Notes on: The Stability and Structure of Matter

1. The New Periodic Table

It is believed that all matter consists of various combinations of the following *elementary particles*.

Basic Building Blocks of Matter

Mass →				
M a	**neutral leptons**	**charged leptons**	**first generation quarks**	**second generation quarks**
s	electron neutrino (ν_e)	electron (e)	up quark (u)	charm quark (c)
s	muon neutrino (ν_μ)	muon (μ)	down quark (d)	bottom quark (b)
↓	tau neutrino (ν_τ)	tau lepton (τ)	strange quark (s)	top quark (t)

Carriers of Force

Electromagnetism	Photon
Strong Force	Gluons
Weak Force	W and Z bosons
Gravitation	Graviton

The elementary particles carry intrinsic properties such as mass, "spin," and electrical charge, and a variety of other "charges." Each of these intrinsic properties results in an interaction. Two massive particles interact gravitationally by "exchanging gravitons." Two electrically charged particles interact electromagnetically by "exchanging photons." And so on. Atoms are familiar microscopic pieces of matter. An atom has a nucleus that has positive electrical charge, and electrons that have negative electrical charge. The atomic nucleus consists of a sea of up and down quarks, all interacting by exchange of gravitons (because quarks carry mass), photons (because quarks carry electrical charge), and gluons (because quarks carry "strong charge"). This sea is not uniform. It is lumpy. The lumps are called protons (two up quarks plus one down quark) and neutrons (two down quarks plus one up quark). All of the quarks in the atomic nucleus interact with the atomic electrons by graviton exchange (because the electrons have mass, too) and by photon exchange (because electrons also have electrical charge).

2. The Atomic Nature of Matter

It is one of the most amazing facts of nature that essentially *everything in the world around us is made from fewer than 100 naturally occurring different kinds of atoms*. In atoms that are **electrically neutral**, the number of electrons equals the number of protons in the nucleus. An **element** is some material that consists of atoms, all of which contain the same number of protons. Thus, atoms with one proton are said to constitute the element hydrogen, atoms with two protons constitute the element helium, and so on. While atoms of a given element all have the same number of protons in their nuclei, they may have *different* numbers of **neutrons**. Two atoms with the same number of protons but different number of neutrons are said to be different **isotopes** of the same element. Different isotopes behave almost identically as far as chemical reactions are concerned, because chemical reactions involve atomic electrons only, not the atomic nuclei.

Protons and neutrons both weigh about 2000 times more than electrons. So most of the "stuff" of an atom resides in its nucleus. Nonetheless, atoms are mostly empty space. The most common isotope of hydrogen consists of one proton and one electron. Suppose we represent the proton in a hydrogen atom by the following dot: •. About how far away from this dot would the electron be on average if this dot were the actual size of the proton? Where the period next to the dot is? Maybe a centimeter? 10 centimeters? A meter? No. Actually, the electron would spend most of its time roughly 100 meters away! The average diameter of the electron orbit in hydrogen is about 100,000 times the diameter of the proton. In atoms with more protons, the electrons spend more time nearer the nuclei, but no matter how many protons and electrons they contain, atoms are mostly empty. Despite that, it is very hard to squeeze the electrons of an atom closer to their nucleus. It is also difficult to make the electrons of two atoms interpenetrate. If that weren't true, it would be impossible for objects to have more-or-less permanent shapes and sizes.

The number of atoms in a macroscopic object may well exceed 10^{20}. The interactions of these vast swarms of atoms lead to qualitatively different **states of matter**. In all materials at all temperatures, the constituent atoms are in ceaseless, disorganized motion. In **solids**, the microscopic agitation of atoms is sufficiently confined that the atoms typically do not exchange places. As a consequence, solids have an essentially permanent shape. In **fluids—**that is, **gases** and **liquids—**however, atoms can pass by each other. This swapping of atomic positions produces the macroscopic phenomenon of **flow** and the microscopic phenomenon of **diffusion** or atomic mixing. Fluids flow around inside solid containers and adopt shapes defined by the containers. Fluids don't have a permanent shape. While solids are characterized by the regular and enduring arrangement of their atoms, fluids are characterized by atomic chaos.

