EXPERIMENT 2000.13.1: Chaos

Topics of investigation: Chaos in physiological systems

Read about this topic in: Supplementary notes

Toolkit: Computer
                    Laboratory interface & software (SW 2.3.2)
                    Heart rate sensor
                    Excel sheet ***Chaosgenerator***

Overview

      Almost all animal and human physiological systems—such as blood pressure, heart rate, brain "wave" activity, and balance, to name just a few—show considerable irregular variability from moment-to-moment, even in a resting state. In fact, some irregularity seems to be necessary for good health. When the irregularity smoothes out, that is often a sign of life threatening illness. Apparently, rigid physiological regularity is associated with loss of adaptation, a condition that places the system at great risk for harm.

      Recently, there has been a concentrated effort to uncover the nature and source of erratic physiological variation, especially in the heart and the brain. There is a dawning recognition that the electrical and mechanical variability observed in these organs is closely allied to the phenomenon of ***deterministic chaos***. Studies strongly suggest that both atrial (Garfinkel, 1992) and ventricular (Witkowski, 1995) fibrillation are chaotic, as is neuronal bursting in the brain (Schiff, 1994). Indeed, using the mathematics of chaos, researchers have shown that it is possible to regularize atrial fibrillation in the heart (Langberg, 1997), suppress (and enhance) electrical irregularity in the brain (Schiff, 1994), and predict with a high degree of reliability when an epileptic seizure is about to ensue (Lehnertz, 1998).

      This laboratory consists of two separate investigations: an experimental study of variability in your own heart rate and a simulation of cardiac chaos and its control.

First Exercise:

E0. Getting started

•E0.1 Turn on your interface box. Start your Mac. Launch Science Workshop. Launch the **Excel spreadsheet Chaosgenerator**. You may be confronted with an Excel window asking you whether or not to "enable macros." Click the "Enable Macros" button. A heart rate sensor probe should be plugged into Analog Channel A.

•E0.2 Program the interface to receive data from an analog heart rate sensor by dragging an analog plug onto Channel A and clicking on Heart Rate Sensor in the sensor window that appears. Drag a graph icon onto the heart rate sensor icon in the set-up window and select Voltage (V). Click and drag a table icon onto the heart rate sensor icon to open a table. Reduce the set-up window to the

control window only by clicking in the resize box, ▱▱.

•E0.3 Click on the Sampling Options button in the control panel window. Increase the Periodic Samples rate to 100 Hz using the scroll bar. Click OK.

E1. Heart Rates

•E1.1 The heart rate sensor monitors flow of blood (not electrical activity as in an EKG). The clip is designed to shine light through a part of the body and record the transmitted intensity as a function of time. The coursing of blood through vessels sandwiched in the clip causes the intensity

of the transmitted light to vary. A small varying voltage is sent to the amplifier box and converted into a signal in the range of 1 to 5 volts. Attach the clip to an ear lobe OR to the web of skin between thumb and index finger. Notice that the amplifier box is equipped with a red light that flashes every time the voltage reaches a certain level. When operating properly the light will flash brightly once every pulse. It takes a couple of seconds for the electronics to settle down after moving the clip, so be patient . To obtain a good signal you must sit or stand quietly. It's helpful to concentrate on the flashing light. Try to get it to flash regularly and brightly. If you can't get a nice repeatable pattern you may have to reposition the clip.

