Quantum Mechanics: Wheeler: Physics 6210

Problems for Chapter 2 by topic

Time dependence

- **S.2.1**: This is the same problem as Sakurai does on pages 76-78, only you will do it in the Heisenberg picture. This means that instead of regarding the kets as time dependent as in eq.(2.1.56) you will work with constant kets (2.1.55) but let the operators S_x, S_y, and S_z become time dependent as in eq.(2.2.10). The answers, of course, must be the same.
- **S.2.8**
- **S.2.23**

Tunneling; coherent states

- **S.2.9**: This problem gives practice with the idea of tunneling without requiring a lot of detail. Sakurai has extracted the essential elements of tunneling: two possible states separated by a barrier, but with terms in the Hamiltonian that let particles move from one state to the other anyway. Work this fully and carefully. It gives good practice with the time-dependent Schrödinger equation and initial conditions. Part e) shows the importance of having Hermitian H (and therefore unitary $U(t, t_0)$.
- **S.2.11**
- **S.2.13 a.**

Path integral and interference

- **S.2.30** This is the start of the derivation of the path integral.
- **S.2.31**: Shows the relationship between the path integral and the Schrödinger equation.
- **S.2.33**: You shouldn't have to work out the full solution to compute what happens to the phase along different paths.
- **S.2.37**

Interactions with magnetic fields

- **S.2.25**: The goal here is to understand something about the idea of flux quantization and the Bohm-Aharonov effect. Don't miss the physical conclusion! In particular, part b raises an issue about which there are frequent misunderstandings. The electron state vanishes inside the inner cylinder and the B-field vanishes everywhere outside the inner cylinder. However, the vector potential, A, does not vanish outside, where the electron state is nonzero. Some people take this as an indication that the vector potential is in some sense "real". However, the magnitude of any measurable effect depends only on the value of B, and not independently on A. Moreover, the effect is not localizable − if you try to claim that the electron followed a particular path it is possible to find a gauge such that the vector potential vanishes along all but one point of that path. Then you are stuck claiming that the interaction took place all at one point! In quantum physics as in classical physics, the vector potential is a mathematical convenience - it is only the E and B fields that have physical meaning.
- **(From S.2.35)**: Consider the Hamiltonian of a spinless particle of charge e. In the presence of a static magnetic field, the interaction terms can be generated by the minimal coupling substitution,

$$\hat{\mathbf{P}} \to \hat{\mathbf{P}} - \frac{e}{c}\hat{\mathbf{A}}$$

where $\hat{\mathbf{A}} = \mathbf{A}\left(\hat{\mathbf{X}}\right)$ is the vector potential operator. Suppose the magnetic field $\mathbf{B}$ is in the z-direction. Show that an interaction term arises that gives the correct expression for the interaction of the orbital magnetic moment $\frac{e}{2mc}\mathbf{L}$ with the magnetic field (i.e., as the dot product of the magnetic field with the magnetic moment)

- **S.2.36**: The substitution $p \to p - eA$ is called minimal coupling. It has a generalization which is a standard technique in field theory. The problem requires recognizing a form of the harmonic oscillator.