: rules of adjacent nodes differ from one another by no more than four clauses.

### Simulation Sets

Our simulations consisted of: (1) establishing *baseline* task-performance success rates for perfect, locally connected, noise-free, synchronous networks; (2) examining the effects of *state noise* in structurally perfect networks; (3) examining the effects of *spatial irregularity* in noise-free and noise-infected networks; (4) constructing large, *mosaic networks* from small functional modules; (5) examining the effects of *3D complex interconnectivity* on network performance; and (6) examining the effects of *asynchronous state updating* in perfect and imperfect, noise-free and noise-infected networks.

### Procedures

(1) Baseline performance: Each of the network types (a)–(d) was “posed” the same set of majority task “problems.” To be consistent with Sipper’s work we used  $15 \times 15$  networks. A problem consisted of assigning to each node a  $-1$  or  $+1$  state at  $t = 0$  and then determining if all nodes had simultaneously attained the correct state in no more than  $t = T = 2 \times 15 \times 15 = 450$  time steps. If so, the task for that problem was declared to be “performed,” and “not,” otherwise. We examined 1000 different, randomly assigned, initial network configurations for each  $-1/+1$  ratio from 1:99 to 99:1, in increments of 1%, recording for each run the percent correctly performed. For each initial ratio, we repeated this procedure 200 times with different random number generator seeds and determined the SDs of the resulting distributions of percent performed. The data are normally distributed so the SDs have their usual interpretation as measures of variance. (2) State noise: We reran the majority task as in (1), but at each time step, from the second on, we first checked whether the network had reached a uniform configuration and, if not, switched each node’s correct output state, with probability  $\eta$ , before the next update. (3) Spatial irregularity: In one experiment, we randomly scrambled Sipper’s rule-heterogeneous networks [type (c), above] to destroy their genetically established spatial placement. In a second, we randomly identified a fraction of the nodes in all of the rule-homogeneous networks [(a), (b), (d)] to be “frozen” into their initially assigned states. In each case, we reran the majority task as in (1) and (2). (4) Mosaic networks: We used Sipper’s rule-heterogeneous  $15 \times 15$  networks as tiles in mosaics consisting of  $15N \times 15N$  nodes. In these mosaics, each module was rewired so that its boundary nodes were connected to nodes in the adjacent module.

Nodes on the edges of the mosaic were connected to nodes on the opposite edges of the entire network in the appropriate directions. (5) 3D complex interconnectivity: One way of modeling 3D effects while preserving the ability of each node to receive five inputs [18] is as follows: (i) assume that the wires connecting nodes are unidirectional and that each node has an output port from which any number of wires can emerge—with one always going to the  $C$  input port (nodes always know their own states); (ii) start with a strictly locally connected 2D architecture, where each node receives one *in-wire* and sends one *out-wire* to each nearest neighbor; and (iii) then sequentially rewire each node in the network by replacing, with probability  $p$ , each of its *in-wires* (except the wire to  $C$ ) with one coming from some other randomly chosen node. After doing this for the entire network, all nodes still are wired to themselves and still have *in-wires* from four other nodes—though not necessarily nearest neighbors. We reran procedure (1) for all networks, stepping through  $p$  in increments of 5%, first without, then with, state noise. (6) Asynchronous state updating: To evaluate the effects of asynchrony, we allowed each node to update to the correct state with probability  $\chi$ , otherwise stay unchanged. *One* effect of asynchrony is that the time necessary for task-performance, *if it happens at all*, is slowed. Thus, in rerunning (1) and (5) again, we examined  $\chi$  values ranging from 99% synchrony down to 1%, and took the cutoff time in each study to be scaled by  $1/\chi$ .

