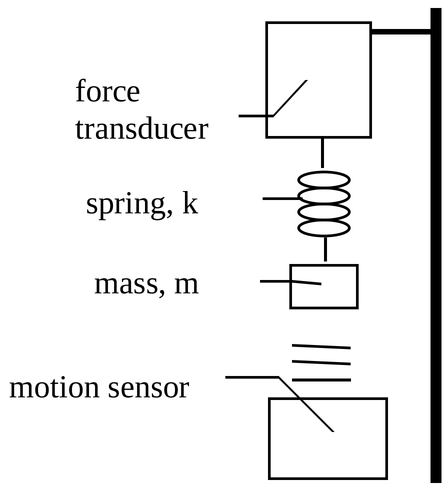
EXPERIMENT 2000.04.1: Harmonic Oscillations

Topics of investigation: Springs and harmonic motion

Read about this topic in: Serway, Ch. 21, C&J, Ch. 10

Toolkit:
- Computer
- Laboratory interface & software (SW 2.3.2)
- Motion sensor
- Force sensor
- Spring
- Masses: two 20 g, 50 g, 100 g, 200 g
- Meter stick
- 1.8 m rod, small rod, table clamp, 90˚ rod clamp

Sketch of set up:



Preliminary Assumption:

The force exerted by a spring obeys Hooke's Law, $F_{spring} = -kx$, where x is the amount by which the spring is stretched ($x > 0$) or compressed ($x < 0$) from its equilibrium length and k is the force constant of the spring.

First Prediction:

By Newton's Third Law, a force applied to a spring is the negative of the force the spring exerts. Thus, a mass, m, suspended in equilibrium by a spring with spring constant k, stretches the spring by an amount x equal to

$$x = \frac{mg}{k}$$

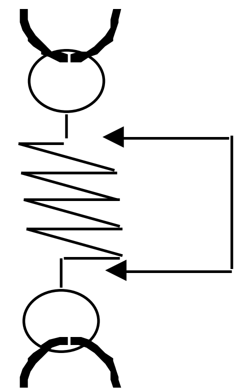
Be sure you understand why the First Prediction is true.

First Exercise:

E0. Getting started

•E0.1 Turn on your interface box. Start your Mac. Launch Science Workshop. Launch the **Excel spreadsheet ForceFit**. A motion sensor probe should be plugged into Digital Channels A and B of the interface and a force sensor should be plugged into Analog Channel A.
•E0.2 Program the interface to receive data from a digital motion sensor and an analog force sensor. Drag a graph icon onto the motion sensor icon in the set-up window and select Position, x (m). Add a second graph of Analog A, Force (N). Click in the Sampling Options box in the control window and set Periodic Samples to 2000 Hz.
•E0.3 Calibrate the force sensor by double clicking on the force sensor icon in the set-up window. With nothing hanging from the force sensor hook, press TARE, release and click in the High Value Read box. Change Force reading to 0 N. Hang a 200 g mass from the hook. Click in the Low Value Read box and set Force reading to -1.96 N. Click OK. Reduce the set-up window to the control window only.

E1. Static measurement of k



•E1.1 **Move the motion sensor aside so that an accidentally falling mass will not hit it**. **In any case, be careful: DON'T DROP ANY MASSES!** Place the spring on the force sensor hook. Hang a 20 g mass from the spring. Carefully measure the length of the spring (in mm, with a meter stick; remember 1 cm = 10 mm)--from the topmost coil of the spring to the bottommost, as shown in the figure to the right. Take this to be the "zero condition"--that is, zero applied force, zero stretch. (To make the spring more "ideal" you have to open the coils up a bit. That's why you start with some mass already on it.) Next, add a second 20 g mass and measure the new length (to the nearest mm). Remove the two 20 g masses and replace with a 50 g mass. Measure the new length. Add a 20 g mass; measure the new length. Finally, remove the 20 g and 50 g masses and replace with a 100 g mass. Measure and record the final length.
•E1.2 Enter your length data into the spreadsheet **ForceFit** in cells B2-B6. The spreadsheet automatically converts all masses (in grams) to forces (in newtons). It then subtracts the original length (with the first 20 g) from each successive length to get stretches, and converts these to m. Finally, it plots applied force as a function of stretch and fits the data with a straight line that goes through the origin. The slope of this line (found in the equation $y = (slope)x$) is k (in N/m). Record it. Incidentally, the fit also reports a quantity R^2. the so-called regression coefficient. R^2 better be near 1; if it isn't there is **human error** (i.e., a screw-up) and you should retake your data. After using **ForceFit** you can quit Excel; **don't save your data, please.**

Second Assumptions:

(1) A mass attached to the end of an ideal spring can be made to oscillate with minimal friction. (2) The mass of a light spring is ignorable.