•E1.2 Have a second member of the team record a few seconds of data. Click on the full data set button, , on the graph window to scale the graph display to a useful size. An example of the kind of graph you should try to obtain is seen to the right .   Once you are able to generate such a graph, record about *two minutes* of heart beat data.
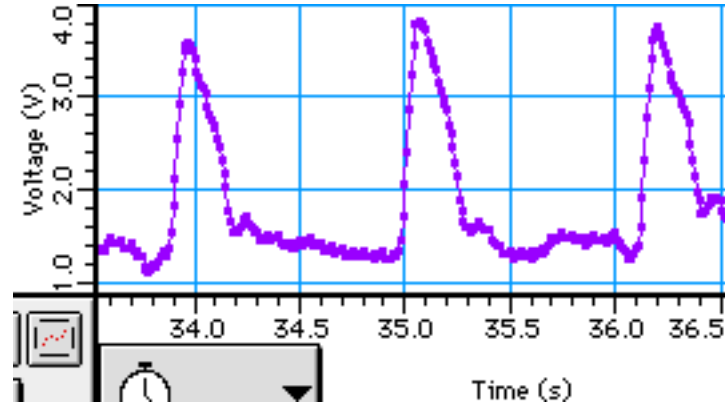
•E1.3 The team member whose pulse was just recorded should remove the clip and engage in a brief period of vigorous exercise (run around the outside of the building or run up two flights of stairs, for example). Then repeat steps E1.1 and E1.2.
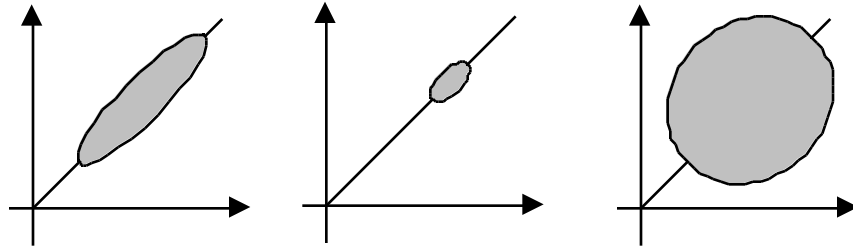
•E1.4 Trade roles until each team member has a resting and a recovery heart rate record.

Analysis

•A1.1 Convert your voltage-time graphs into heart rate-time graphs by clicking on the analog sensor icon on the vertical axis of your graphs and selecting "Heart Rate, rate (b/m)." ScienceWorkshop calculates the effective heart rate by measuring the interval between beats and taking the reciprocal. Thus, you will observe one data value per beat. That is, instead of 100 data values per second you will now see about one per second. You should observe that the rate after exercise gradually slows down as the body recovers. For both sets of data, you should see quite a bit of variation from second-to-second. Don't be alarmed: that variability is a actually sign of good health. One way physiologists analyze heart rate data is by making a **first return map**.

A first return map is a plot of the next entry in a data set as a function of the previous. That is, if x_k is the kth entry in a data set and x_{k+1} is the $(k+1)$th entry, then a first return map is a graph of all pairs (x_k, x_{k+1}). The graph has next values along the y-axis, previous values along the x-axis. To do this with your data, select one of the resting sets. Bring the table window to the foreground. Again, change the display from voltage-time to heart rate-time. With your mouse arrow, select the entire data set (it should be about 100 entries long). Copy the data (⌘-c). Bring **Chaosgenerator** to the foreground. Click on the worksheet (bottom of the window) entitled "firstreturn." If there are data on the sheet click in the "Clear" button. Click on cell A1. Paste your data into column A (⌘-v). The first three cells of column A will have header material. The first data value will be in A4. Click in the "Run first return" button. The sheet is set up to copy from A5 to the bottom then paste that in column B starting with B4. Thus, B4 will have A5 in it, B5 will have A6, and so on. Check to see that is right. The graph automatically plots column B as a function of column A.

A typical graph of resting heart rate data might look like an oval cluster stretching along the diagonal line $x_{next} = x_{previous}$, as in the first graph to the right. If the heart rate is very regular, the data are all about the same and the first return map looks almost like a dot, as in the second graph. That's not a good condition. If the heart rate is very irregular then the data spread out willy-nilly, and the first return map is more like a circular blob (last graph). Such a situation is often observed in various kinds of arrhythmia (including fibrillation). What does a graph of your data look like? Repeat for all the resting sets of your team.

When done with this part, quit ScienceWorkshop. DON'T SAVE any data sets. Turn off your interface.

Second Exercise:

We now turn to the analysis of some simulations of deterministic chaotic behavior. Chaotic behavior is erratic, it never repeats. At first glance, chaotic behavior seems to be random—without any underlying rules. But, in fact, chaotic behavior is governed by rigid, **deterministic** rules. In a random process the future is not determined by the past. In a chaotic process, the future is rigidly determined by the past. We distinguish between **low-dimensional** and **high-dimensional** chaos. In low-dimensional chaos, the next datum is determined by only a few prior data. In high-dimensional chaos, many prior data values are required to determine the next outcome. Low-dimensional chaos can be discovered by a few easy tricks. High-dimensional chaos is as good as random. In fact, there is as yet no operationally effective way to differentiate between the two, and most of what is usually called random (rolling dice, shuffling cards, molecules colliding in a beaker of gas, turbulence, ...) is probably really high-dimensional chaos.

