Advanced Quantum Mechanics: Wheeler: Physics 671

Assignment 7

Start reading Chapter 3

1. Problem 2.18. This problem shows various properties of coherent states.

2. Time evolution of a (slightly altered) three-state system: Suppose the Hamiltonian of a 3-state system is given by

\[
\hat{H} = \begin{pmatrix}
\frac{\Delta}{\sqrt{2}} & 0 & \frac{\Delta}{\sqrt{2}} \\
0 & \frac{\Delta}{\sqrt{2}} & -\frac{\Delta}{\sqrt{2}} \\
\frac{\Delta}{\sqrt{2}} & -\frac{\Delta}{\sqrt{2}} & 0
\end{pmatrix}
\]

Find the following (the normalizations aren’t quite as nice as the exam problem):

(a) The energy eigenvalues, in increasing order: \( E_1 \leq E_2 \leq E_3 \)

(b) The corresponding normalized energy eigenvectors, \( |E_1\rangle, |E_2\rangle, |E_3\rangle \)

(c) Express the state \( |\alpha\rangle = \frac{1}{\sqrt{2}} (1, 0, 1) \) as an explicit linear combination of the energy eigenkets \( |E_1\rangle, |E_2\rangle, |E_3\rangle \)

(d) Find the time evolution, \( |\alpha, t\rangle \) of the state \( |\alpha\rangle = \frac{1}{\sqrt{2}} (1, 0, 1) \) by acting on \( |\alpha\rangle \) with the time-evolution operator, \( \hat{U}(t, 0) = e^{-i\hat{H}t} \).

(e) Find the probability of finding the value \( E_1 \) for the state \( |\alpha, t\rangle \), at any time \( t \).

(f) Find the probability of finding the state \( |\alpha, t\rangle \) in the initial state, \( |\alpha\rangle \), at any time \( t \).

3. Neutrons in a gravitational field. This experiment has shown a purely quantum mechanical effect of the gravitational field. A beam of neutrons travelling at constant velocity, is split. One half is made to travel at height \( h_1 \) and the other at height \( h_2 \) in a gravitational field where the potential may be treated as \( V = mgh \). After each beam has travelled a distance \( L \), the two beams are brought back together and allowed to interfere. Show that the phase difference between the two beams when they rejoin is given by

\[
\Delta \phi = \frac{m^2gL}{\hbar^2k} (h_2 - h_1)
\]

As the difference in heights is varied, the interference pattern will vary through constructive and destructive interference.