

EXPERIMENT 2000.06.1: Thermal Equilibrium

Topics of investigation: Temperature, heat, heat capacity, entropy, irreversibility

Read about this topic in: Serway, Chs. 16, 17, 18; C&J Chs 13, 14, 15; also, read the attached supplemental reading, NOW!

Toolkit: Computer
Laboratory interface & software (SW 2.3.2)
Two thermal probes
Three aluminum cylinders
Insulated sample holder
Refrigerator
Spreadsheet software

Assumptions:

- (1) Interactions between atoms in a solid are consistent with the conservation of energy and momentum, but involve the transfer of units (or “quanta”) of energy delivered at random intervals.
- (2) Temperature--on the macroscopic level--is a measure of the average energy per atom.
- (3) A macroscopic body consists of a large number of atoms.

Prediction:

When two macroscopic solids are in thermal contact, heat flows from the one at the higher temperature to the one at the lower temperature. This transfer of energy continues until the two solids are at the same temperature.

Exercise 1: Physical determination of thermal equilibrium between two solids containing the same number of atoms

- E1.1 Place two of the three aluminum cylinders in the freezer of the refrigerator.
- E1.2 Launch the Excel spreadsheet **Heat Sim**. Launch **Science Workshop**. Insert temperature probe leads into analog channels A and B. **Don't touch the tips of the two probes. Handle the probes by the plastic wands.** Program the software to create two thermal sensor instruments by: (a) clicking and dragging an analog probe icon onto analog channel A, (b) clicking on “Temperature Sensor” in the “Choose an analog sensor” window, (c) then repeating (a) and (b) for channel B. Click and drag a digital display (“Digits”) onto sensor icon A; repeat for B. Resize the setup window by clicking in the resize box (upper right hand corner). Click on the digital displays and drag them apart so you can observe both at the same time. Click on “Monitor” in the control panel window. You should observe two digital readings corresponding to temperatures in degrees Celsius.
- E1.3 Both probes should be reading the same temperature, since they are both in thermal equilibrium with the air in the room--but frequently they won't. That's because it is fairly difficult to calibrate thermometers exactly. Record the average temperature displayed for A and for B, to identify by how much A is offset from B.
- E1.4 Delete both digital displays. Resize the control window to full size. Click and drag a graph icon onto sensor A. Make a second graph by clicking on the pull down icon on the right in the second row below the sensor A icon in the graph window. Click on sensor B and Temperature probe. Identify which probe is physically connected to channel A and which to B. Resize the control window back down. Click on “Sampling Options” and set “stop time” to 600 seconds.
- E1.5 Within the insulated sample holder position the room temperature aluminum cylinder so that when sensor A is fed through the hole in the insulator it inserts into the slot in the cylinder. After it

has been in the freezer for about ten minutes, remove one cylinder from the freezer and quickly place it inside the insulator on top of the room temperature cylinder, with its slot pointing away from the room temperature cylinder. Feed sensor B through the insulator lid and fit it in good contact with the cold cylinder. Make sure the lid fits snugly on top of the sample holder. Wait about thirty seconds for sensor B to come into thermal equilibrium with the cold cylinder, then click on “Record.” Since you will collect data for ten minutes you should go on to the next exercise. Science Workshop will continue operating in the background. Make sure the cylinders and the sensors continue to make good contact throughout this exercise. One person should probably apply a small amount of force to keep the sensors in contact with the cylinders. If a sensor does not make good contact an insulating layer of air between it and the cylinder can cause the temperature readings to fluctuate wildly.

Exercise 2. Simulation of heat transfer between two equal size solids

- E2.1 Under the Finder icon (right hand most icon in the top menu bar), toggle to Excel. You should see, among other things, a sheet with two cells outlined in green, a graph window, and a Run Sim button. The program you are about to run simulates heat transfer from one body to a second in good thermal contact with it by flipping a set of “spinners.” At initiation, the program supplies each cell in column A (that is, each atom in body A) with one energy unit. The units are labeled 1 through N_R , where N_R is the number of rows. At the same time the simulation sets all cells (atoms) in column (body) B to zero. Then, in each time step, it flips one spinner selecting, at random, one of the N_R energy units to move. Subsequently it flips a second spinner assigning at random a direction--up, down, left, right. The selected energy unit is then moved according to the selected direction. If at any time the selected unit is in a cell in column A and is supposed to move left, it stays put. Similarly, if it is in B and is supposed to move right it stays in place. The same rule applies for units at the top (and selected to move up) and the bottom (and selected to move down) of columns. The random motion of energy units back and forth between atoms on sides A and B models what happens in the two aluminum cylinders that are swapping heat.
- E2.2 Click once in cell F2 (# of rows), type “20,” then hit “return.” If the number of columns is not 2, click once in F4, type “2,” then “return.” Click on the Run Sim button. The simulation software first clears any data from a previous run. At initiation each atom on side A has 1 energy unit while each on side B each has zero. Eventually data will begin to appear on the graph. The filled diamonds are values of average number of energy units per atom on side A, the open squares, average energy units per atom on side B.
- E2.3 Data are sampled and plotted every 50 time steps for a total run time of 5000 time steps. The software calculates average values for sides A and B for time steps 3000 to 5000, as well as the associated standard deviation. Record the averages and the standard deviation. What is your overall impression of the plot of data? Click on Run Sim and repeat the simulation a total of three times, recording average A and average B and standard deviation each time.
- E2.4 The laws of thermodynamics--in this case, heat always goes from a hot body to a cold body--are true for *very* large numbers of atoms (for example, 10^{20}). The simulations in the previous steps involve only 40 atoms (20 on each side), and in any time step you probably observe energy to go from one side to other “willy-nilly.” But 40 atoms is a pretty puny number. So rerun the simulation with 100 rows. (Click once in F2, type “100,” and “return.”) Do you see a difference between the simulations with 20-by-20 atoms and with 100-by-100 atoms? (Note the standard deviation.) The willy-nilliness should be less. Is that what you see? Run the simulation three times to get a sense of how it works with 100-by-100 atoms. Finally, increase the number of rows to 500 and rerun. The data should look even smoother and more monotonic. Do they?
- E2.5 In the case of equal numbers of atoms on both side of the interface equilibrium is established when both sides have the common temperature $T_{equilibrium} = \frac{1}{2}(T_{hot} + T_{cold})$. Assuming that temperature is a monotonically increasing function of the average energy per atom in a macroscopic body, what should the condition for equilibrium be for the simulations done above in terms of