High-dimensional chaos is pretty messy. It's hard (at this point, impossible) to tell what the behavior will do next. Low-dimensional chaos, on the other hand, has several attractive features. Once the rules are known, accurate (short-range) forecasting can be done and the behavior can be controlled. Both of these have profound practical implications (think about the weather, the stock market, and certain diseases, like cardiac arrhythmia). One distinction between low- and high-dimensional chaos is what first return maps of the two look like. Low-dimensional chaos produces fairly simple first return maps; high-dimensional chaos makes unstructured blobs.

E2. Chaos in the Ricker Equation

E2.1 The Ricker Equation, $x_{k+1} = bx_k\exp(-x_k)$, was developed in the 1950's to describe populations of fish and insects. In it, x_k is supposed to represent the number of members of a given population in generation k divided by the *carrying capacity* of the environment. Carrying capacity is the largest number of members that can coexist under optimal nutritional conditions in the environment. The Ricker Equation is an example of an **iterated function**. The way it works is, you choose a value for b and a starting value x_0. Plug x_0 into the right hand side of the equation and obtain as a result x_1. Then *iterate*. That is, insert x_1 into the right hand side (keep b fixed) and obtain x_2. Then iterate again. And so on. The result is a sequence of x values—x as a function of time. **Try calculating a few with a hand calculator to make sure you see what is going on. Try $b = 10$ and $x_0 = 1$, for example**.
•E2.2 Irrespective of the original intent of this model it turns out that the Ricker Equation actually closely describes the time behavior of certain chemical reactions and certain electronic circuits. Thus, its formal mathematical characteristics are of interest in many areas of science, not just to

population biologists. In its biological applications, the parameter b is the average number of births per individual per generation. In its other uses, the meaning of b is less obvious, but is related to such system characteristics such as ion concentrations, voltages, and the like. We will imagine, here, that the Ricker Equation can be a useful cartoon for describing, in qualitative terms, heart rate data. Thus, we will assume that the x_k is the interval between the (k-1)th and kth heartbeats. We suppose that the parameter b, in our cartoon, is related to the chemical state of the heart.

Click on the sheet "ricker" in **Chaosgenerator**. The sheet is set up to calculate a sequence of x values using the Ricker Equation once it is supplied with a parameter value (b) and a starting x value. The graph to the right displays the results with x plotted as a function of time. First try a parameter value of **6** and any starting value between 0 and 100, say. (What does the Ricker Equation produce with a starting value of zero?) Click the Run **ricker** button. What do you observe the late time behavior to be? You should see that the value of x becomes constant after awhile. A constant value of x is called a **fixed point**. Estimate the value. Try another starting value, quite different from the first. What is the late time behavior now? A parameter value of 6 is an unhealthy situation in cardiac language. Repeat, with parameter values 7-14. What happens? (When a value of x repeats every other time it is called a **two-cycle**, every three times a **three-cycle**, and so on. Describe what you observe, using the language of N-cycles.)

•E2.3 Now try a parameter value equal to 15. You should observe a very different kind of graph. At $b = 15$, the output of the Ricker Equation is chaotic. To see more clearly how this transition comes about try $b = 14.5$. What does that produce? Then try $b = 14.7$. What you are seeing is a sequence of ever-longer repeat times: fixed point, two-cycle, four-cycle, eight cycle, and so on. As b approaches 14.77..., the repeat time of x becomes so long it doesn't repeat. It becomes chaotic. We see this so-called **period doubling route to chaos** all over the place: in electronic circuits, fluids, heart cells, wood wind instruments, arms races, and even babies crying!

•E2.4 Set $b = 18$, a nice chaotic parameter. Take any nonzero x_0. Run **ricker**. Observe the wild output fluctuations associated with this parameter value. Click at the of column B to select the whole column. Copy. Click on the sheet **firstreturn**. Click at the top of column A then paste the contents of the clipboard (the data from **ricker**). Run **firstreturn**. The first return map of the Ricker Equation is a simple shape (it's the graph of $bx\exp(-x)$). Every previous x has a unique x following it. Because x_{k+1} depends on x_k only, the Ricker Equation is said to produce **one-dimensional chaos.** (Thus, it is not a totally accurate model of cardiac behavior. Look back at your on heartbeat first return maps.)

