Time in conformal general relativity

Gravity theories based on the conformal group give general relativity augmented by local dilatational covariance. In one of these theories, biconformal gravity, the dimension of the original space doubles, giving a symplectic manifold with conformal general relativity on a lagrangian submanifold.

The restriction of the Killing form to the biconformal space is non-degenerate and signature-changing. This opens several possibilities, from time arising only within solutions to a Euclidean gravity theory, to novel approaches to the AdS/CFT correspondence, or the natural emergence of an SO(n) Yang-Mills field on gravitating spacetime. We briefly discuss some of our investigations into these options.

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Gauge theory: classical approach

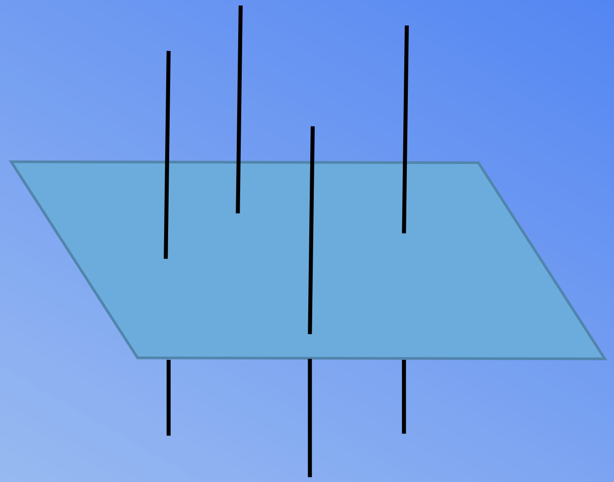
Globally invariant Lagrangian

Fiber bundle (noticed in passing)

Locally invariant generalization

(including kinetic term for the gauge fields)

Dirac equation: $(i\gamma^{\alpha}\, \partial_{\alpha} - m)\psi = 0$



Dirac equation coupled to E&M

$$(i\gamma^{\alpha}\, \partial_{\alpha} + e\gamma^{\alpha} A_{\alpha} - m)\psi = 0$$

$$\partial_{\alpha}F^{\alpha\beta} = e\, \psi^{t*}\gamma^{0}\gamma^{\alpha}\, \psi$$

