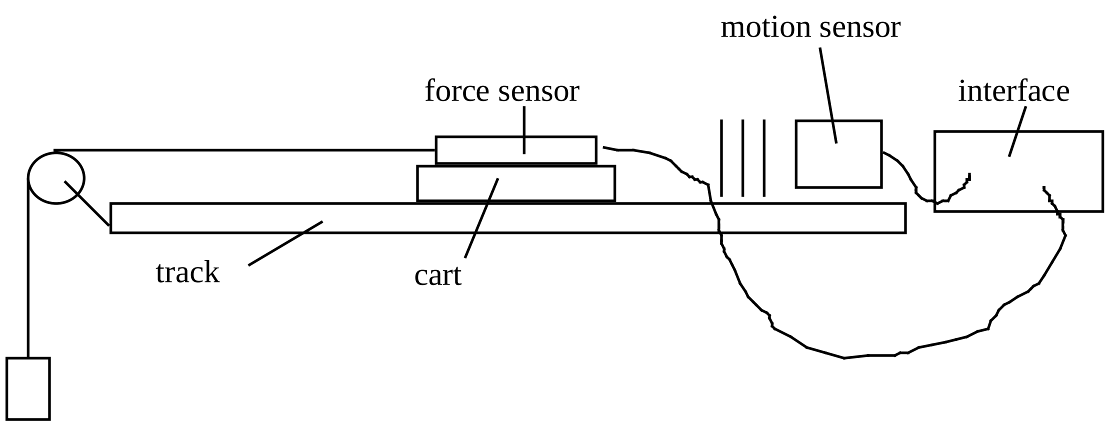
EXPERIMENT 2000.03.1: Work and Energy

Topics of investigation: The relation between force and acceleration

Read about this topic in: Serway, Ch. 7, 8; C&J Ch. 6

Toolkit:
Computer
Laboratory interface & software (SW 2.3.2)
Motion sensor
String and pulley
Masses: 20 g, 50 g, 100 g; two massive bars
Flat track
Cart

Sketch of set up:



First Assumptions:

(1) The leveled track in this exercise can be used to compensate for gravity. (2) The light string and pulley have negligible affect on the dynamics of the masses in the setup. (3) Frictional forces on the cart are negligible.

First Prediction:

If the mass of the cart plus force sensor is M and the mass attached to the "falling" end of the string is m, then, given the above assumptions, the work done on the cart plus force sensor is due to the tension in the string only. Under the assumptions above, the tension, T_0, is constant (as long as the cart is accelerated by the string only) and, therefore, the net work done on the cart plus force sensor as they undergo a displacement Δx is given by

$$W_{net\ on\ cart} = T_0 \Delta x .$$

At the same time the cart is undergoing a displacement Δx the falling mass is undergoing a displacement Δy, equal to $-\Delta x$. The net work on the falling mass during the same interval, then, is given by

$$W_{net\ on\ falling\ mass} = (T_0 - mg)\Delta y = -(T_0 - mg)\Delta x$$

The work-energy theorem states that the net work done by all forces acting on a body equals the change in the body's kinetic energy: $W_{net} = \Delta K$, where $K = \frac{1}{2} mv^2$. Consequently, we expect that

$$T_0(x_f - x_i) = \tfrac{1}{2}M(v_f^2 - v_i^2)\text{, for the cart}$$

and

$$-(T_0 - mg)(x_f - x_i) = \tfrac{1}{2}m(v_f^2 - v_i^2)\text{, for the falling mass.}$$

