Midterm Exam Review

February 27, 2013

Chapter I

Basics of bra-ket notation

1. Kets and bras; dual space, inner products $\langle \alpha | \beta \rangle$, conjugation $\langle \alpha | \beta \rangle = \langle \beta | \alpha \rangle^*$
2. Basis bras, discrete and continuous $\langle a| , \langle x|$
3. Conversion between vectors and kets, and between operators and matrices

$$\langle a | \beta \rangle \quad \leftrightarrow \quad \begin{pmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_n \end{pmatrix}$$

$$|a_2\rangle \langle a_3| \quad \leftrightarrow \quad \begin{pmatrix} 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 \end{pmatrix}$$

Identity operator (completeness),

$$\hat{1} = \sum_i |a_i\rangle \langle a_i|$$
$$\hat{1} = \int dp \, |p\rangle \langle p|$$

Orthonormality,

$$\langle a_i | a_j \rangle = \delta_{ij}$$
$$\langle p | p' \rangle = \delta (p - p')$$

Quantum interpretation

Measurements, observables, eigenstates. Hermiticity, diagonalization, compatible observables. Probability and probability density.

*Diagonalization and eigenkets

Find the eigenvalues and eigenkets for a three-state system

*Spin operators; Stern-Gerlach; sequential measurement

Anything to do with $\hat{S}_x, \hat{S}_y, \hat{S}_z$, and their eigenkets; use of general eigenkets, $\left|\mathbf{n}\cdot\hat{\mathbf{S}}, \pm\frac{1}{2}\right\rangle$

Continuous bases

Conversion between $\langle x|, \langle p|$ as needed $\langle x|p\rangle = \frac{1}{\sqrt{2\pi\hbar}}e^{\frac{i}{\hbar}px}$; inserting identity to evaluate operators, e.g.

$$\hat{P}|\alpha\rangle = \int dp\hat{P}|p\rangle\langle p|\alpha\rangle$$

etc. Use of

$$\langle x'|\hat{P}|x\rangle = -i\hbar\frac{\partial}{\partial x'}\delta(x'-x)$$

Chapter II

Time evolution

Time evolution operator $\hat{\mathcal{U}}(t,t_0)$ and its generator, $\hat{H}$. Schrödinger equation. Use of

$$\hat{\mathcal{U}}(t,t_0) = e^{-\frac{i}{\hbar}\hat{H}t}$$

Energy eigenkets. Time evolution of a two-state system.

$$\begin{aligned}\hat{\mathcal{U}}(t,t_0)(a|E_1\rangle + b|E_2\rangle) &= e^{-\frac{i}{\hbar}\hat{H}t}(a|E_1\rangle + b|E_2\rangle) \\ &= ae^{-\frac{i}{\hbar}E_1t}|E_1\rangle + be^{-\frac{i}{\hbar}E_1t}|E_2\rangle\end{aligned}$$

Solving the Schrödinger equation

*Simple harmonic oscillator

Everything about $\hat{a}, \hat{a}^\dagger, \hat{N}, \hat{H}$ operators and the $|n\rangle$ eigenkets. Time evolution.

*Schrödinger equation

The time dependent and time independent Schrödinger wave equations. Solutions for piecewise constant potentials. Probability interpretation. Conservation of probability.