Biological materials typically share features of both the solid and fluid states. (This delicate balancing act is encapsulated in the colorful phrase, "**life exists on the edge of chaos**") For example, biological membranes that surround cells or sub-cellular components are basically two-dimensional highly ordered structures that also have a large degree of mobility within them. Their constituent phospholipid molecules tend to be aligned parallel to each other, but can move about within the plane of the membrane quite rapidly by diffusion. Such highly ordered, but yet fluid structures are termed **liquid crystals**. A second significant example is the **gel**-like nature of cellular cytoplasm. Gels have some of the properties of solids, including a rigidity, but can be greatly deformed as well. Cytoplasm is a complex material consisting of thousands of different macromolecules, including proteins, nucleic acids, phospholipids, polysaccharides, as well as smaller organic molecules and salts. Under the control of several filamentous proteins that supply an internal structural rigidity, the cytoplasm can be changed back-and-forth between conditions that are more fluid-like and more solid-like.

2. Mass, Density, and the Size of Atoms

Mass is a fundamental property of matter. For now, it is sufficient to think of mass as a measure of the substance of a body. Mass can be measured by an ordinary bathroom or grocery market scale, if the body whose mass is being measured is of moderate size, and by more sophisticated scales if the body is either too large or too small. The SI unit of mass is the

kilogram (kg). A kg is roughly the mass of a rock the size of a grapefruit. A kg weighs about 2.2 pounds. It is possible to determine the masses of individual atoms with a delicate scale called a "mass spectrometer." Compared with a rock an atom doesn't have very much mass. A rule for assessing the approximate mass of an atom is to look up the number of "atomic mass units per atom" (designated u/atom) for the atom of interest on a periodic table, then multiply by 1.66×10^{-27} kg/u. The number of atomic mass units per atom is essentially the average number of protons plus neutrons in all isotopes of the element in question found on Earth.

We wish to demonstrate how knowledge of macroscopic properties sometimes can be converted into knowledge about atoms. Let's start with the question, how many atoms are contained in a 1 kg mass of known composition? Suppose, for example, we are told that the mass is solid gold. A periodic table tells us that gold has about 197 u/atom. So the mass in kg of a gold atom is 197 u/atom x 1.66×10^{-27} kg/u = 3.27×10^{-25} kg/atom. The 1 kg is some number of atoms times the mass per atom, so if we divide the latter value into 1 kg we find that 1 kg of gold contains 1 kg/3.27×10^{-25} kg/atom = 3×10^{24} atoms. That's a typical number for solids: 1 kg of a solid contains from about 10^{24} to about 10^{26} atoms.

A macroscopic measurement of **density** of a solid body allows us to answer the question, how far apart are atoms in the body? Average density, ρ, is defined as mass/volume. If we divide the density of a solid body by the mass per atom we get atoms per unit volume. If we take the reciprocal of that, we get volume per atom. Now, if we pretend that each atom is a little cube of side d_0, the volume per atom is d_0^3. Thus, d_0 is the cube root of the volume per atom; it is also the average distance between adjacent atoms. For iron we have 7900 kg/m^3/(55.8 u/atom x 1.66×10^{-27} kg/u) = 8.53×10^{28} atoms/m^3. The volume per atom in solid iron is then (8.53×10^{28} atoms/m^3)$^{-1}$ = 1.16×10^{-29} m^3/atom, and the cube root of that, 2.26×10^{-10} m, is the average atomic spacing. The distance 10^{-10} m recurs frequently when considering atoms. You will sometimes find 1×10^{-10} m referred to as 1 ångstom = 1 Å, though in keeping with the SI conventions it is more fashionable these days to use the nanometer: 1×10^{-9} m = 1 nm. Thus, 2.26×10^{-10} m is either 2.26 Å or 0.226 nm.