Note that if the last two equations are added, the result is

$$mg(x_f - x_i) = \tfrac{1}{2}(m + M)(v_f^2 - v_i^2).$$

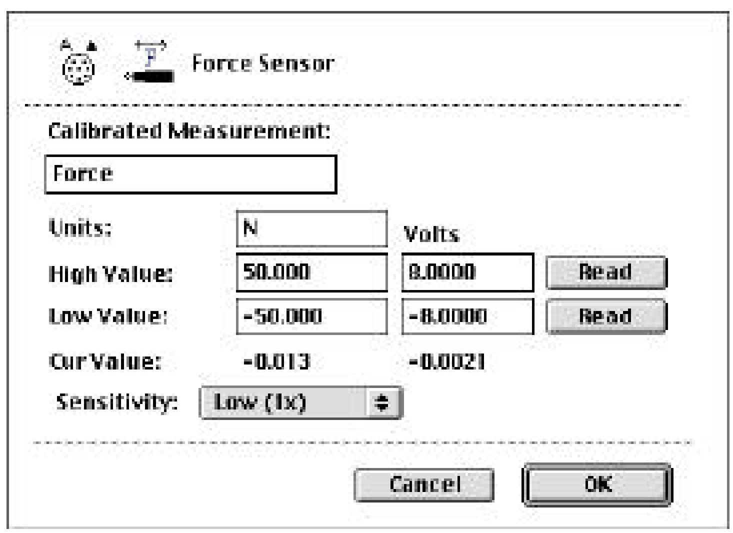
Defining the change in gravitational potential energy as $\Delta U_{grav} = mg\Delta y$, the latter becomes a statement of conservation of mechanical energy $0 = \Delta(K + U)$.

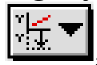
Before proceeding: Make sure you understand how each of the equations in the Prediction comes about.

Exercises:

E0. Getting started

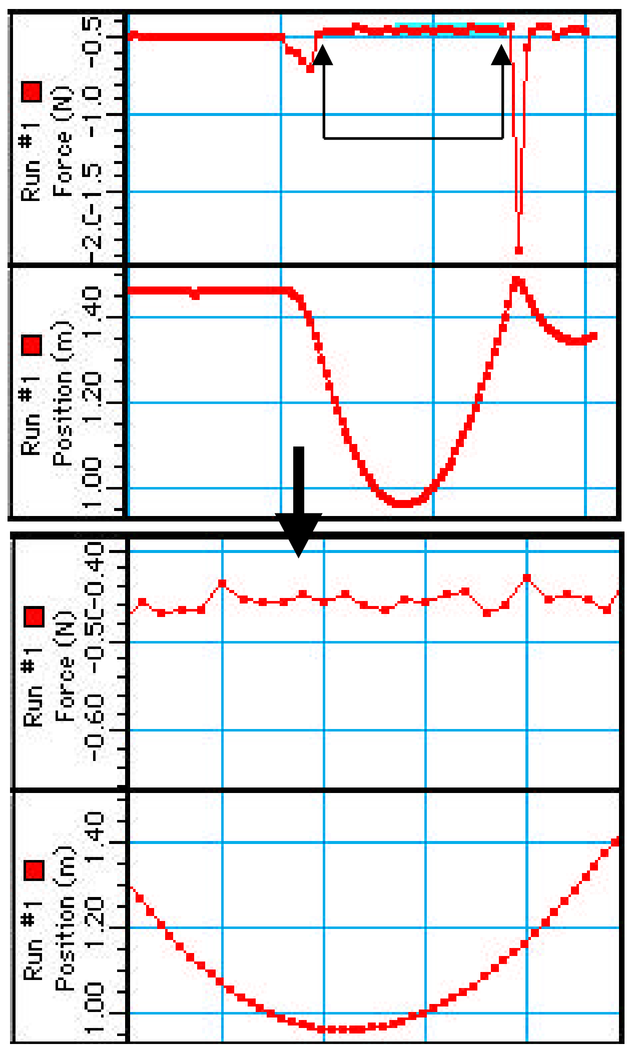
•E0.1 Weigh your cart, force sensor, and all additional masses (2 bar masses per cart). Record all values.
•E0.2 Level your air track if necessary. A bubble level should be available to check your track. Each station should have a string with loops on both ends that is about 1 m long.
•E0.3 Turn on your interface box. Start your Mac. Launch Science Workshop. Make sure the motion sensor leads are properly inserted in the interface box (yellow in first digital port, black in the second). Make sure the lead from the force sensor is inserted in analog port A.
•E0.4 Program the software to recognize a motion sensor (drag a digital plug icon onto the first Digital Channel, then define it in the Choose a digital sensor window) and a force sensor (drag an analog plug icon onto analog port A, then define it in the Choose an analog sensor window).
•E0.5 Calibrate your force sensor. To do this, double click on the force sensor icon in the set-up window. A window like the one shown to the right should appear. The range of forces, -50 N to +50 N, is too coarse for the purposes of this lab. (A negative force is a pull, a positive one a push.) With the force sensor lying in the cart bed (as shown in the sketch above) and with no strings attached, press the TARE button on the side of the sensor for about a count of ten. Release. The Cur Value under Volts should now be very close to zero. **Click on the High Value Read button**. That will insert the current voltage output of the force sensor in the Volts box. **Replace the High Value 50.000 N with 0**. That will identify the current voltage output of the force sensor with a force of 0 N. Attach a string with the 100 g mass on one end to the hook. Feed the string over the pulley and allow the cart to rest gently against the stop. Cur Value Volts should now be some value such as -0.1 or -0.2. **Click on Low Value Read and replace the -50.000 N with -0.98**. The force sensor is now calibrated for "pulls" between 0 N and 0.98 N. Click OK.