## 3. SIMULATION RESULTS

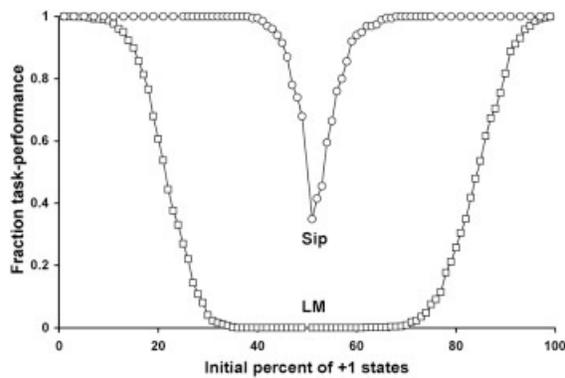
In the following we refer to “easy” and “hard” majority task problems. An easy problem is one that is solved correctly in no more than  $M$  time steps, where  $M$  is the number of nodes in the network (that is, half the cutoff time). Otherwise (i.e., correct solution but only after a long period, incorrect solutions, or unresolved final network configurations), the problem is hard. The density of hard problems generally increases as the ratio  $-1:+1$  approaches 50:50.

### Baseline Performances

All networks we examined perform well for highly one-sided initial state ratios, but greatly differ from one another for initial state ratios close to 50:50. Figure 2 contrasts the performance of an LM network with that of one of Sipper’s GA-found rule-heterogeneous networks. Clearly, the LM/autonomous-node strategy for performing the majority task only works for extreme initial state ratios. All other networks we examined out-perform LM over a broad range of initial state ratios and all perform similarly to the Sipper example seen in Figure 2. All perform at close to 100% efficiency for ratios above 70:30, but experience a sharp downturn in performance for ratios near 50:50.

To probe network performance for “50:50” problems, we looked at another 1000 problems for each network drawn from a Gaussian distribution of initial  $-1:+1$  ratios centered

**FIGURE 2**



Comparison of majority task-performance by  $15 \times 15$  locally connected, noise-free networks. Each point is the average performance for 1000 randomly assigned initial configurations, each with the corresponding density of *ones*. Shown are LM and GA-found rule-heterogeneous (“Sip”) networks.

on 50:50 with a SD about 6%. We see in Table 1 that, though their performance rate is fairly high, none of the genetically found networks examined out-performs the 2DGKL network for perfect, locally connected network conditions.

When a network solves a majority task problem correctly, it is usually accomplished quickly (i.e., easily) with little evidence of patchiness. For hard problems, on the other hand, initially random distributions of states develop into intermixed patches of all  $-1$ , all  $+1$ , and  $-1/+1$  checkerboards. An example is shown in Figure 3(a). As time goes on, these patches sometimes resolve to the correct state, sometimes to the incorrect state, and sometimes (most often) they get stuck in unresolved configurations. Invariably, when hard problems are solved correctly the patches move coherently across the network. Reminiscent of stomatal experiments, it is never possible (except for trivial situations) to predict which solution scenario will appear for a given problem. (In fact, we have observed that switching the signs of a single  $-1/+1$  pair can make an easy problem hard or vice versa.) Coherent motion is apparently essential to solving hard problems; an LM network never solves hard problems because the patches formed in them do not propagate. As mentioned previously, patchiness in artificial majority task networks and in real stomatal networks are remarkably

**TABLE 1**

Successful Performance Rates for the Majority Task by Various  $15 \times 15$  CA Networks

Simulation set	Network	Performance (% correct)
Baseline	(a) LM	All: $39 \pm 1$ , 50:50: 0
	(b) 2DGKL	All: $93 \pm 1$ , 50:50: $68 \pm 1$
	(c) SipHET	All: $90 \pm 1$ , 50:50: $62 \pm 1$
	(d) SipHOM	All: $85 \pm 1$ , 50:50: $52 \pm 1$
State noise	(a) LM	Max: $5 \pm 1$
	(b) 2DGKL	Max: $75 \pm 2$
	(c) SipHET	Max: $71 \pm 2$
	(d) SipHOM	Max: $62 \pm 2$
Spatial irregularity	SipHET (random)	All: $87 \pm 1$ , 50:50: $55 \pm 1$
	2DGKL, 5% frozen	All: $51 \pm 3$ , 50:50: $1 \pm 1$
	2DGKL, 5% frozen, 2% noise	All: $79 \pm 3$ , 50:50: $47 \pm 2$
Mosaic	Same Sipper, no noise	All: $81 \pm 1$ , 50:50: $45 \pm 1$
	Random Sipper, no noise	All: $55 \pm 5$ , 50:50: $5 \pm 3$
	Same Sipper, 0.5% noise	All: $84 \pm 3$ , 50:50: $51 \pm 3$
	Random Sipper, 0.5% noise	All: $65 \pm 3$ , 50:50: $20 \pm 3$
Asynchrony	2DGKL, 1% synchrony	All: $82 \pm 3$ , 50:50: $48 \pm 3$
	2DGKL, 1% synchrony, 1% noise	All: $95 \pm 3$ , 50:50: $73 \pm 3$

