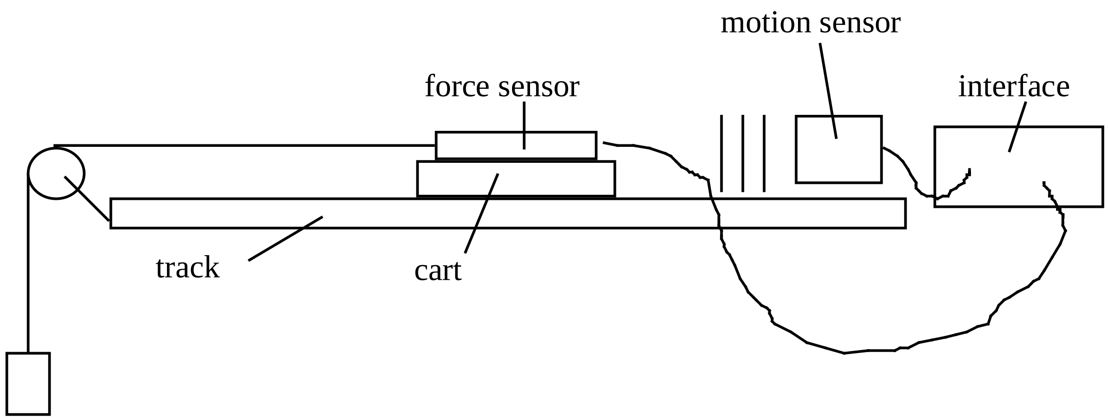
EXPERIMENT 2000.02.1: Newton's Second Law of Motion

Topics of investigation: The relation between force and acceleration

Read about this topic in: Serway, Ch. 4, 5; C&J Ch. 4

Toolkit:
- Computer
- Laboratory interface & software (SW 2.3.2)
- Motion sensor
- String and pulley
- Masses: 20 g, 50 g, 100 g; two massive bars
- Flat track
- Cart

Sketch of set up:



First Assumptions:

(1) The leveled track in this exercise can be used to compensate for gravity. (2) The light string and pulley have negligible affect on the dynamics of the masses in the setup. (3) Frictional forces on the cart are negligible.

First Prediction:

If the mass of the cart plus force sensor is M and the mass attached to the "falling" end of the string is m, then, given the above assumptions, Newton's Second Law predicts that when the cart is being pulled by the string only the magnitude of its acceleration will be

$$a_0 = g\frac{m}{m+M},$$

a constant, irrespective of how m and M are initially moving. (The subscript "0" designates "no friction.") Similarly, Newton's Second Law predicts the tension in the string is to be

$$T_0 = Ma_0 = \frac{mM}{m+M}g,$$

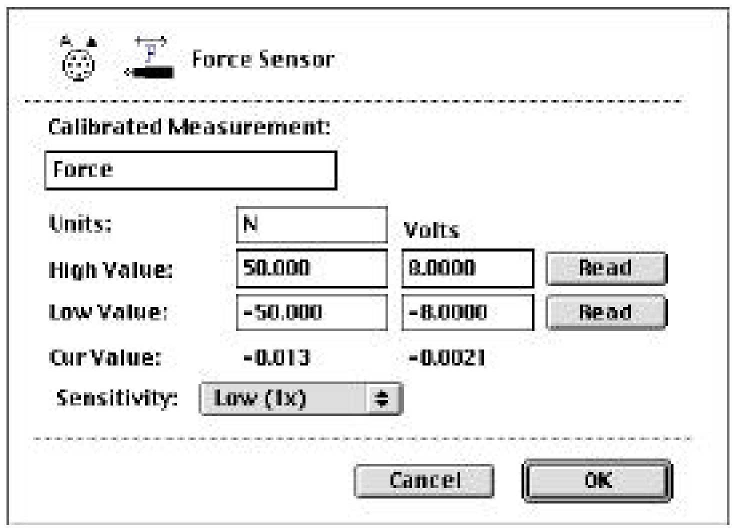
irrespective of how m and M are moving.

Before proceeding: Derive the predicted acceleration and tension by using Newton's Second Law of Motion applied to each mass in the system. What assumptions do you have to make in order to get the stated result?

