

# Intermediate Laboratory - PHYX 3870-3880

## Glossary of Important Terms in Data And Error Analysis

### Measurement Theory

*Precision* - A measure of the reproducibility of a measurement. If an experiment has small random errors, it is said to have high precision.

*Accuracy* - A measure of the validity of a measurement. If an experiment has small systematic errors, it is said to have high accuracy.

*Discrepancy* - The difference between two measured values of the same quantity.

*Uncertainty* - The outer limits of confidence within which a given measurement "almost certainly" lies. It is important to specify what criteria are used to determine the confidence limits.

*Absolute Uncertainty* - The uncertainty of a quantity expressed in the same units as the quantity. For example, a measured length might be 1.0 +/- 0.1 m, that is a length of 1.0 m with an uncertainty of 0.1 m.

*Relative (Fractional) Uncertainty* - The ratio of the absolute uncertainty to the estimated value of a measured quantity. The example above has a relative uncertainty of 0.1 or 10%.

*Systematic Errors* - Errors which are characterized by their deterministic nature. Such errors are frequently constant. An improperly zeroed meter results in a typical systematic error.

*Random (Statistical) Errors* - Errors which are due to random or stochastic phenomenon which are characterized by the property that repeated occurrences of the phenomenon do not always lead to the same observed outcome. The uncertainty in the number of radioactive decays per unit time from a standard source is an intrinsically random error.

*Illegitimate Errors (Mistakes, Blunders)* - Errors resulting from procedural errors on the part of the experimenter. Computational errors are included in this category. A common illegitimate error is to read the wrong scale of a meter.

*Percent Error* - The ratio, expressed as a percentage, of the difference between two values of an unknown to the

average value of the quantity.

*Propagation of Errors* - A method of determining the error inherent in a derived quantity from the errors of the measured quantities used to determine the derived quantity.

*Significant Figures* - A notation convention for writing the value of measured quantities. In general, the measured quantity should have only as many significant figures as warranted by its absolute uncertainty.

*Rounding* - A method of truncating numbers, particularly useful in the context of significant figures. By standard convention, numbers to be rounded should be truncated for trailing numbers less than 5, rounded up for trailing numbers over 5, and rounded to the nearest even number for a trailing 5.

*Dimensions* - The fundamental quantities used to express physical quantities independent of the system of units used. The basic dimensions are length (L), time (T), mass (M), and electric current (A).

*Dimensional Analysis* - The use of dimensions of physical quantities to verify calculations and formulas. For example, Newton's Second Law states that  $F = ma$ . Dimensional analysis shows that  $F \underline{=} MLT^{-2}$  represents the dimensions of force. Note the symbol  $\underline{=}$  represents the equality of dimensions. Dimensional analysis is not capable of completely determining an unknown functional relationship, but it can delimit the possibilities and, in some cases it can give the complete relationship to within a constant of proportionality.

*Units* - An arbitrary set of measurement standards used to compare physical quantities. Common systems of units include the meter-kilogram-second (MKS or SI) system, the centimeter-gram-second (CGS) system, and the foot-pound-second (English) system.

*Rejection of Data (Chauvenet's Criterion)* - An algorithm of determining when to reject a measurement as unrealistic. Chauvenet's criterion states that if the probability of the value in question deviating from the mean value of a series of N such measurements is less than  $\frac{1}{2}N$ , then the data should be rejected.

### Probability and Statistics

Consider an **example set** of  $N=9$  measurements (2,5,5,5,6,6,7,8,9) in the definitions below.

**Mean (average)** - The average value of  $N$  independent

measurements  $x_i$  is given by  $\bar{x} = \langle x \rangle = \frac{1}{N} \sum_{i=1}^N x_i$

Standard notation denotes average values with a "bar" over the symbol or surrounded by angular brackets. The mean of the example set of nine measurements is 5.9.

**Median** - The median is the measured quantity that is as frequently exceeded as not. The median of the example set is 6.

**Mode** - The most probable measured value. The mode of the example set is 5.

**Standard Deviation (Root-mean-squared deviation)** - The standard deviation is related to the average error of **each** measurement. It is the square root of the average value of the square of the individual deviations from the mean. For  $N$  independent measurements, the standard deviation is

$$\sigma_x = \sqrt{\frac{1}{N-1} \sum_{i=1}^N (x_i - \bar{x})^2}$$

The standard deviation of the sample set is 1.9.

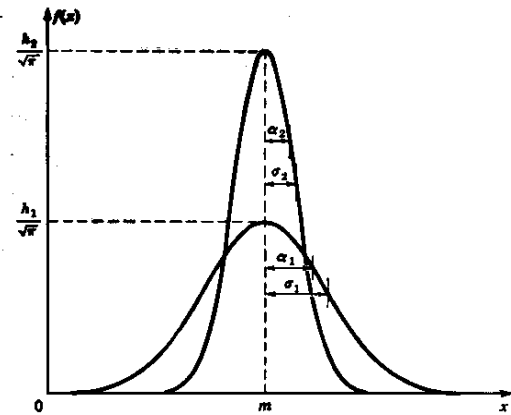
**Variance** - The square of the standard deviation, that is the average value of the square of the individual deviations from the mean. The variance of the sample set is 3.6.