Second Prediction:

A mass, m, suspended by a massless, ideal spring with spring constant k, oscillates **harmonically**--that is, in such a way that its instantaneous position relative to the equilibrium position is

$$x(t) = A\cos(\omega t + \varphi)$$

where A is the amplitude of the motion (maximum displacement from equilibrium) and ω is the angular frequency of the motion ($\omega=2\pi f$, where f is the ordinary frequency). The "phase factor" φ that appears in the argument of the cosine is determined by where the mass is and how it is moving at $t = 0$. If the spring is fully stretched at $t = 0$, $x(0) = A$ and $\varphi = 0$. The angular frequency is determined by k and m via

$$\omega = \sqrt{\frac{k}{m}}\,.$$

Note that if the expression above for the mass' position as a function of time is valid, its velocity is given by $v(t) = -\omega A\sin(\omega t + \varphi)$, and its acceleration is given by $a(t) = -\omega^2 A\cos(\omega t + \varphi) = -\omega^2 x(t)$.

Make sure you understand how all of the relations in the Second Prediction come about.

Second Exercise:

•E2.1 Click on the TARE button to reset the force sensor. Attach a 50 g mass to the end of the spring. Reposition the motion sensor under the hanging mass. **LIFT the mass 1 or 2 cm vertically. Drop it from rest**. **DO NOT PULL THE MASS DOWN AND RELEASE. Doing that can cause the mass to disengage from the spring and possibly damage the motion sensor (as well as mangling the mass).** Click on MON to ensure that the motion sensor is positioned properly.
•E2.2 Acquire data for about 10 complete oscillations.
•E2.3 Inspect your graphs of position and force versus time. Both should appear to be sinusoidal curves. Using the statistics tool, Σ, fit both data sets with "sine series." The default setting for the sine series fit is a single sine function, which is what you want. If the fit is poor, try re-fitting by selecting 2 or 3 complete oscillations. Eventually, by judicious selecting, you should be able to obtain a high quality fit. (High quality means small "chi^2," typically much less than 1. The noise in the force sensor data may be fairly high, so chi^2 for that set may be quite a bit higher than for the position data.) Record the frequencies associated with these fits. (They should be very close.) You may have to expand the size of the graph to see all of the statistical output. (Click and drag on the tab in the lower right hand corner of the graph window.)

Analysis of Second Exercise:

•A2.1 The motion recorded is supposed to originate from Hooke's Law. In Hooke's Law the force exerted by the spring is always 180˚ out of phase with respect to the mass' position. Your force sensor measures the force exerted by the spring on the force sensor hook, not the force exerted on the mass. The difference is the direction of the force. Thus, **if Hooke's Law really is behind the motion observed, the force measured by the force sensor should be exactly *in phase* with the mass' position**. (Make sure you understand this point.) Position the cross hairs tool exactly at a maximum on the position-time graph. Does the vertical hair hit a maximum on the force-time graph below? Look at several maxima and minima. How well do you think this prediction holds up?
•A2.2 Change the force-time graph to a velocity-time graph by clicking on the force sensor icon along the vertical axis of that graph and selecting Digital 1, Velocity, v(m/s). The Prediction says that when position is a cosine function, velocity is a negative sine function. In other words, where velocity is zero and going from positive to negative, position should be a maximum. Where velocity is zero and going from negative to positive position should be a minimum. (Be sure you