•E0.6 Create a graph of force sensor readings by dragging a graph icon onto the force sensor icon. Using the add graph button, , add to the first graph two graphs of motion sensor output, Position, x (m), and Velocity, v (m/s).

E1. Work and kinetic energy

•E1.1 Central to the First Prediction is the idea that once the cart is moving under the influence of the string only, tension is independent of cart motion. This aspect of the prediction may be investigated by launching the cart toward the motion sensor and collecting force-time data for a round trip. If the acceleration is constant throughout, a graph of the output of the force sensor as a function of time will be constant. Let's try it. Place the 50 g mass on the end of the string. Start with the cart at the end of the track farthest from the motion sensor. Practice setting the cart in motion with a **gentle** push so that the 50 g mass comes almost up to the pulley before falling back down. The member of the team who is putting the car in motion should also carefully hold the force sensor lead cord up so that it doesn't rub or bind on the table. Don't pull the cart with the cord. When ready to record data, remove the string from the force sensor hook while at the same time pressing the TARE button for a count of ten. (That resets the force sensor to its original calibration. **You have to "TARE" each time you are ready to take a new set of data.**) Place the string back on the hook. Another member of the team clicks on REC to start collecting data. Push the cart into motion. Continue collecting data until the cart hits the restraining bumper.



•E1.2 Your data set will include moments when the hand is pushing the cart, moments when the string only is affecting the cart's motion, and moments when the cart is colliding with the bumper. It is only the middle portion of the data for which the Prediction might be valid. To see how close the tension is to being constant, click on the Σ symbol in the graph window. This should open up a statistics panels for both graphs. In the pull down menu of the force-time panel, select Mean and Standard Deviation. With the magnifying glass tool select the portion of the force-time graph that corresponds to the smoothly curved position-time data, e.g., between the two arrows in the top graph on the right. The result should look like the bottom two graphs on the right. Select the force data from the leftmost time to the time when the velocity graph goes through zero. Record the mean force and standard deviation for this time interval. Then select the force data from the time when the velocity graph goes through zero to the rightmost time. Again record the mean force and the standard deviation. Can you tell any difference? (**You can't if the two force ranges, [mean-stddev, mean+stddev], overlap.**)

•E1.3 With the cross hairs tool on the graph window identify and record the position of the cart at the leftmost time (call it x_i). Similarly identify and record the position of the cart when its velocity is zero (call it x_f).

Analysis

•A1.1 Using the average of the values of the tension in the string obtained in E1.2 and the initial and final positions obtained in E1.3 calculate the work done on the cart by the tension in the string in the interval you selected. Similarly, calculate the net work done (by tension and gravity) on the falling mass in the same interval.

•E1.4 Repeat E1.3 but now using x_f measured there as a new initial position x'_i. Take the new final position (x'_f) to be the position of the cart at the rightmost time on the graphs.

•A1.2 Repeat A1.1 using x'_i and x'_f.

•E1.5 Identify and record the velocity of the cart at x_i and x_f.

•A1.3 Calculate the change in kinetic energy of the cart between x_i and x_f. Calculate the change in kinetic energy of the falling mass over the same interval. Compare the net works calculated in A1.1 with the appropriate kinetic energy changes. Are they (almost) equal?
•A1.4 Add the two kinetic energy changes calculated in A1.3. Calculate separately the gravitational potential energy change of the falling mass in the interval selected. Add the total kinetic energy change to the gravitational potential energy change. Do you get zero as the Prediction says?

•E1.6 Repeat E1.5 for x'_i and x'_f.

•A1.5 Repeat A1.3 and A1.4 for x'_i and x'_f.