LM, network executing the local majority rule; 2DGKL, network executing a 2D extension of the 1D GKL rule; SipHET, average of four heterogeneous networks executing different rules at different sites; SipHOM, average of four homogeneous networks executing the predominant rule from each of the SipHET networks; SipHET (random), average of four heterogeneous networks executing the respective SipHET rules but with their placement randomly scrambled. Mosaic networks consist of five  $15 \times 15$  tiles on a side; Same Sipper, each tile is the same SipHET network; Random Sipper, each tile is one of the four SipHET networks chosen randomly; Percent frozen, percent of nodes with fixed state; Percent “noise,” percent of states randomly flipped before each iteration; percent synchrony, percent of nodes updating in each time step; All, percent correct for 200 runs each consisting of 1000 initial configurations, for state ratios ranging from 1:99 to 99:1 in steps of 1%; 50:50, percent correct restricted to initial configurations with state ratios near 50:50; Max, maximum performance for 50:50 problems with different levels of added state noise.

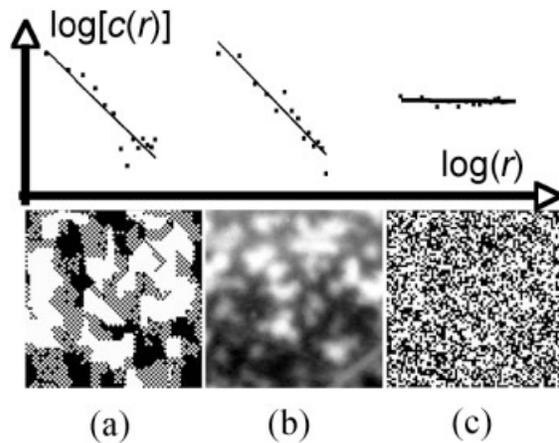
**FIGURE 3**

Image (a) shows coherent state patchiness in a very weakly rewired 2DGKL network. Above the image is the corresponding state-state pair correlation function, showing power law correlation. The slope of the best-fit line is about  $-1$ . Image (b) shows similar coherent patchiness for stomatal apertures on a leaf of the plant *Xanthium strumarium* L. The corresponding correlation function is a power law with about the same exponent as in (a). Image (c) shows very little patchiness in a massively rewired LM network. The corresponding correlation function is flat.

similar. Figure 3(b) shows an example of stomatal patchiness to compare with the CA patchiness in Figure 3(a). Respective pair correlation functions are displayed directly above images (a) and (b); they, like many other spatiotemporal statistics, are essentially indistinguishable [10].

### State Noise

Adding large amounts of state noise ( $\eta \geq 10\%$ , say) destroys the ability of all networks to perform the majority task for 50:50 problems. On the other hand, adding small amounts of noise ( $\eta < 5\%$ ) *improves* the performance of the majority task by all networks—though the LM network still performs exceedingly poorly for such problems. This is shown in Table 1. The entry “Max” indicates the maximum performance level attained for 50:50 problems with added state noise (the amount of which differed for different networks).