The average distance between atoms in any elemental solid is *roughly* the same as for iron. Now here is a very familiar result: it is exceedingly difficult to increase the density of a solid by squeezing it. In other words, in a solid, the atoms are crammed together about as closely as possible. This fact and the fact that the average spacing of atoms is about 0.2 to 0.3 nm for all elemental solids tells us the very interesting and surprising result that **all atoms are about the same size**—despite the fact that their atomic masses vary by a factor of over 200!

You might be tempted to conclude that because liquids flow and have no permanent shape that the spacing of atoms in liquids would be a lot larger than in a solid. Let's see. How about in water? Water is a molecular liquid. The u/molecule for water is about 18 (2 for the two hydrogen atoms and 16 for the oxygen atom). Since there are three atoms per molecule, the average mass per atom is 6 u. The density of water is about 1000 kg/m^3. Consequently, the average molecular spacing in water is about 0.22 nm—more-or-less the same value as in solids. The remarkably different physical properties of solids and liquids arise from only very small differences in how their atoms are spaced.

What can we say about atomic spacing in gases? The most familiar gas is air, a mixture of primarily nitrogen and oxygen molecules. Let's say that the average u/molecule for air is about 29. Because nitrogen and oxygen molecules contain two atoms, the average u/atom for air is about 14.5. The density of air at room temperature and at sea level atmospheric pressure is 1.29 kg/m^3—a value that is something like 1000 times less than water. The average atomic spacing in air is about 2.7 nm, that is, about 10 times greater than in a solid or liquid. If we squeeze a quantity of air down to 1/1000th of its normal volume, it becomes a liquid; the densities of liquid oxygen and liquid nitrogen are almost exactly 1000 times that of air.

Because biological materials have properties midway between the solid and liquid states the spacing of atoms in them is about 0.2 to 0.3 nm. We can use this idea to assess how many atoms one might find in a typical biological cell. Cells have somewhat different sizes, but a typical cell is roughly about 20×10^{-6} m = 20 micrometers = 20 μm on a side. That is, a cell has a volume roughly about 8×10^{-15} m^3 (obtained by cubing 20 μm). If a typical atom spacing is 0.25 nm, the volume occupied by an atom is about $(0.25 \text{ nm})^3 = 1.5 \times 10^{-29}$ m^3/atom. Consequently, the number of atoms per cell is about $(8 \times 10^{-15}$ m^3/cell)/(1.5×10^{-29} m^3/atom) = 5×10^{14} atoms/cell.

A cell has lots of stuff in it. All cells contain DNA, for example. Drawings of pieces of DNA in textbooks show it as a long, double helix structure. But, just how long is it? DNA consists of multiple subunits called base pairs ("C-G" and "A-T"). The number of atoms per pair is 27. Typical animal cells have about 5×10^9 pairs in their DNA. That corresponds to about 1.4×10^{11} atoms. Suppose that all of the atoms in the DNA molecule were strung end-to-end in a linear chain. The chain would be about (1.4×10^{11} atoms)x(2.5×10^{-10} m/atom) = 35 m long! Of course, clumping atoms into base pairs of about 30 atoms each saves space. Even so, if the pairs were strung out in a linear chain, the DNA would still be about 1 m long. Obviously, DNA in a cell can't be a linear chain because it would burst through the cell membrane. It must be stored in a tight coil when "not in use" and only small portions must be pulled apart when transcription or replication occur. Similar conclusions can be made about other important ingredients of a cell, such as large proteins, for example.

3. Center-of-Mass

A universe that consisted of purely stationary mass would be pretty boring. Time requires change—both for its definition and for its measurement. Thus, in a static universe time would have no meaning. In such a universe there certainly would be no life, no intelligent thought, no social behavior. But motion alone doesn't automatically ensure that things will be interesting. For example, a universe in which some of the mass moved at a constant speed in one direction would be only slightly more complicated than a purely static universe. There still wouldn't be any critters walking around or music being played. In order for there to be genuinely complex behavior, there has to be *change in motion*—matter speeding up and slowing down and going around corners. When matter speeds up, slows down, or goes around corners it is said to be **accelerating**. *Complexity requires acceleration*.

