

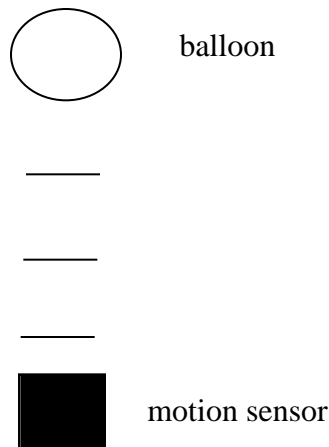
## EXPERIMENT 2000.12.1: Terminal Velocity

**Topics of investigation:** Newton's Second Law, velocity dependent friction

**Read about this topic in:** Serway, Ch 4, C&J Ch 4

**Toolkit:** Computer  
Pasco universal laboratory interface  
Science Workshop 2.3.3  
Excel sheet "balloon"  
Motion sensor  
Balloons, paper clips

### Sketch of apparatus



### Assumption:

When an air-filled balloon moves through air, it experiences a frictional force whose magnitude is of the form  $kAv^p$ , where  $A$  is the cross-sectional area of the balloon. The coefficient  $k$  depends on the properties of the air and its value as well as that of the dimensionless exponent  $p$  are unknown and to be determined.

### Prediction:


If a balloon is dropped from rest, it will eventually achieve a constant "terminal" velocity the value of which is determined by  $v_{terminal} = \left(\frac{mg}{kA}\right)^{1/p}$ .

### Exercises:

E0. Getting started

E0.1 Turn on your interface box. Start your Mac. Launch Science Workshop (SW). Launch the Excel sheet "balloon."

E0.2 Two leads from a motion sensor with phono jack plugs should be inserted in Digital Channels 1 and 2 of the interface. Be sure that the yellow-banded lead is in Channel 1. To program SW to send and receive motion sensor signals, click on the digital probe icon in the SW set-up window and drag the icon onto Channel 1. When the "Choose a digital sensor" window appears, click on "Motion Sensor" and "OK." Click on "OK" in the motion sensor calibration window that immediately appears. Using these default settings, the motion sensor will send out 20

pulses per second. Drag a graph icon onto the Motion Sensor icon appearing below the digital channels. Select “Position,  $x$  (m)” and click on “Display.” Click the resize button  on the set-up window to shrink the window down to just the control panel.


E0.3 Inflate three balloons, if they haven’t already been supplied. One should be about 10 cm in diameter, a second about 14 cm, and the third about 18 cm.