•E1.7 Place the two massive bars on the cart and repeat all of the previous steps.

E2. Desperately seeking friction, again

Amended Assumption:

Friction, while small, is not negligible. We assume, here, the frictional force, *f*, is constant.

This leads to a

Second Prediction

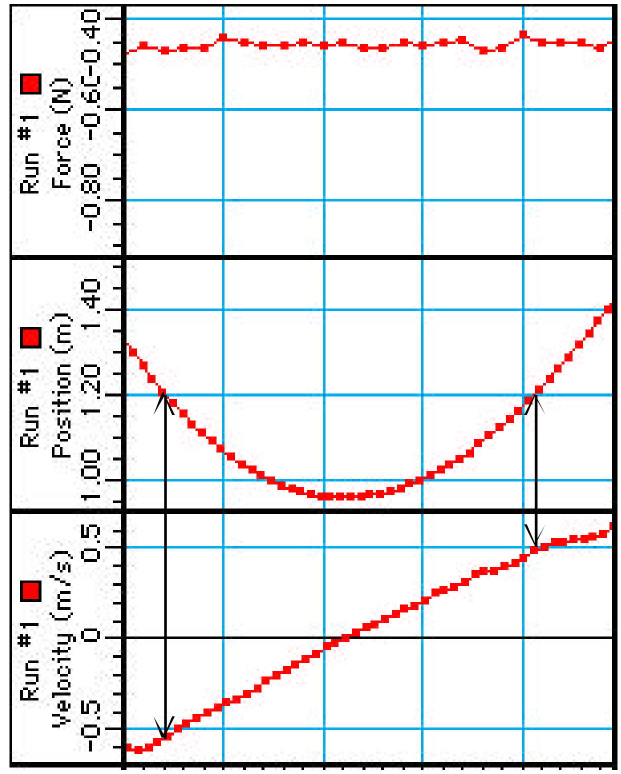
The work done in a round trip by tension and by gravity will be zero, but the work done by friction in the round trip will be

$$W_{friction} = -fd, \text{ where } d \text{ is the total distance the cart travels.}$$

Note that the work done by friction is always negative. As a result
$\Delta(K+U) = \Delta K = W_{friction}$, in a round trip.

You should verify these predictions before proceeding.

•E2.1 For the first data set collected in E1, pick a round trip such as the one shown by the arrows in the graphs to the right. With the cross hairs tool identify and record the positions at the beginning and end of the trip and the position when the velocity is zero. Similarly, identify and record the velocities at the beginning and end of the trip.



•A2.1 Use the Second Prediction to calculate the magnitude of the frictional force.

•E2.2 Repeat E2.1 for the second data set.

•A2.2 Repeat A2.1 for the second data set.
•A2.3 Do the frictional forces calculated in A2.1 and A2.2 obey the law for kinetic friction, $f = \mu_k N$?

Worksheet:

exercise 1
mass: cart+FS (g) =
falling mass (g) = 50
predicted accel. (m/s^2)
predicted tension (N)

a3	accel. (m/s^2)	tension (N)

average accel. ± stddev (m/s^2) =
average tension ± stddev (N) =

exercise 3
mass: cart+FS+1 bar (g) =
falling mass (g) = 50
predicted accel. (m/s^2) =
predicted tension (N)

a3	accel. (m/s^2)	tension (N)

average accel. ± stddev (m/s^2) =
average tension ± stddev (N) =

exercise 2
mass: cart+FS (g) =
falling mass (g) = 50
predicted accel. (m/s^2)
predicted tension (N)

a3	accel. (m/s^2)	tension (N)

average accel. ± stddev (m/s^2) =
average tension ± stddev (N) =

exercise 4
mass: cart+FS+2 bars (g) =
falling mass (g) = 50
predicted accel. (m/s^2) =
predicted tension (N)

a3	accel. (m/s^2)	tension (N)

average accel. ± stddev (m/s^2) =
average tension ± stddev (N) =

exercise 5

au (m/s^2)	ad (m/s^2)	f (N)

average f =

exercise 7

au (m/s^2)	ad (m/s^2)	f (N)

average f =

exercise 6

au (m/s^2)	ad (m/s^2)	f (N)

average f =

exercise 8

au (m/s^2)	ad (m/s^2)	f (N)

average f =