Maybe you can convince yourself that acceleration is interesting but unchanging motion isn't by recalling how it feels to ride in a car traveling along a straight, flat highway that has recently been resurfaced. If the car's speedometer is fixed at a constant reading you can close your eyes and not know you are moving at all, no matter how fast the speedometer says you are moving. Of course, roads aren't straight and flat for very long stretches. You feel clues that you are moving from the little bumps and turns the car makes. Riding in an elevator may be a better example. Once the elevator gets going, only the flashing floor numbers give any hint that anything is happening—no matter how fast the elevator is traveling or whether you are going up or down. When the elevator starts up or slows to a stop, you feel that. But not the constant speed parts of the motion. Unchanging motion feels exactly like standing still. It's boring. Unchanging motion doesn't demand much attention. But, interesting, changing motion does. Acceleration always has a cause; it doesn't happen spontaneously.

Our goal here is to begin to study the motion of real world objects like an *E. Coli* swimming in a person's gut or a jaguar dashing after a gazelle or a satellite being launched into orbit from the cargo bay of the Space Shuttle. Each of these examples involves bodies that are made of huge numbers of atoms and occupy an extended region of space. Each somehow involves motion of the whole as well as motion of parts (flagella wiggling, tails snapping, antennae rotating). In addition, if we were able to look on a microscopic level, we'd see that all of the atoms in each of the bodies would also be jiggling furiously—independently of their over all macroscopic motion. The description of all of these various motions can be extremely complicated. To make progress we need a simple starting point. That's provided by the so-called **center-of-mass** (CM) of the body of interest.

Center-of-mass is a measure of where the "average" mass is. Let's think a bit about how you calculate an average. For example, suppose you've scored 70 on one exam and 100 on a second. If the two exams are of equal importance—that is, of *equal weight*—the average is (70+100)/2 = 85, a value that lies exactly midway between 70 and 100. On the other hand, suppose the exam on which you scored 100 is twice as important as the other exam; it has *twice the weight* of the other exam. What would the average be then? That situation would be equivalent to having taken *three* exams, *one* with a score of 70 and *two* with scores of 100. The average would then be (1•70+2•100)/3 = 90. Note that (1•70+2•100)/3 is the same as ((1/3)•70+(2/3)•100). That is, the 100 point score is 2/3 of the total weight of the scores and the 70 point score is 1/3 of the total weight. So the average can be calculated by multiplying 70 by its fraction of the total weight (1/3), 100 by its fraction of the total weight (2/3), and adding the results. Note also that the difference between 100 and 70 is 30; the average value 90 is 10 points below 100 and 20 points above 70. That is, 90 is 1/3 the total difference separating the scores away from the score whose weight is 2/3 and 2/3 the total difference away from the score whose weight is 1/3.

Center-of-mass is reckoned similarly. Suppose we are looking down at two ants walking on the ground. From our perspective, the ants appear as small dots—for all intents and purposes what one might call "point masses." The CM of the two ants is defined as a point on the line connecting them. That point will be closer to the more massive of the ants and farther from the less massive one—in just the same way that the weighted average of two scores is positioned between the two. Thus, if the ants have the same mass, the CM is at the point exactly halfway on the line connecting them (Figure 3.1 (a)). On the other hand, if one ant has twice the mass of the second, the CM is the point on the line connecting them that is 1/3 away from the ant whose mass is 2/3 the total mass and 2/3 away from the ant whose mass is 1/3 the total (Figure 3.1 (b)).