understand this.) Check this by magnifying about three complete cycles with the magnifying glass tool. Position the cross hairs tool on zeroes of velocity. What do you conclude?
•A2.3 The Prediction also says that when x is a cosine, acceleration is a negative cosine. In fact, $a = -\omega^2 x$. To check this, change the velocity-time graph to acceleration-time, then change the independent variable from time to position. (Click on the clock icon and select Digital 1, Position, x (m).) What should the acceleration-position graph look like? Change the statistics option for the acceleration-position graph to a linear fit. Record the slope of the best line fit.
•A2.4 Calculate the mean of the frequencies recorded in E2.3. Use this value to determine an experimental value of angular frequency, ω. Compare your experimental value with the theoretically predicted value in the Second Prediction. (That is, use your values for k and m.) Are they close?
•A2.5 Is ω^2 close to the value of the magnitude of the slope of the best fit line found in the last part of A2.3?

•E2.4 Reset the second graph to force-time. Remove the spring from the force sensor and press TARE to reset the force calibration. Add a 20 g mass to the 50 g mass. Repeat E2.1-E2.3.

•A2.5 Repeat A2.4 for the data collected in E2.4.

•E2.5 Remove the spring from the force sensor and press TARE to reset the force calibration. Replace the previous hanging masses with a 100 g mass. Repeat E2.1-E2.3.

•A2.6 Repeat A2.4 for the data collected in E2.5.
•A2.7 You will probably notice that your theoretical values for ω are systematically a little higher than your corresponding experimental values. Can you think of a reason that might be true? One reason may be because the spring actually has mass, too. That is, when the attached mass accelerates, mass in the spring does also. So the total available energy is shared between the attached mass and the mass in the spring. But not all of the mass in the spring is moving with the same speed--the part near the point of attachment at the force sensor hardly moves at all, while down near the bottom, there's lots of motion. So we expect that some fraction of the spring's mass ought to be added to the falling mass. Let's try to find out what this fraction is. For each of the three data sets taken, determine m', the *effective mass* required to make the experimental ω equal to the theoretical ω: that is, $m' = \dfrac{k}{\omega^2_{experimental}}$. Subtract the hanging mass in each case from m' to determine how much of the spring's mass participates in the dynamics of the motion: that is, let $m' = m + \Delta m$, where m is the hanging mass, and Δm is the amount of the spring's mass that should be added to the hanging mass. Find the average of Δm for your three data sets. Weigh your spring and evaluate what fraction of the spring's mass participates in the dynamics: *fraction* = $\Delta m_{\text{average}}/m_{\text{spring}}$.

Third Exercise:

•E3.1 There is remarkably little friction in the set-up you've used so far. To see the effects of friction on oscillatory motion use a rubber band. A rubber band seems like a nice elastic material, but it actually isn't. Hang a 200 g mass from a rubber band (or string of rubber bands) and set the mass in motion as you did before.
•E3.2 Record position versus time starting immediately after releasing the mass.
•E3.3 Make a graph of position versus time.

•A3.1 There is a marked difference between your spring data and your rubber band data. What is it?
•A3.2. Why do the rubber band data appear as they do? Try the following: Place the rubber band on your lips. Feel its temperature. Now vigorously stretch and shrink the rubber band a few times and quickly place it on your lips again. Do you feel a difference? Rubber is made of polymers that in the relaxed state are tangled. Pulling the rubber band taut straightens the polymers out. Relaxing tangles them up again. The process of straightening and tangling converts external, mechanical energy into internal energy--just like friction. This behavior is called "internal friction." Internal friction is a property of many polymers found in biological organisms.

A general question: The mass on the end of the spring moves vertically in this experiment, but gravity doesn't appear in the Second Prediction. How come?

When done quit Science Workshop. Don't save any data. Turn off your interface and shutdown your computer.

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Worksheet:

First Exercise

Mass (g)	Length (mm)
20	
40	
50	
70	
100	

k, slope from ForceFit: (N/m)

Second Exercise

Hanging mass (kg)	Freq from position (Hz)	Freq from Force (Hz)	Average freq (Hz)	Angular freq exp'ment (rad/s)	Angular freq theory (rad/s)	Effective mass m' (kg)	Added mass Δm (kg)

Slope of acceleration-position (50 g data) =

ω^2 (experimental value, 50 g data) =

Spring mass = (kg)

Fraction of spring mass = $\Delta m_{average}/m_{spring}$ =