The geodesic equations are:

$$
\begin{aligned}
0 &= \frac{du^0}{d\tau} - \frac{a}{r^2\left(1+\frac{a}{r}\right)}u^0u^1 \\
0 &= \frac{1}{c^2}\frac{du^1}{d\tau} - \left(1+\frac{a}{r}\right)\frac{a}{2r^2}u^0u^0 + \frac{a}{2r^2c^2\left(1+\frac{a}{r}\right)}u^1u^1 - \frac{r}{c^2}\left(1+\frac{a}{r}\right)u^2u^2 - \left(1+\frac{a}{r}\right)\frac{r}{c^2}\sin^2\theta u^3u^3 \\
0 &= \frac{du^2}{d\tau} - \sin\theta\cos\theta u^3u^3 + \frac{2}{r}u^2u^1 \\
0 &= \frac{du^3}{d\tau} + \frac{2\cos\theta}{\sin\theta}u^2u^3 + \frac{2}{r}u^3u^1
\end{aligned}
$$

We also have the relation given by the line element,

$$
\frac{1}{c^2}\frac{ds^2}{d\tau^2} = -\left(1+\frac{a}{r}\right)\left(\frac{dt}{d\tau}\right)^2 + \frac{1}{c^2\left(1+\frac{a}{r}\right)}\left(\frac{dr}{d\tau}\right)^2 + \frac{r^2}{c^2}\left(\frac{d\theta}{d\tau}\right)^2 + \frac{r^2}{c^2}\sin^2\theta\left(\frac{d\varphi}{d\tau}\right)^2
$$

Consider the case of a bound null orbit. Restrict the motion to constant r and $\theta = \frac{\pi}{2}$. Then

$$
\begin{aligned}
0 &= \frac{du^0}{d\tau} \\
0 &= +\left(1-\frac{2m}{r}\right)\frac{m}{r^2}u^0u^0 - \left(1-\frac{2m}{r}\right)\frac{r}{c^2}u^3u^3 \\
0 &= \frac{du^3}{d\tau}
\end{aligned}
$$

The first and third show that the 4-velocity is constant, while the second becomes

$$
\begin{aligned}
0 &= \frac{mc^2}{r^3}\left(u^0\right)^2 - \left(u^3\right)^2 \\
u^3 &= \sqrt{\frac{mc^2}{r^3}}u^0
\end{aligned}
$$

We also have the line element,

$$
\begin{aligned}
0 &= -\left(1-\frac{2m}{r}\right)\left(u^0\right)^2 + \frac{r^2}{c^2}\left(u^3\right)^2 \\
&= -\left(1-\frac{2m}{r}\right)\left(u^0\right)^2 + \frac{r^2}{c^2}\frac{mc^2}{r^3}\left(u^0\right)^2 \\
0 &= -1 + \frac{2m}{r} + \frac{m}{r} \\
r &= 3m
\end{aligned}
$$