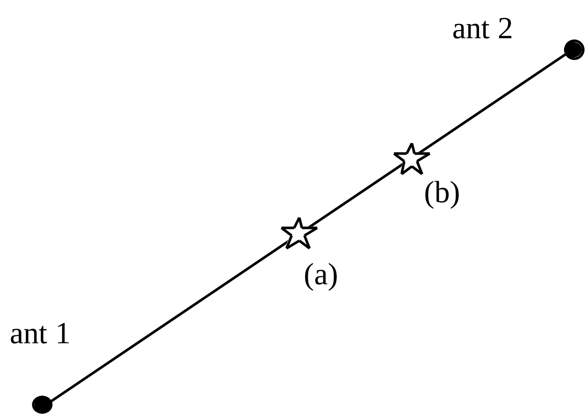

Figure 3.1 The center-of-mass of two ants when the mass of ant 2 is (a) the same as the mass of ant 1 and (b) twice as big as the mass of ant 1.

Example 3.1 Suppose one ant has a mass of 22 mg and the second has a mass of 3 mg. At one instant the two ants are 10 cm apart. Where is the CM at that instant?

Solution: The two ants' total mass is 25 mg, so the fraction of the total mass associated with the more massive ant is 22/25 = 0.88 and with the less massive 3/25 = 0.12. Consequently, the CM is the point on the line connecting the ants that is (3/25)•10 cm = 1.2 cm away from the more massive ant and (22/25)•10 cm = 8.8 cm away from the less massive ant.

The center-of-mass of a collection of more than two ants (or any point masses) can be determined geometrically in the following way. Imagine connecting the ants to each other in pairs. An ant can belong to one pair only. Thus, if you have an even number, *N*, of ants there will be half as many pairs, *N*/2. If there is an odd number of ants then there will be one ant left unpaired. Next, find the CM for each pair. Treat each CM as if it were an ant with the total mass of the two it came from. Then repeat the process again and again until there is only one CM.

Figure 3.2 shows such a construction for four ants and for three ants, all of equal mass. In each case, the CM of a pair is assigned the sum of the masses of the pair members. That's why in the three-ant example the final CM is closer to the CM of the first paired ants than the single unpaired ant. You should prove to yourself that it doesn't matter what pairs you choose to start with by taking different pairs from the ones shown in the Figure; you always get the same final CM.

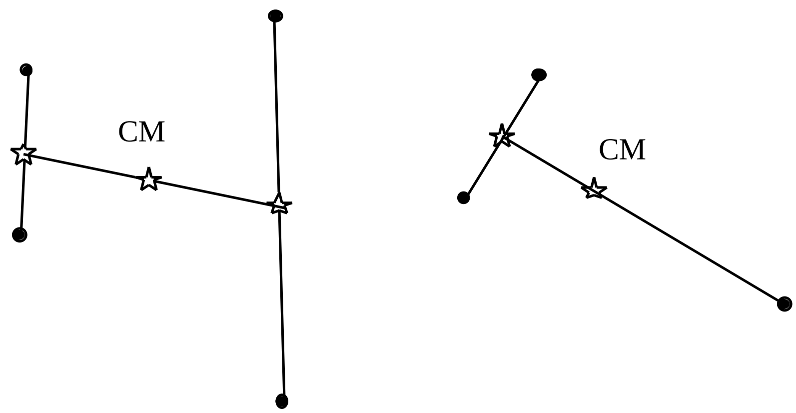

Figure 3.2 The centers-of-mass of a four ant cluster and of a three ant cluster. In each case the masses of the individual ants are equal. The CM of two ants has an equivalent mass that is twice that of each ant.

Now, as the ants move about, the CM of the collection of ants moves, too. Imagine recording the positions of the ants for a few seconds, say, by using a video camcorder. Such a recording is actually a sequence of still frames. For standard American video the frames are 1/30th of a second apart. At first we assume that the camera is held fixed. Figure 3.3 is a cartoon of 12 successive frames superposed on top of each other—i.e., it's a "time lapse photograph." What is shown is a "platoon" of eight ants, all marching directly north at the same, constant speed. The streaks are the sequential positions of the respective ants. There are also 12 pictures of a rock. But since the rock and the camera