### Spatial Irregularity

The distributions of local rules in Sipper’s genetically found heterogeneous networks have strong spatial correlations. If these correlations are eradicated by rearranging the rule placement, network performance degrades—but not by much. As shown in Table 1, networks with the same numerical distribution of rules but with randomly scrambled spatial placement (“SipHET-random”) perform only slightly less well than their corresponding original networks. The decline in performance is totally accounted for by the 50:50

problems, indicating that the precise placement of the rules in the unscrambled networks is a result of genetic fine tuning for that purpose alone. Note that networks with randomly scrambled rule placement still (slightly) out-perform homogeneous networks (SipHOM) using a single, reasonably successful, genetically found rule.

Inclusion of frozen nodes has a profoundly detrimental effect on task performance via collective dynamics. In these simulations, successful performance of the majority task was taken to be the following: all nodes other than the frozen ones reach the correct state before the cutoff time. As shown in Table 1, if 5% of the nodes in a 2DGKL network are fixed in unchanging states, overall performance is almost halved and virtually no 50:50 problems are solved. On the other hand, adding 2% state noise to such damaged networks increases the overall performance back to 85% of what it would be in a perfect, noise-free situation (and to  $>70\%$  for 50:50 problems). Similar results were found for the other networks in this study.

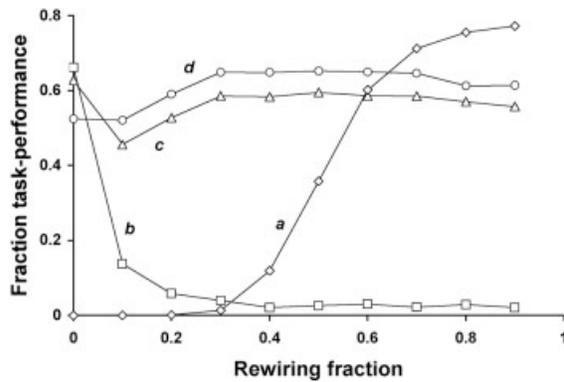
### Mosaic Networks

Despite the fact that Sipper’s  $15 \times 15$  node networks are wired in wraparound fashion, when they are “pulled apart” and wired sequentially in a large array, the resulting network is still able to perform the majority task fairly well. As the number of tiles (modules) in such a mosaic increases, the performance decreases, but not precipitously. The decrease is due to 50:50 problems only. Table 1 shows the performance for mosaic networks, five modules on a side ( $75 \times 75$  nodes), both where each module is the same Sipper rule-heterogeneous network and where all four Sipper networks are used in random arrangement. The same-module structures perform only slightly less well than do their component individual modules. When state noise is added the performance in the  $75 \times 75$  networks rises to almost the same level as in the  $15 \times 15$  modules. Remarkably, even the random-module structure performs the majority task more often than not, doing so more frequently when noise is present.

### Complex Connectivity

Figure 4 shows the effect on the performance of the majority task for 50:50 problems of introducing random rewiring in the four network types we studied. Though the LM network essentially never works well for 50:50 problems in strictly locally connected networks, it begins to perform better and better as the concentration of long-range wires increases beyond a threshold value of  $\sim 30\%$  (plot *a*; a conclusion corroborated by studies on 1D networks [18]). On the other hand, the high performing, locally connected 2DGKL network suffers a monotonic degradation of performance as the fraction of long-range wires increases (plot *b*). Curiously, Sipper’s genetically found, rule-heterogeneous networks have an intermediate response to rewiring (plot *c*).

**FIGURE 4**



Task-performance by the (a) LM, (b) 2DGKL, (c) SipHET, and (d) SipHOM networks for 50:50 problems only on noise-free  $15 \times 15$  networks as functions of network rewiring. Small amounts of rewiring cause degradation in performance by the 2DGKL, SipHET, and SipHOM networks; the LM network performs extremely poorly for small rewiring. For large amounts of rewiring SipHET, SipHOM, and LM perform well, whereas 2DGKL performs extremely poorly.