Exercises:

E0. Getting started

•E0.1 Weigh your cart, force sensor, and all additional masses (2 bar masses per cart). Record all values.
•E0.2 Level your air track if necessary. A bubble level should be available to check your track. Each station should have a string with loops on both ends that is about 1 m long.
•E0.3 Turn on your interface box. Start your Mac. Launch Science Workshop. Make sure the motion sensor leads are properly inserted in the interface box (yellow in first digital port, black in the second). Make sure the lead from the force sensor is inserted in analog port A.
•E0.4 Program the software to recognize a motion sensor (drag a digital plug icon onto the first Digital Channel, then define it in the Choose a digital sensor window) and a force sensor (drag an analog plug icon onto analog port A, then define it in the Choose an analog sensor window).
•E0.5 Calibrate your force sensor. To do this, double click on the force sensor icon in the set-up window. A window like the one shown to the right should appear. The range of forces, -50 N to +50 N, is too coarse for the purposes of this lab. (A negative force is a pull, a positive one a push.) With the force sensor lying in the cart bed (as shown in the sketch above) and with no strings attached, press the TARE button on the side of the sensor for about a count of ten. Release. The Cur Value under Volts should now be very close to zero. Click on the High Value Read button. Replace the High Value 50.000 N with 0. Attach a string with the 100 g mass on one end to the hook. Feed the string over the pulley and allow the cart to rest gently against the stop. Cur Value Volts should now be some value such as -0.1 or -0.2. Click on Low Value Read and replace the -50.000 N with -0.98. Click OK.

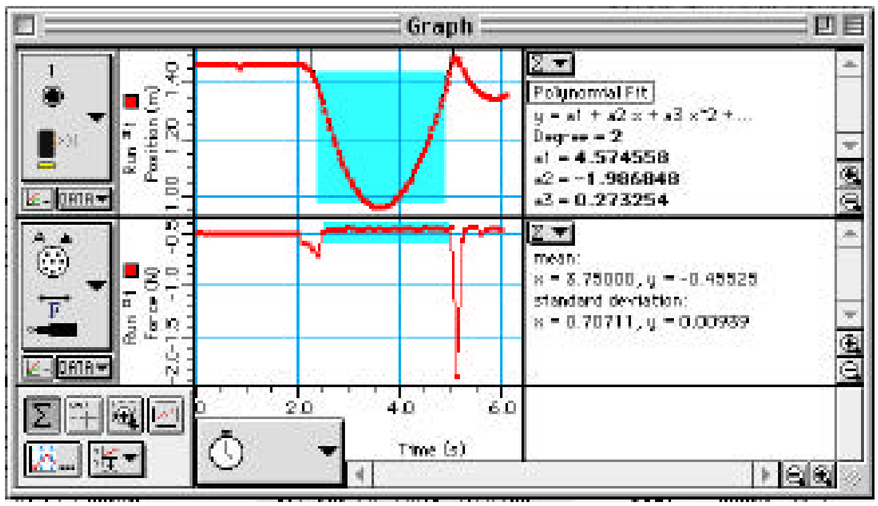


•E0.6 Create a graph of motion sensor readings by dragging a graph icon onto the motion sensor icon and selecting Position, x (m). Using the add graph button, , add to the first graph a graph of force sensor output.

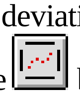
E1. Acceleration and force

•E1.1 Central to the First Prediction is the idea that once the cart is moving under the influence of the string only, both acceleration and tension are independent of cart motion. This aspect of the prediction may be investigated by launching the cart toward the motion sensor and collecting position-time data for a round trip. If the acceleration is constant throughout, a graph of position of the cart as a function of time will have a tell-tale parabolic shape. (**Why**?) Let's try it. Place the 20 g mass on the end of the string. Start with the cart at the end of the track farthest from the motion sensor. Practice setting the cart in motion with a **gentle** push so that the 20 g mass comes almost up to the pulley before falling back down. The member of the team who is putting the car in motion should also carefully hold the force sensor lead cord up so that it doesn't rub or bind on the table. Don't pull the cart with the cord. When ready to record data, remove the string from the force sensor hook while at the same time pressing the TARE button for a count of ten. (That resets the force sensor to its original calibration.) Place the string back on the hook. Another member of the team clicks on REC to start collecting data. Push the cart into motion. Continue collecting data until the cart hits the restraining bumper.

