Quantum Mechanics: Wheeler: Physics 6210

Assignment 9

READ: Sakurai, chapter 3, sections 3.1 - 3.3, pp 152 - 187, and 3.5. (We can come back to 3.4 later) We begin a detailed discussion of angular momentum, in general, and in quantum mechanics. I agree fully with the statement in the opening paragraph: "The importance of angular momentum in modern physics can hardly be overemphasized." The basic principles underlie all of modern field theory. The first three sections introduce the basic concepts, including spin and the Euler angles. Section 3.4 digresses to discuss the density matrix. Note especially, equation 3.2.44.

PROBLEMS:

- **1:** Work out the neutron interferometry experiment described on pages 161-163, and derive result given in eq.(3.2.25).
- **S.3.2**: The denominator in this expression is a bit mysterious − it is a matrix! It can be simplified by multiplying both the numerator and denominator by

$$a_0 + i\boldsymbol{\sigma} \cdot \mathbf{a}$$

  and taking advantage of 3.2.41.
- **S.3.3**: The notation here is new. The operator $S_{|e_-\rangle}$ acts only on $|e_-\rangle$, and similarly for the $|e_+\rangle$ state and operators. Thus, for example:

$$\left(S_{|e_-\rangle} S_{|e_+\rangle}\right)\left(|e_-\rangle\,|e_+\rangle\right) = \left(S_{|e_-\rangle}\,|e_-\rangle\right)\left(S_{|e_+\rangle}\,|e_+\rangle\right)$$

- **S.3.13**: This is a good problem, but let me clarify it a little. These Gs are precisely the ones I chose in class, so they should look a bit familiar. To prove that this ε_{ijk} form satisfies the commutation relation you will have to use the identity,

$$\varepsilon_{ijk}\varepsilon_{mnk} = \delta_{im}\delta_{jn} - \delta_{in}\delta_{jm}$$

  The second question is merely asking about the transformation that diagonalizes G_3. Find this transformation and show that it is unitary. What is its determinant? Since the transformation is unitary, it corresponds to some rotation (finding the axis and angle of the rotation may be a bit involved, but try it if you like). The last part of the problem, relating the result to

$$V_n \to V_n + \varepsilon_{nij} n_i V_j$$

  or, in vector notation,

$$\overrightarrow{V} \to \overrightarrow{V} + \hat{n} \times \overrightarrow{V}$$

  is a bit vague. First note that this is the standard classical form for an infinitesimal rotation. The point of the question is to show that this classical form is the same as the quantum rotation generators we have been dealing with, i.e., the Js. What you are to do is to write this expression for V in terms of the generators $[G_i]_{jk} = -ih\varepsilon_{ijk}$. Then, since you have already shown that this form of the Gs may be diagonalized by a unitary transformation, you have completed the proof that the usual classical rotation formula is one representation of the full generator of rotations in terms of J.