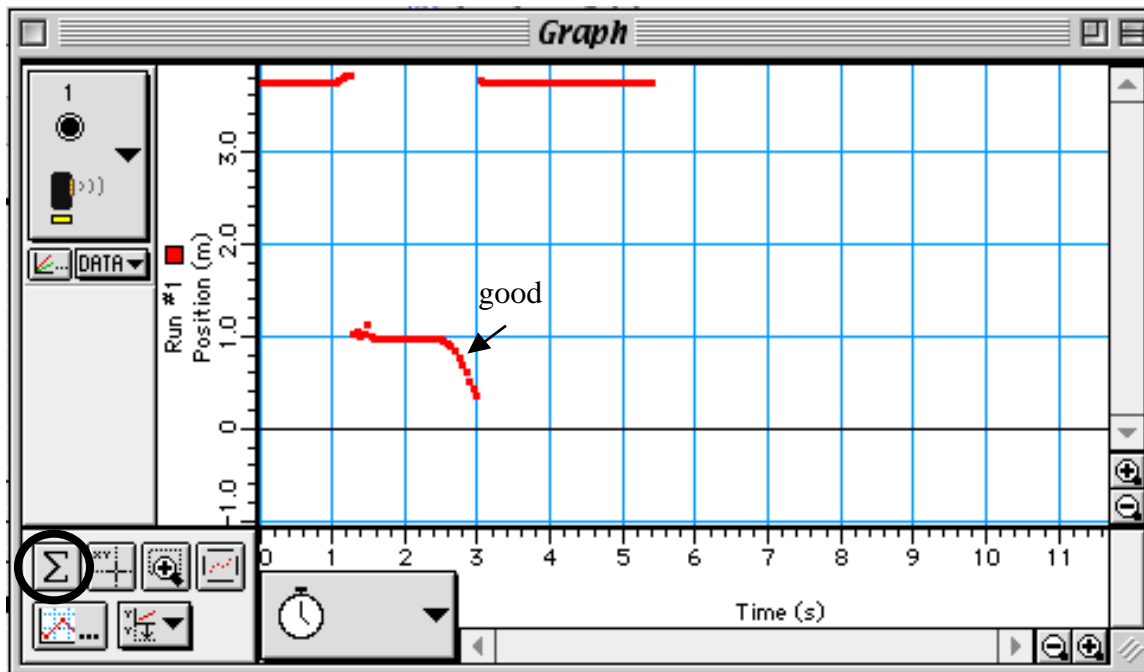
#### E1. Terminal velocity measurements



E1.1 In this experiment you will try to determine the unknown coefficient  $k$  and the unknown exponent  $p$  in the force law for air resistance. To do so, you will drop a series of balloons of different cross-sections and weights vertically onto a motion sensor. Recall that the coordinate system defined by the motion sensor and the software has an origin at the sensor face and a positive  $x$ -axis pointing away from the face. Using that coordinate system, write down Newton’s Second Law in  $x$ -component form for the falling balloon. If the balloon starts from rest, what is its acceleration the moment it is released? How does the acceleration change with time? Make sure you understand why the balloon eventually achieves a constant speed. Make a sketch of position versus time for the balloon in this experiment assuming it reaches terminal velocity well before hitting the motion sensor. Compare this sketch with a second in which the object is in “free fall.” Free fall occurs when  $mg \gg kA$ . (Why is that the condition for free fall?)

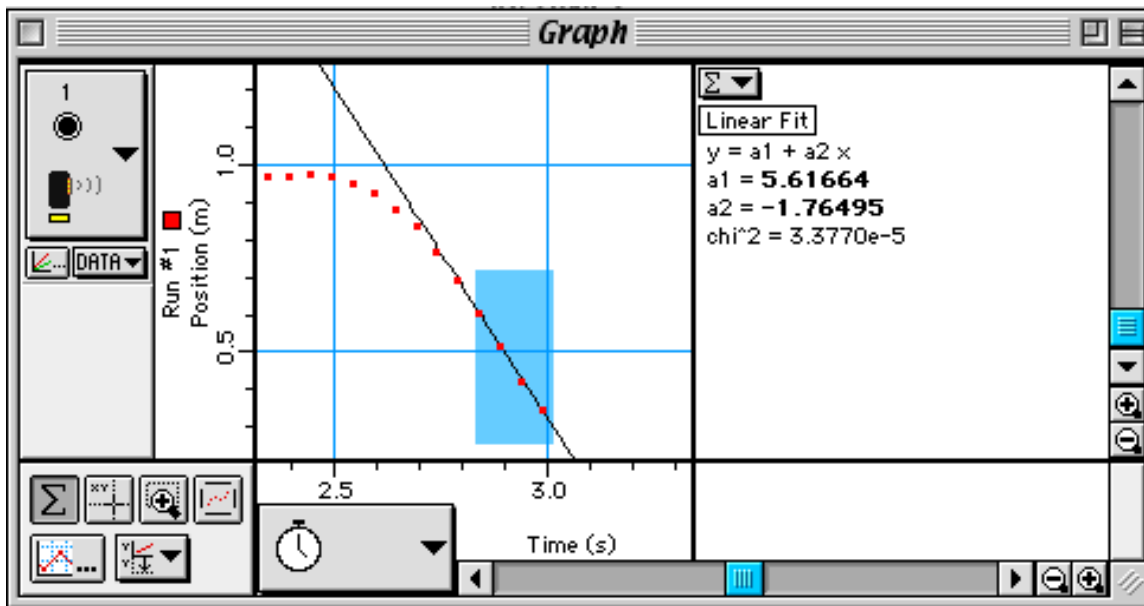
E1.2 Carefully weigh the three balloons without added paper clips and record the results. Carefully weigh your paper clips and record the results. Carefully measure the diameters of the three balloons and record the results. (This part may require a little thought as to how best measure the diameters.)

