Quantum Mechanics I: Physics 6210: Wheeler

Assignment 2

READ: Finish reading Sakurai, Chapter 1.

PROBLEMS:

1. S.1.11: For this problem, it's not hard to find the eigenvalues and eigenkets and the condition that has to hold when $H_{12} = 0$, using standard matrix techniques. The only trick is to recognize that the hamiltonian

$$\begin{aligned} H &= H_{11} \left|1\right\rangle \left|1\right\rangle + H_{22} \left|2\right\rangle \left|2\right\rangle + H_{12} \left(\left|2\right\rangle \left|1\right\rangle + \left|1\right\rangle \left|2\right\rangle \right) \\ &= H_{11} \left|1\right\rangle \otimes \left|1\right\rangle + H_{22} \left|2\right\rangle \otimes \left|2\right\rangle + H_{12} \left|1\right\rangle \otimes \left|2\right\rangle + H_{12} \left|2\right\rangle \otimes \left|1\right\rangle \end{aligned}$$

is just the matrix

$$H = \begin{pmatrix} H_{11} & H_{12} \\ H_{12} & H_{22} \end{pmatrix}$$

where either $\left|1\right\rangle \left|2\right\rangle$ or $\left|1\right\rangle \otimes \left|2\right\rangle$ denotes the outer product of the two vectors $\left|1\right\rangle$ and $\left|2\right\rangle$. If these vectors (in some arbitrary basis) have components

$$\begin{aligned} \left|1\right\rangle &= (a, b) \\ \left|2\right\rangle &= (c, d) \end{aligned}$$

then the direct product is

$$\left|1\right\rangle \otimes \left|2\right\rangle = \begin{pmatrix} ac & ad \\ bc & bd \end{pmatrix}$$

When $\left|1\right\rangle$ and $\left|2\right\rangle$ *define* the basis, then we have

$$\begin{aligned} \left|1\right\rangle &= (1, 0) \\ \left|2\right\rangle &= (0, 1) \end{aligned}$$

and the outer product singles out a particular component of the matrix:

$$\left|1\right\rangle \otimes \left|2\right\rangle = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

It is easy to write a general matrix as a sum of such terms.

2. S.1.12: Use the hint for problem 11! This problem is nicely physical.

3. S.1.13: Another good physical problem. Work through step by step.

4. S.1.14: Straightforward linear algebra.

5. S.1.15: This seems simple, but which answer is right? Careful thinking will get you to the answer quickly.

6. S.1.16: The proof can be written in one line.

7. S.1.17: You might need three or four lines.