average energy per atom in A and in B? Is that what you observe? (The last simulation may take longer than 5000 time steps to come to equilibrium.)

Return to the data collected by Science Workshop:

Analysis of E1.

- A1.1 The expression above for the equilibrium temperature is derived from an energy balance:

$$C_A T_A = -C_B T_B, \quad (1)$$

where C_A and C_B are the heat capacities of bodies A and B, respectively. Heat capacity depends on what atoms the body is made of and how many atoms there are. Since each cylinder is made of the same kind of atoms the changes in temperature of each cylinder will depend only on the cylinder

masses: $T_A = - T_B \frac{M_B}{M_A}$. Weigh your two cylinders, making sure to note which is "A" and which is "B." Examine the data on your Science Workshop graphs. Is the latter prediction corroborated?

- A1.2 Derive $T_{equilibrium} = \frac{1}{2}(T_{hot} + T_{cold})$ from $T_A = - T_B \frac{M_B}{M_A}$. (What are the assumptions about M_A and M_B in E2?)

Don't delete this data set yet. You'll use it later to answer Question 4, below.

Exercise 3: Physical determination of thermal equilibrium between two bodies containing unequal numbers of atoms

- E3.1 Quickly retrieve the second aluminum cylinder from the freezer. Place it on top of the other two cylinders and make sure that sensor B is good thermal contact with it and that sensor A is good contact with one of the other two cylinders (and the lid is snugly in place). After about thirty seconds click on Record. Keep the sensors in good contact for about ten minutes.

Exercise 4: Simulation of heat transfer between two unequal size bodies

- E4.1 On the Excel spreadsheet Heat Sim, click in F2, type "200," and "return." Click in F4, type "3," and "return." Now the program is set to simulate heat transfer between body A consisting of 200 atoms (1 column, 200 rows), and body B consisting of 400 atoms (2 columns, 200 rows). Initially, there are 200 energy units on side A, and none on side B. Click on Run Sim and observe the plotted data. Repeat for a total of three runs.

- E4.2 Equation (1) can be used in this case, but now $C_B = 2C_A$. (Why?) What should the equilibrium temperature be in this case (in terms of T_{hot} and T_{cold})? What does that translate into in terms of energy per atom on side A and on side B? Is that what you observe (approximately)?

Return to the data in Science Workshop:

- A3.1 Weigh your third cylinder. Do you find that $T_A = - T_B \frac{M_B}{M_A}$? Is that what you see E4 also?

Questions:

1. Temperature is a macroscopic concept. What is an equivalent statement of **thermal equilibrium** from a microscopic point of view? That is, how would the prediction be rewritten in the language of atomic energies? (What is the atomic equivalent of thermal contact, heat flow, and temperature?)
2. Macroscopically, **heat** is defined as the net energy that is transferred between bodies because of temperature differences only. What is an equivalent microscopic definition of heat?
3. One macroscopic version of the **Second Law of Thermodynamics** says that heat travels from a hotter body to a colder body only, *never* the reverse. Give an equivalent microscopic version of the Second Law. (Hint: What do you think the microscopic equivalent of *never* should be?)
4. Another macroscopic version of the Second Law is that in all spontaneous processes **entropy increases**. Macroscopically, the entropy change for a body is defined as the incremental heat flow into or out of the body per unit temperature: $S = \frac{Q}{T}$. The total entropy change for a system of bodies equals the sum of the entropy changes for each. To see how entropy changes during the heat flow you measured in E1 launch the Excel spreadsheet Entropy. In Science Workshop, select your first data set. Make a table by going back to the set-up window and dragging a table icon onto analog sensor A. In the table select and copy the first 500 values. Paste these data into Entropy by clicking on cell A1 and Paste. Return to Science Workshop. Change the table data to Analog B. Select and copy the first 500 values. Paste into Entropy by clicking on cell B1 and Paste. The graph should automatically plot the first 500 values of the total entropy of cylinders A and B. Does the graph agree with your expectation about entropy and the Second Law?