E1.3 Practice dropping the smallest balloon from a height of about 1.5 meters onto the motion sensor. To take data, click in the “REC” button of the control panel window. Start collecting data before the balloon is dropped and continue until after it has hit the motion sensor. After creating

your first graph click on the icon . Click on the “Connected Points” option in the window that appears, to remove the check. This will remove lines connecting the data points on the graph and make the following procedure easier. The graph of the data will probably look something like that shown below. The relevant data are contained in the small patch labeled “good.”



The point of the exercise is to measure the terminal speed. To do so, click on the statistics button “ $\Sigma$ ” (circled). The new graph window will be divided into two panels, one containing the data, the other an empty statistics panel. Click and drag on the menu to Curve Fit, then Linear Fit. Click on the  icon to show all data. To magnify the “good” data click on the magnifying glass tool , then by clicking and dragging on the graph, expand the region desired. Finally using the mouse cursor, select the last three or four data values in the “good” region to fit a straight line to. The result of all of this should look something like the following.



The equation displayed under the box “Linear Fit” represents the best straight line the software can fit to the data points selected. If the balloon is really moving at constant speed then its position as a function of time will be of the form  $x = x_0 + v_{0x}t$ . The equation is written in the form  $y = A + Bx$ , because the software doesn’t know what is plotted on the  $x$ - or  $y$ -axes. For your data, position ( $x$ ) is plotted vertically, and time ( $t$ ) is plotted horizontally. As a consequence the quantity  $B$  must correspond to  $v_{0x}$ . Record the magnitude.

E1.4 Repeat E1.3 for the same balloon four more times. The terminal speeds should all be about the same. If they are not you need more practice.

E1.5 Repeat E1.3 and E1.4 for all of the balloon/paper clip combinations in the worksheet.

### Analysis and questions

A1. For each set of terminal speeds find the average value. Enter the mass and diameter of each balloon trial in the appropriate cell in the Excel sheet “balloon.” Enter the average terminal speed for each case in the appropriate cell in “balloon.” “Balloon” automatically creates a graph of the data entered, plotting  $mg/A$  along the  $y$ -axis and  $v_{terminal}$  along the  $x$ -axis. Because the prediction implies that  $mg/A = k(v_{terminal})^p$ , a power law fit to the data (that is,  $y = Cx^p$ ) should yield both  $k$  and  $p$ , directly. Note that the graph displays the best-fit power law equation and a quantity called  $R^2$ . The latter is called the *regression coefficient* and should be near 1 if the data are good. What do you find for  $k$  and for  $p$ ? What are the units of each?

A2. Terminal velocity has an important use in biological applications. It is the reason that molecules can be sorted by size in *gel electrophoresis*. In electrophoresis, a mixture of molecules of different sizes is placed in a gooey gel and exposed to an external electric field. Because most biomolecules have a small amount of charge when in solution, they are accelerated by the electric field just as the balloons in this experiment are accelerated by the Earth’s gravitational field. In electrophoresis the friction of the gel on the moving molecules eventually causes them to reach a terminal velocity. Assume that all molecules in an electrophoretic separation experiment have the same charge and, consequently, feel the same electric force. Assume that  $p$  is 1 for electrophoresis. Derive an expression for  $v_{terminal}$  in terms of the electric force,  $F_E$ ,  $k$ , and  $A$ . In electrophoresis the small molecules get ahead of the large ones, but in the balloon analogy of electrophoresis, the large balloons would get ahead of the small ones (if we dropped a mixture of both all from the same height). Explain why.

Worksheet for “Terminal Velocity”

Masses in kg:

Small balloon (SB) =  
Large balloon (LB) =

Med. balloon (MB) =  
Paper clip (PC) =

Diameters in m:

SB =

MB =

LB =

Measured terminal speeds

SB	MB	LB
Ave	Ave	Ave
SB + PC	MB + PC	LB + PC
Ave	Ave	Ave
LB + 2PC		
Ave		