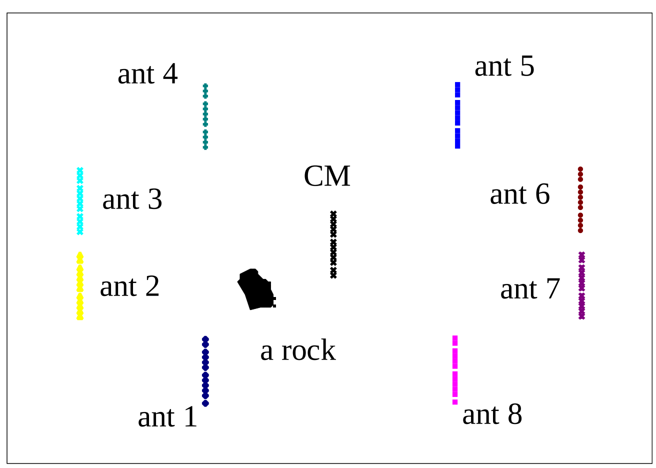

Figure 3.3 Eight identical ants marching directly north at the same speed. Their center-of-mass also travels north at the same speed.

are not moving relative to each other all 12 rock pictures are in the same place.

Let's assume that the ants all have the same mass. The CM of the platoon is sketched on each frame of the superposition shown in Figure 3.3. At every instant the CM is in the middle of the platoon. Thus, as the platoon marches north the CM moves north at the same rate as the ants. We give the motion displayed in Figure 3.3 a special name: **rigid translation**. A rigid translation is a motion involving a collection of elementary units (e.g., the ants) in which each moves in the same direction by the same amount in a given time.

It is interesting to note what an "observer" moving along with the CM would see for the case of a rigid translation. This can be accomplished by moving the camera northward at exactly the same rate that the platoon is moving. A superposition of 12 successive frames made in such a way is shown in Figure 3.4. In Figure 3.3 an observer moving with the CM takes one step north at exactly the same moment that all eight of the ants do so. Thus all eight ants remain in place relative to this observer ant, and thus their positions in this moving **frame of reference** are fixed. The rock, on the other hand, is seen to move southward relative to the camera. The main point of this little thought experiment is that *to describe a rigid translation one need only describe the motion of the CM*.

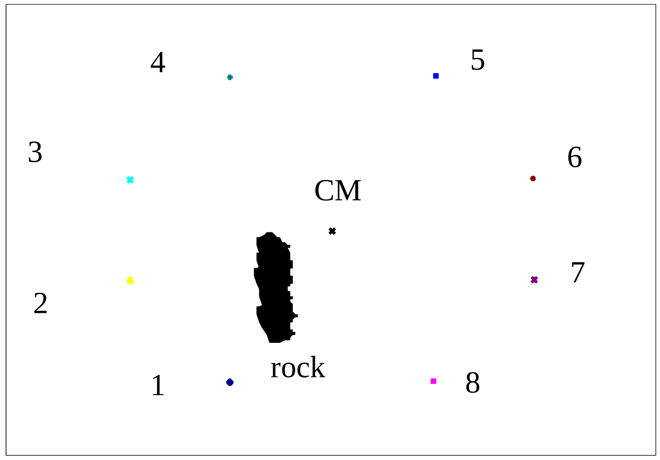

Figure 3.4 The positions of eight ants relative to an observer moving along with the CM for the rigid translation shown in Figure 3.3. Note that none of the positions change.

No collection of real ants moves as rigidly in lockstep as depicted in Figure 3.3. A more realistic pattern of ant motion might involve each of the individual ants moving north at the same *average* speed as in Figure 3.3 but also performing little idiosyncratic departures from the constant speed northward trend. An ant might speed up or slow down a little from time-to-time, and also possibly dart east and west a bit. Let's assume that these variations are done without rhyme or reason (i.e., they are "random"), but are always small. Figure 3.5 shows how the platoon and the CM might move under these conditions. Note that the motion of each individual ant is quite complicated and somewhat erratic. Nonetheless, the