When you have completed all measurements and all simulations, PLEASE QUIT BOTH EXCEL AND SCIENCE WORKSHOP WITHOUT SAVING ANY FILES. Thanks.

Supplemental reading:

A macroscopic body is composed of a fantastic number of atoms: a liter of water contains about 10^{25} atoms, while even a liter of air contains about 10^{22} . In principle, the motion of a composite body can be described as the motion of the body's center of mass, plus motion of the composite pieces relative to the center of mass. The center of mass motion of a macroscopic body is pretty easy to analyze, because the center of mass moves like a single particle. Keeping track of it involves acquiring and updating only 6 pieces of information -- 3 position components and 3 velocity components. Motion about the center of mass entails keeping track of more variables. Sometimes the motion can be approximated as a kind of **collective** activity. For example, the atoms that make up a "rigid" body move collectively together in such a way that the distances between atoms remain approximately fixed. This restriction greatly reduces what individual atoms can do and, as a result, we can describe the motion of a rigid body about its center of mass by only a few additional variables (such as angular velocity, rotational inertia, and so on). The atomic motions in fluid bodies are much less restricted, but even there collective behaviors such as smooth flow and wave propagation can exist. Like rigid motions, flows and waves typically require just a few variables to describe.

A careful study of the atoms in any real body shows that in addition to whatever organized, collective motion there might be, there is quite a bit of apparently disorganized, individualistic motion as well. For many purposes we can ignore these latter motions. For example, when you toss an apple into the air how it moves is almost exclusively dictated by collective behavior: motion of the center of mass plus spin around the center of mass. On the other hand, the disorganized atomic motions we just ignored for the flight of the apple are actually extremely important for other purposes: they are the source of such macroscopic quantities as temperature and heat--the stuff of thermodynamics.

Now, imagine the problem of trying to understand the thermal properties and interactions of macroscopic bodies from first principles. To do so, we would have to acquire something like 10^{25} pieces of position and velocity data, then update those data every 10^{-13} or so seconds (that being the typical time between "collisions" of atoms in a macroscopic body). Such a task is phenomenally implausible: we can perform neither the required measurements nor the required computation. But even if we could accomplish this task, how would we interpret the results? How would the incredible deluge of numbers that would result from such an analysis tell us anything about the thermal interactions we seek to understand?

Remarkably, there *is* a way to connect the microworld of the atoms that make up a body with the macroworld of temperature and heat. The trick that lets us make progress is *to give up*. More precisely, we say that the atomic interactions are just too many and too complicated to keep track of in detail, so, instead, we say that the atoms move about effectively **randomly**.

The word random needs amplification. *Random* means that the next outcome of some process cannot be predicted, even in principle. The unpredictability of a random event is not merely the result of our ignorance. No matter how much information we gather about the process, if it is truly random, we will never be able to predict the next outcome. In a random process, the future is not determined by the past. On the other hand, "random" does not mean "anything goes." For example, if the flip of a coin is random (is it? that's a question that has yet to be resolved), the outcome is constrained to be a head or a tail, but we don't know which it will be. Even if the flip of a coin isn't truly random in a rigorous sense, the process of flipping may be so complicated that for practical purposes the outcome can be taken to be *effectively* random.

Although detailed prediction of the future for a random process is impossible, we *can* say something useful about such a process. In the case of the coin, we could take lots and lots of

flipping data and then present a statement about **average** behavior--for example, "on the average, if you flipped this coin a very large number of times you would expect about half of the outcomes to be heads." Such a statement says little about small data sets. It certainly says nothing at all about a single flip. If you flip a coin 10 times, you might get 10 heads; 10 isn't a "very large number." On the other hand, if you flip a coin 100,000 times, it's pretty safe to say you won't get 100,000 heads, or 80,000, or even 60,000 (although each of those is possible, since long strings of heads are not forbidden by any known laws of physics). The statement says that you'll most likely get something close to 50,000. (Here's an interesting question: Is 7 heads out of 10 flips more unusual than 51,000 heads out of 100,000? **Statistics** is a branch of mathematics that tries to quantify the degree to which you should feel a departure from average in a given set of data is unusual.)

Like the coin example, when we model the internal atomic motions of a macroscopic body as random, not anything can go. Certain laws of physics must be obeyed: for example, no microscopic event that fails to conserve energy (or momentum, or angular momentum, or a bunch of other quantities as well) will be permitted. The ultimate goal of our random model of atomic motions is to say something about the average outcome of large numbers of events. And since, in a typical experiment on a macroscopic body, we are dealing with 10^{20} to 10^{30} atoms and 10^{10} to 10^{20} events for each, **LARGE** is the operational word!!! The enterprise of making average statements about the internal energetics of macroscopic bodies is sometimes called **statistical mechanics**. In this laboratory, we will be doing a kind of statistical mechanics analysis.