General Relativity Midterm

March 6, 2015

Take home problem:

A static, spherically symmetric line element may be put in the form

$$ds^2 = -f^2(r)\,dt^2 + h^2(r)\,dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2$$

where $f(r)$ and $g(r)$ may be any functions. For a general choice of these functions, the resulting metric

$$g_{\alpha\beta} = \begin{pmatrix} -f^2 & 0 & 0 & 0 \\ 0 & h^2 & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2\theta \end{pmatrix}$$

does not satisfy the Einstein equation. For your test, compute all nonvanishing components the Riemann curvature tensor,

$$R^\alpha_{\ \beta\mu\nu} = \Gamma^\alpha_{\ \beta\mu,\nu} - \Gamma^\alpha_{\ \beta\nu,\mu} - \Gamma^\alpha_{\ \rho\mu}\Gamma^\rho_{\ \beta\nu} + \Gamma^\alpha_{\ \rho\nu}\Gamma^\rho_{\ \beta\mu}$$

where

$$\Gamma^\alpha_{\ \beta\mu} = \frac{1}{2} g^{\alpha\nu}\left(g_{\nu\beta,\mu} + g_{\nu\mu,\beta} - g_{\beta\mu,\nu}\right)$$

The result for components of $R^\alpha_{\ \beta\mu\nu}$ will depend on f, h and their first and second derivatives.

Compute by hand using the formulas above. You may use the Notes from class, and email me for help at any time. Consulting me will not detract from your score. When you return after break, be prepared to turn in your solution.

Be careful to check and recheck your expressions for $\Gamma^\alpha_{\ \beta\mu}$ before proceeding to compute the curvature.