Quantum Mechanics I: Physics 6210: Wheeler

Assignment 2

READ: Finish reading Sakurai, Chapter 1.

PROBLEMS:

1. S.1.11: For this problem, it’s not hard to find the eigenvalues and eigenkets and the condition that has to hold when \( H_{12} = 0 \), using standard matrix techniques. The only trick is to recognize that the hamiltonian

\[
H = H_{11} |1\rangle |1\rangle + H_{22} |2\rangle |2\rangle + H_{12} (|2\rangle |1\rangle + |1\rangle |2\rangle)
\]

is just the matrix

\[
H = \begin{pmatrix}
H_{11} & H_{12} \\
H_{12} & H_{22}
\end{pmatrix}
\]

where either \(|1\rangle |2\rangle\) or \(|1\rangle \otimes |2\rangle\) denotes the outer product of the two vectors \(|1\rangle\) and \(|2\rangle\). If these vectors (in some arbitrary basis) have components

\[
|1\rangle = (a, b) \\
|2\rangle = (c, d)
\]

then the direct product is

\[
|1\rangle \otimes |2\rangle = \begin{pmatrix}
ac & ad \\
bc & bd
\end{pmatrix}
\]

When \(|1\rangle\) and \(|2\rangle\) define the basis, then we have

\[
|1\rangle = (1, 0) \\
|2\rangle = (0, 1)
\]

and the outer product singles out a particular component of the matrix:

\[
|1\rangle \otimes |2\rangle = \begin{pmatrix}
0 & 1 \\
0 & 0
\end{pmatrix}
\]

It is easy to write a general matrix as a sum of such terms.

2. S.1.12: Use the hint for problem 11! This problem is nicely physical.


5. S.1.15: This seems simple, but which answer is right? Careful thinking will get you to the answer quickly.

6. S.1.16: The proof can be written in one line.

7. S.1.17: You might need three or four lines.