Q1. (5 points each) Short and Simple.
(a) Is the function \( e^x + e^{-x}/2 \) even or odd? Why?

(b) What is the Gibbs phenomenon?

c) Calculate \( \lim_{x \to 0} \frac{\sin(nx)}{x} \).

(d) Consider the two vectors \( \mathbf{a} = \begin{pmatrix} 1 \\ i \\ 2 \\ -i \end{pmatrix} \) and \( \mathbf{b} = \begin{pmatrix} 1 \\ 2i \\ 2 \\ 1 \end{pmatrix} \) that live in a four dimensional complex vector space. Calculate the inner product \( (\mathbf{a}, \mathbf{b}) \).

(e) Calculate the integral \( \int_{-\infty}^{\infty} (3 + 2x + x^2) \delta(x - 1) \, dx \).
Q2. (5 points) Starting with \[ q(x,t) = \frac{1}{2} \left[ a(x + ct) + a(x - ct) + \frac{1}{c} \int_{x-ct}^{x+ct} b(x') dx' \right] \], show that \( a(x) \) is the initial displacement.

Q3. (10 points) Consider a vector space of functions defined on the interval \( 0 \leq x \leq L \). Find the norm of the function \( f(x) = x^2 \).
Q4. The function $q(x,t) = Ae^{-(x-ct)^2/\alpha^2}$ is a solution to the wave equation.

(a) (10 points) Describe in words, as completely and precisely as possible, the nature of this solution.

(b) (10 points) Find the initial conditions $a(x)$ and $b(x)$ that give rise to this solution.
Q5. Consider the 1D wave equation on the interval \(0 \leq x \leq L\) with boundary conditions 
\[ q(x,t) = q(L,t) = 0. \]

The general solution can be written as 
\[ q(x,t) = \sum_{n=1}^{\infty} \sin\left(\frac{n\pi x}{L}\right)\left[b_n \cos(\omega t) + d_n \sin(\omega t)\right]. \]

(a) (5 points) What is the relationship between \(\omega\) and \(n\pi/L\)?

(b) (15 points) Starting with the equation above, find an expression for \(d_n\) in term of the initial conditions \(a(x)\) and \(b(x)\). Carefully show your work.
Q6. the function \( f(x) = \begin{cases} e^{-Ax} & x \geq 0 \\ 0 & x < 0 \end{cases} \).

(a) (15 points) Calculate the Fourier transform \( h(k) \) of \( f(x) \). Express your answer for \( h(k) \) in the form \( h_R(k) + ih_I(k) \), where \( h_R(k) \) and \( h_I(k) \) are both real functions.

(b) (5 points) Find \( \int_{-\infty}^{\infty} |h(k)|^2 \, dk \).