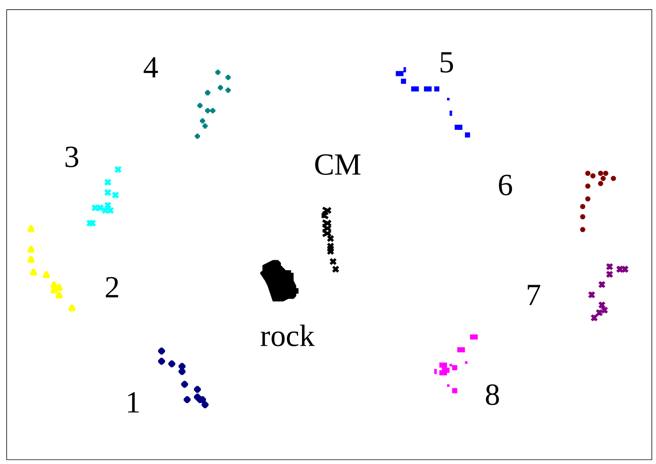

Figure 3.5 Eight identical ants moving north with the same average speed as in Figure 3.3 but with very small, random changes in motion. The CM moves to the north almost as steadily as in Figure 3.3.

CM progresses northward almost as steadily as in Figure 3.3. As far as the motion of the CM is concerned, it's as if the erratic part of the ants' motions cancel out. Indeed the motion of the CM becomes smoother and more regular as the number of members of the collection increases.

Again, it is instructive to ask what an observer moving with the CM sees for the motion of Figure 3.5. In this case, an observer moving along with the CM will *not* see the eight surrounding ants remain in place. Instead, from the perspective of an observer moving with the CM each of the surrounding ants performs a little random wiggle—north-south, east-west. This is shown in Figure 3.6.

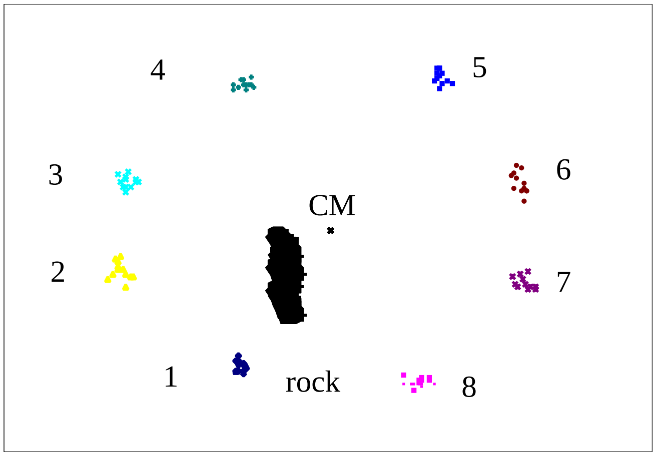

Figure 3.6 The motion of the eight ants of Figure 3.4 as seen by an "observer" moving along with the CM.

One can summarize the results of Figures 3.3-3.6 by saying that *the motion of a collection of elementary units is reducible to the motion of the collection's center-of-mass plus the motion of the collection relative to the center-of-mass***.**

The atoms from which any macroscopic body is made are similar to the ants of Figures 3.5 and 3.6. Like the northward drifting ants, the atoms can move coherently together over macroscopic distances (distances measured by meter sticks, for example). And, like the idiosyncratic random ant dartings, the atoms can also move incoherently over microscopic distances (about the size of an atom). Because all macroscopic bodies consist of huge numbers of atoms, this internal, atomic turmoil cancels out as far as the motion of the CM goes—and so, for now, we will ignore it. Our discussion of the motion of macroscopic bodies starts with the *approximation* that the constituent atoms are fixed in place relative to the CM and that a macroscopic body can be made to move in a rigid translation. In rigid translation the body does not tumble or rotate or vibrate, and its motion can be completely described by the motion of its CM. In other words, a rigidly translating body can be treated as if it were a single point mass, with all of the mass of the body located at the CM.