**Standard Error (Standard Deviation of the Mean)** - The standard error is a measure of the uncertainty in the average value  $\langle x \rangle$  of  $N$  measurements and is given by:

$$\sigma_{\bar{x}} \equiv \frac{\sigma_x}{\sqrt{N}} = \sqrt{\frac{1}{N(N-1)} \sum_{i=1}^N (x_i - \bar{x})^2}$$

**Confidence Level** - The range of uncertainty within which an error of a quantity lies. For instance, one expects that 68% of any subsequent measurements  $x$  which obeys a normal distribution, will fall within the range  $\langle x \rangle \pm \sigma_x$ . Likewise, the stated value of  $\langle x \rangle \pm 2\sigma_x$  has a 95% confidence level.

**Normal (Gaussian, Bell-shaped) Distribution** - A normalized, symmetrical probability distribution function centered at  $\langle x \rangle$  with a width  $\sigma_x$  given by



Gauss distributions for two values of  $h$  with the same  $m$ ;  $h_2 = 2h_1$ . Positions of  $\sigma$  and  $\sigma$  for the two curves are shown.

$\frac{1}{\sigma \cdot 2 \cdot \pi} e^{-\frac{(x-\bar{x})^2}{2 \cdot \sigma^2}}$ . If a measurement is subject to many small sources of random error and negligible systematic errors, then the measured values will follow a normal distribution. The best estimate of a quantity obeying a normal distribution is  $\langle x \rangle$  with an uncertainty of  $\sigma_x$  at the 68% confidence level.

**Binomial Distribution** - A normalized probability distribution function which gives the probability of having  $v$  "successes" in  $n$  trials, where  $p$  is the probability of

success in a single trial:  $\binom{n}{v} \cdot p^v \cdot (1-p)^{n-v}$ . The

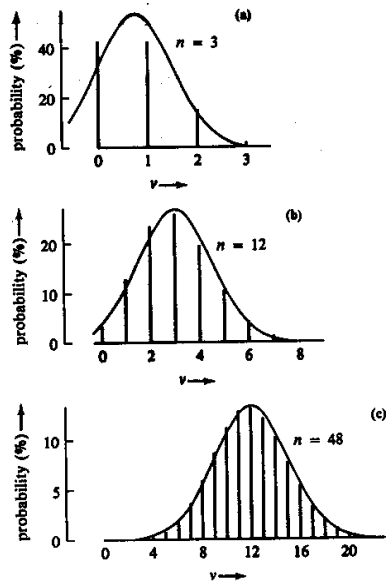
average number of successes is  $\langle v \rangle = np$  with standard deviation  $\bar{v} = \sqrt{np(1-p)}$ . Note this distribution

predicts discrete outcome and is not symmetric. For any value of  $p$  and for large  $n$  this distribution can be approximated by a Gaussian distribution.

**Poisson Distribution** - A normalized probability

distribution function  $e^{-\mu} \cdot \frac{\mu^v}{v!}$  which is a special case of

the binomial distribution for a large number of trials and a small probability of success. The expected value is  $\langle v \rangle = \mu$  where  $\mu$  is the average number of successes per unit time. This distribution describes the results of experiments where one counts events that occur at random, but at a definite



The binomial distributions with  $p = \frac{1}{2}$  and  $n = 3, 12, \text{ and } 48$ . The continuous curve superimposed on each picture is the Gauss function with the same mean and same standard deviation.

average rate. It is especially important in atomic and nuclear physics, where one counts the disintegration of unstable atoms and nuclei.

*Addition in Quadrature* - The combination of numbers by taking the square root of the sum of the squares, that is  $X_{sum} = \sqrt{X_1^2 + X_2^2}$ . This is used to combine errors for two or more measurements subject to random errors, obeying a normal distribution.

*Full Width at Half Maximum* - A measure of the width of a peak or distribution at half of the maximum intensity. For a normal distribution,

$$FWHM = 2 \cdot \sigma_x \cdot \sqrt{2 \cdot \ln(2)} = 2.35 \cdot \sigma_x$$

*Linear Regression* - An analytical method for finding the best straight line to fit a series of experimental data. Also referred to as the least-squares fit for a line.

*Least Squares Fit* - An analytical technique for solving for the coefficient of a general functional form to fit a series of data by minimizing the sum of the squares of the difference between the data and function.

*Chi Squared Test* - A statistical test to determine whether a series of measured data is consistent with an expected outcome.

$$\text{Generally, } \chi^2 = \sum_{i=1}^N \frac{(\text{observed value} - \text{expected value})^2}{(\text{error in expected value})^2}$$

and suggests agreement when  $\chi^2 \leq n$ .

*Covariance* - A statistical test to determine the independence or correlation of two variables x and y. The covariance is given by

$$\sigma_{xy} = \frac{1}{N} \cdot \sum_{i=1}^N (x_i - \langle x \rangle)(y_i - \langle y \rangle).$$

*Correlation Coefficient* - A statistical test to determine if two variables x and y are linearly related. The correlation coefficient  $r = \sigma_{xy} / \sigma_x \sigma_y$ . A value of  $r = 0$  indicates that x and y are completely independent, while  $r = |1|$  indicates that x and y are perfectly linearly related with a positive ( $r = +1$ ) or negative ( $r = -1$ ) slope.

*Degree of freedom* - In statistics, the number of degrees of freedom is the number of observed data minus the number of parameters calculated from the data and used in the calculation.