•E1.2 Your data set will include moments when the hand is pushing the cart, moments when the string only is affecting the cart's motion, and moments when the cart is colliding with the bumper. It is only the middle portion of the data for which the Prediction might be valid. To see how well the data are described by a quadratic function, click on the Σ symbol in the graph window. This should open up a statistics panels for both graphs. In the pull down menu of the position-time panel, drag to Curve Fit then to Polynomial Fit. The default setting with this option is to try to fit a second order (quadratic) polynomial to the data. The fit is of the form $y = a_1 + a_2 x + a_3 x^2$ where y is the dependent variable (position relative to the motion sensor) and x is the independent variable (time). The constants represent values of initial position, initial velocity, and 1/2 the acceleration, respectively. (**Make sure you understand why.**) For our purposes here, it is only the latter that is of interest. As in the figure to the right, select a portion of the position-time data set that appears to be parabolic by clicking on the graph and dragging. Record the value of a_3. Multiply by 2 to get the acceleration associated with this quadratic fit. Record the acceleration.



•E1.3 In the statistics panel for the force sensor data click and drag on the pull down menu and select Mean and Standard Deviation. Select the same time interval as for the position data. Record the mean value of the tension. Note the standard deviation value. That is a measure of how accurately the force sensor measures force. Most of the values in the interval shown will be within plus or minus one standard deviation from the mean value.

Remember, any time you want to view all data click in the  button.

Analysis

•A1.1 Using the expressions in the Prediction and the values of the masses you measured, calculate a_0 and T_0. **You will need a value of g; in Logan, to good approximation, it is 9.801 m/s^2**. Compare these calculated values with the values obtained in steps E1.2 and E1.3. (**Reminder:** 1 g = 0.001 kg.)

•E1.4 Repeat steps E1.1-E1.3 four more times, each time recording best fit values of acceleration ($2a_3$) and tension. Don't delete any good data sets; you'll be using them again later.

•A1.2 You will undoubtedly observe that the value of the best fit acceleration varies from run-to-run. That is an inevitable characteristic of real measurements. No one measurement is "right." There are many reasons for measurement variation, most having to do with how the instrumentation is designed and constructed. Sometimes beginning experimenters clump all such variations into a single category they call "human error"--suggesting that humans are measuring devices with built-in variability. With contemporary electronic data collection techniques the subjective role of the human observer is vastly reduced. The concept of "human error" is a kind of cop-out and should be abandoned, unless, of course, by "human error" is meant "screw-ups." If the latter is infecting your experimental results you should make every effort to STOP DOING IT!

In any case, intellectual responsibility requires that a statement of how reliable the value is accompany the report of a measured value. One way of assessing reliability is to repeat the measurement many times, each time performing the measurement the same (screw-up free) way. Then, the reported measurement should be the mean value from all trials and the reliability should reflect the variations actually observed. One such measure of reliability is the "standard deviation." Standard deviation is calculated as follows: Start with a set of measurements $\{x_1, x_2, \ldots, x_N\}$. Determine the mean: $<x> = (x_1+x_2+\ldots x_N)/N$. Subtract $<x>$ from each measurement to find the "deviation of the measurement from the mean:" $d_i = x_i - <x>$. Square each deviation (so that each deviation is converted into a positive number), and find the mean of them (the mean of the deviations without squaring is exactly zero): $<d^2> = (d_1{}^2+d_2{}^2+\ldots+d_N{}^2)/N$. Finally, take the square root to determine the standard deviation: $\sigma = \sqrt{<d^2>}$. A properly reported measured value is then given as $<x>\pm\sigma$.

From the five trials you have run in E1.1-E1.4 with m = 20 g determine the mean value of acceleration ($<a>$) and its standard deviation (σ_a). Likewise determine the mean value of the tension ($<T>$). Take as the uncertainty of the tension the mean value of all the standard deviations (σ_T). Now, we can answer the question, Are the First Predictions consistent with the measured values? The answer is YES if a_0 lies in the range $<a>-\sigma_a$ to $<a>+\sigma_a$ and T_0 lies in the range $<T>-\sigma_T$ to $<T>+\sigma_T$, and (probably) NO otherwise. So which is it?