•E2.5 One of the signature characteristics of chaos is its *sensitive dependence on initial conditions*, popularly known as the *Butterfly Effect*. In ordinary, tame behavior, two close starting states that evolve by the same rules stay close. That is **not** true for chaotic behavior, however. Click on the sheet **butterfly**. Use a parameter value of 14 (periodic behavior) and two starting values that are really close (e.g., 2 and 2.000000000001). The graph displays the difference between the outputs of the Ricker Equation for the two different starting values. You should observe that when the output is periodic, "close stays close" (actually, "gets closer"). Now change to a parameter value of 15 (chaotic output). Do the two outputs begin to diverge? Make the chaos is even more complex by using a parameter value of 18. Notice the scale of the difference. In chaos, two close values only stay close for awhile. Later they can get as far apart as the dynamics takes them.

•E2.6 The Ricker Equation can easily be modified to demonstrate higher dimensional chaos (as is see in real cardiac data). Click on the sheet **ricker2**. In **ricker2** the function that is iterated is $x_{k+1} = bx_k\exp(-x_k) - cx_{k-1}$. That is, x_{k+1} now depends on two previous values of x, x_k and x_{k-1}. This new equation produces two-dimensional chaos (for the right values of the parameters b and c). Try x *param* (that is, b) equal to **16**, y *param* (c) equal to **0.32**, x *start* (x_0) equal to **4**, and y *start* (x_1) equal to **2**. Run **ricker2**. The graph displayed is a first return map of the x values. Do you see that making the dynamics 2-d instead of 1-d increases the complexity of the first return map? Play around with different starting and parameter values. Notice that unlike the first Ricker Equation, this modified one can lead to nasty blow-ups. They are accompanied with the error message in Column H "CARDIAC ARREST! AT STEP # .."

•E2.7 Adding one more term to the Ricker Equation makes the chaos even more complicated. Click on **ricker3**. The equation used in **ricker3** is $x_{k+1} = bx_k\exp(-x_k) - cx_{k-1} + dx_{k-2}$. That is, x_{k+1}

now depends on three past values of x. This equation yields three-dimensional chaos. Use for parameters **20.5**, **0.43**, and **0.31**, respectively, and for start conditions **3**, **2**, and **2**, respectively. The graph is a first return map of x values. Compare it with what you got for data from **ricker** and **ricker2**.

•E2.8 Though chaos appears unruly and essentially random, it is actually filled with order. If chaos is nothing else, it is a continual attempt to be periodic—it's just that the periodicity doesn't ever stabilize. This means that chaotic behavior is episodically simple (periodic). For example, the 1-d Ricker Equation in full blown chaos ($b = 18$, for example) tries again-and-again to take on its fixed-point value. It just doesn't stay there very long. Because chaos is episodically simple, it can be controlled. The way control works is you train a computer to look for recurrent simplicity in the output signal. Once the computer recognizes it, it is programmed to watch for another recurrence of the same simple behavior, and, when it comes back to within some specified tolerance level, to make very slight adjustments to the parameters of the system to keep the behavior near the unstable, periodic behavior. (Control of cardiac chaos consists of injecting a bit of current into the heart at just the right times.) The sheet **control** demonstrates this technique. Click on **clear** to remove any previous data. Use a parameter value of **18** and pick any nonzero staring value you please. Use a *tolerance* level (fraction of the fixed-point value to be controlled to) of **0.05**. Control will be attempted when x gets to within $x_{fixed} \pm$ *tolerance*. Select the number of steps control will be applied (e.g., 300 or so). After this number of steps have passed, the control will be turned off. For the first 100 steps, the computer searches for evidence of an unstable fixed point. At step 101 the computer examines the output for the possibility of implementing control. When x gets close enough to the fixed point, the computer turns on a control mechanism consisting of making very small changes in b selected to keep x near x_{fixed}. The two graphs show x as a function of time and the change in parameter as a function of time. Note how few parameter changes are needed to maintain control and how small the parameter changes are. Try other values of *tolerance* (e.g., 0.2, 0.1, 0.01, 0.001). **Clear** before each run. How do the graphs change as the tolerance is changed? The control method demonstrated here was used in the cardiac and brain control experiments mentioned in the overview above, with pretty much the same results.

References

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Lehnertz, K., and Elger, C.E., "Can Epileptic Seizures be Predicted? Evidence from Nonlinear Time Series Analysis of Brain Electrical Activity," Physical Review Letters, vol. 80, 5019-5022 (1998)