For low rewiring probability, their performance also degrades relative to the locally connected performance, reaching a minimum at about 10% rewiring. Beyond that, however, their performance begins to increase again, rising back to about the perfect network level at ~30% rewiring. The robustness of Sipper’s heterogeneous networks is *not* due to the diversity of local rules employed in them. Homogeneous networks created from each heterogeneous network’s predominant rule respond identically to rewiring (plot *d*).

Although small amounts of long-range wiring never enhance performance of the majority task in the 2D networks we examined, the deleterious effects of rewiring can be offset, to some degree, by the beneficial effects of state noise. For example, introducing 5% rewiring without state noise causes the 2DGKL performance on 50:50 problems to drop from 68 to 25%, but adding 0.5% state noise produces a recovery to 65%.

### Asynchrony

Irrespective of whether their nodes’ states are updated synchronously, configurations of all  $-1$  and all  $+1$  are fixed points for all networks examined here. Asynchronous updating inevitably produces a higher proportion of “stuck” attractor configurations that do not resolve to a correct solution prior to the cutoff time. On the other hand, exactly like the negative effects of rewiring, this condition can be overcome by adding small amounts of state noise (see Table 1). Though Table 1 only includes data for the locally connected 2DGKL network, we observed the same outcome for all perfect and rewired networks. In all cases, the performance levels of the corresponding networks in both syn-

chronous and asynchronous mode are statistically identical for state noise levels  $\eta \approx 1\%$ , regardless of connection topology.

### 4. DISCUSSION

The observations presented above suggest a number of implications for understanding and designing CPU-less technological networks that perform the “identify and correct” (majority) task and also for better understanding stomata. Let’s address stomata first. Coherently propagating patches—whatever their function—can only occur, we find, in networks with relatively low density of long-range wires [a so-called “small world” topology [19]; see Figure 3(c)]. Thus, long-range coordination of stomata—through signals carried by veins, for example—cannot be very important for optimal gas exchange. In addition, in order to (finally) develop a (to date elusive) quantitatively predictive model of stomatal behavior, it is essential to account for stomatal interactions. In this regard, it is equally essential to experimentally elucidate these interactions, in particular, their ability to produce coherent collective activity.

Now, let’s turn to implications for the design of “identify and correct” networks. (1) Attempting to perform the “identify and correct” task at the individual node level (like the LM rule) does not produce task performance at the level of the network over a wide range of conditions. (2) Interactions between nodes that are optimized for task performance in perfect, locally connected networks (like the 2DGKL rule) can fail in imperfect networks containing frozen nodes and wiring irregularity; on the other hand, interactions that are suboptimal in perfect networks (like Sipper’s GA-found rules) can sometimes exhibit high quality performance in damaged networks. (3) Networks in which the nodes operate slightly differently from site-to-site (again, like Sipper’s rules) can out-perform those that are perfectly homogeneous. (4) Large, functional networks can sometimes be constructed from small, functional modules, even though the modules are not designed to be wired as in the large array. And, (5) small amounts of wiring irregularity typically degrade performance in networks that perform well with perfect, local connectivity; on the other hand, some such networks exhibit high quality performance when the wiring irregularity is massive. Though the discussion here is based on our CA simulations, we note that our conclusions are more general: we have repeated all of the experiments reported in this work with analogous continuous time, continuous state networks (i.e., *cellular nonlinear networks*) and find qualitatively identical results (unpublished results).

Finally, the overarching lesson of our studies is that small amounts of temporal noise often facilitate task performance and can sometimes help to overcome structural defects. As noted previously, the primary failure mode for “identify and correct” networks is the formation of unresolving, minority patches. A fundamental characteristic of the networks that

perform this task competently is that such stuck patches are fragile. That is, small perturbations on the boundaries between stuck patches can spawn massive changes that eventually sweep through the whole network, similar to how “avalanches” of dynamical activity are triggered in self-organized critical (SOC) systems [11]. This is why state noise can sometimes improve task performance in these systems. It can be expected, we believe, that for at least some artifi-

cial and biological networks, not only is suppression of noise not necessary, its presence may actually be essential to optimizing network function.

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