Topics for third exam

November 13, 2019

1 Mathematical methods

1. Divergence (div)
2. Gradient (grad, del)
3. Divergence theorem
4. Laplacian ($\boldsymbol{\nabla} \cdot \boldsymbol{\nabla} f = \nabla^2 f$)
5. Change of coordinates
   (a) Chain rule
   (b) Implicit differentiation
6. Separation of variables
7. Special functions:
   (a) Fourier modes $\left(e^{ikx}\right)$
   (b) Bessel functions ($J_n(x)$)
   (c) Legendre polynomials ($P_l(x)$)
   (d) Associated Legendre polynomials $P_l^m(x)$
   (e) Spherical Bessel functions ($j_n(x)$)
   (f) Spherical harmonics, $Y_l^m(\theta, \varphi)$
8. Power series solution to differential equations

2 Physical applications

1. Normal mode solutions; functions as vectors in an orthonormal basis
2. Time dependence of normal modes
3. Plane waves
4. Laplace equation (eg., electrostatics in 2 and 3 dimensions)
5. Cylindrical waves (2 and 3 dimensions)
6. Spherical waves; radiation
7. Laplace, Helmholz, Schrödinger

3 List of topics discussed

Cartesian coordinates:

1. Three dimensional wave equation
2. div and grad
3. Divergence theorem
4. Laplacian
5. Plane waves (solutions in Cartesian coordinates)
6. General solution in Cartesian coordinates
7. Initial conditions for the general solution
8. Separation of variables (Cartesian)

Cylindrical coordinates

1. Change of coordinates; new basis vectors
2. Gradient
3. Laplacian
4. Separation of variables
5. Solving for R, Φ, Z
6. Bessel functions
7. Relations between separation constants
8. Solutions to the cylindrical wave equation

Spherical coordinates

1. Spherical coordinates and basis vectors
2. Gradient
3. Laplacian
4. The Laplace equation in spherical coordinates
5. Solving for R, Θ, Φ; spherical harmonics
6. Legendre polynomials; series solution
7. Associated Legendre polynomials
8. Spherical harmonics
9. Solutions to the Laplace equation
10. Separation of variables
11. Solutions for T, R; spherical Bessel functions
12. Solutions to the wave equation (Helmholz equation)
13. Dipole waves