Gauge theory: quotient group approach

Choose Lie group G and symmetry subgroup H

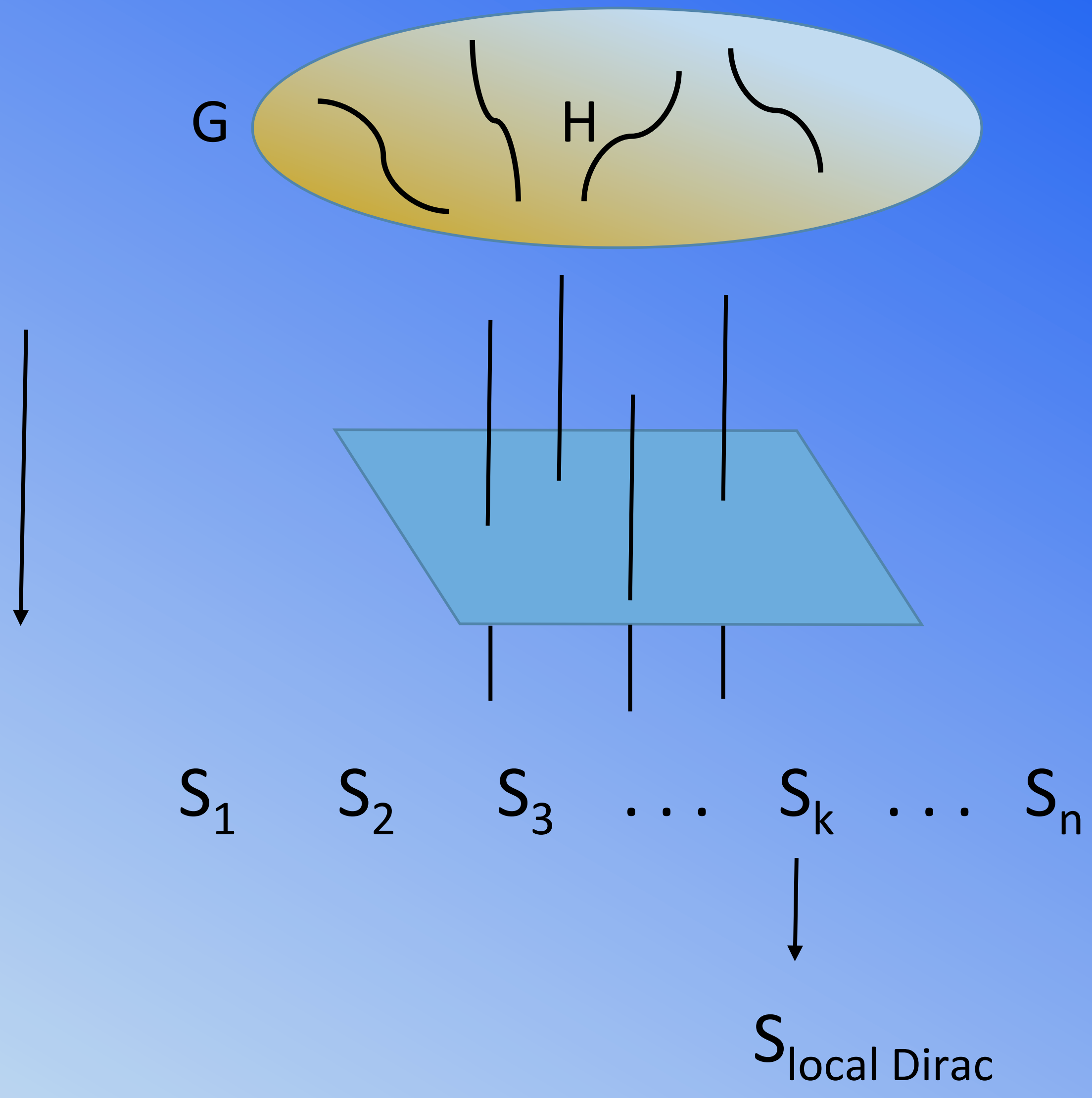
Fiber bundle as G/H

Generalize to curved connection

Generalize manifold

Find resulting tensors;

Build many possible locally invariant theories



Biconformal gauging: construction

G = conformal group of n-space (signature (p,q))

H = homogeneous Weyl group

G/H $\longrightarrow$ local $\mathcal{M}^{2n}$ x H principal fiber bundle

This becomes the local model for a Cartan geometry

Biconformal gauging: properties

Symplectic form

Non-degenerate Killing form $\longrightarrow$ Metric

Complex structure

Permits an action linear in the curvature

ALL of these properties follow from group properties.

Biconformal space: The signature theorem

Spencer, J. A. and Wheeler, James T., *The existence of time*, International Journal of Geometric Methods in Modern Physics, Vol. 8 No. 2 (2011) 273-301.

Using the metric and symplectic form of a signature (p,q) biconformal space, we show:

- There exist orthogonal, metric, Lagrangian submanifolds if and only if the original gauged space is Euclidean or signature zero.
- In the Euclidean cases, the resultant configuration space is necessarily Lorentzian.

In these models, time is a derived property of general relativity.

Properties arising from the underlying groups

Hazboun, Jeffrey S. and Wheeler, James T., *Time and dark matter from the conformal symmetries of Euclidean space,* Classical and Quantum Gravity 31.21 (2014) 34 pages.

Two new tensors, including a vector determining the timelike direction

Metric: $g_{ab} = e^{\phi}\eta_{ab}$

Trivial Weyl vector: $W_{\mu} = \partial_{\mu}\sigma$

In general, ϕ and σ do not vanish together,

$$v_{\mu} = \partial_{\mu}(\sigma - \phi)$$

is gauge invariant.

The development of a timelike direction stems from group properties. The infinitesimal generator for special conformal transformations (translations of infinity),

$$K_a = (2x^{\alpha}x^{\beta} - x^2\delta^{\alpha\beta})\,\partial_{\beta}$$

leads to the form of the Killing metric on a Lagrangian submanifold

$$\begin{aligned} g^{\alpha\beta} &= 2v^{\alpha}v^{\beta} - v^2\delta^{\alpha\beta} \\ &\sim (v^2)\,\mathrm{diag}(1, -1, \dots, -1) \end{aligned}$$

AdS/CFT

Jeffrey S. Hazboun

Coleman-Mandula

An alternative to supersymmetry

Benjamin Lovelady and JTW