•E1.5 Replace the 20 g mass with the 50 g mass and repeat steps E1.1-E1.4 and the analyses in A1.1 and A1.2. Does the average acceleration increase as predicted? Are the new Predictions compatible with the new results?
•E1.6 Place one of the massive bars in the bed on the back of the force sensor. Repeat E1.5.
•E1.7 Place the second massive bar in the bed on the back of the force sensor. Repeat E1.5.

E2. Desperately seeking friction

If you examine some of your position-time data sets carefully, you will probably notice that the parts of the graphs that are supposed to be parabolas are not exactly symmetric around the turning points. So, we have

Amended Assumption:

Friction, while small, is not negligible. We assume, here, the frictional force, f, is constant.

This leads to a

Second Prediction

Let the subscript "u" refer to motion in which m goes up and "d" to motion in which m goes down. Then

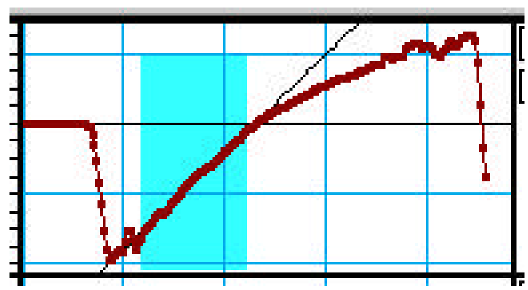
$$a_u = a_0 + \frac{f}{M+m} \text{ and } a_d = a_0 - \frac{f}{M+m}$$

or, what is the same thing,

$$f = \tfrac{1}{2}(M+m)(a_u - a_d)$$

You should verify these predictions before proceeding.

•E2.1 The Second Prediction states that acceleration during the time the cart is being pulled by the string only is **not** constant. It is greater when m is going up than when m is falling. Make a sketch of what you think velocity as a function of time would look like under these conditions. One way of testing this prediction is to reanalyze the data you have already taken. Double click on the first data set taken. Instead of displaying position-time display velocity-time. (Click on the motion sensor icon in the graph window, drag to Digital 1, then to Velocity, v (m/s).) In the statistics pull down menu of the graph change from Polynomial Fit to Linear Fit. The parameter a_2 in a linear fit of velocity versus time **is acceleration.** (Why?) Select data over the first portion of "string only" motion as shown to the right. It should be clear that the velocity versus time does **not** have a single slope (as it should if the acceleration were constant throughout). Record a_2 (= a_u). Select data over the second phase of the "string only" motion. Record a_2 (= a_d).



•A2.1 Calculate f from your values of m, M, a_u, and a_d.

•E2.2 Repeat E2.1 for all data sets collected in step E1.

•A2.2 Repeat A2.1 for all values recorded in E2.2.
•A2.3 Calculate average values of f for each separate combinations of masses. If f were due to sliding friction f would equal μMg. Does f change as M changes the way the latter expression says it should?

Worksheet:

exercise 1
mass: cart+FS (g) =
falling mass (g) = 20
predicted accel. (m/s^2)
predicted tension (N)

exercise 3
mass: cart+FS+1 bar (g) =
falling mass (g) = 50
predicted accel. (m/s^2) =
predicted tension (N)

a3	accel. (m/s^2)	tension (N)	a3	accel. (m/s^2)	tension (N)

average accel. ± stddev (m/s^2) =
average tension ± stddev (N) =

average accel. ± stddev (m/s^2) =
average tension ± stddev (N) =

exercise 2
mass: cart+FS (g) =
falling mass (g) = 50
predicted accel. (m/s^2)
predicted tension (N)

exercise 4
mass: cart+FS+2 bars (g) =
falling mass (g) = 50
predicted accel. (m/s^2) =
predicted tension (N)

a3	accel. (m/s^2)	tension (N)	a3	accel. (m/s^2)	tension (N)

average accel. ± stddev (m/s^2) =
average tension ± stddev (N) =

average accel. ± stddev (m/s^2) =
average tension ± stddev (N) =

exercise 5 | exercise 7

au (m/s^2)	ad (m/s^2)	f (N)	au (m/s^2)	ad (m/s^2)	f (N)

average f = | average f =

exercise 6 | exercise 8

au (m/s^2)	ad (m/s^2)	f (N)	au (m/s^2)	ad (m/s^2)	f (N)

average f = | average f =