Exercises using the wedge product

1. Find the dimension of the space of $p$-forms, $\Lambda_p$, on a space of $n$-dimensions.

2. Find the dimension of the space of forms,

$$\dim(\Lambda_0 \otimes \Lambda_1 \otimes \Lambda_2 \otimes \cdots \otimes \Lambda_n)$$

This is just the sum of dimensions of the $\Lambda_p$,

$$\sum_{p=0}^{n} \dim(\Lambda_p)$$

3. We may define the wedge product,

$$\wedge: \Lambda_1 \otimes \Lambda_1 \to \Lambda_2$$

as an antisymmetric, linear, and associative mapping given by antisymmetrizing the outer product:

$$\omega \wedge \eta = \frac{1}{2} (\omega \otimes \eta - \eta \otimes \omega)$$

Use these properties on the pairwise product to show that if $\omega$ is a $p$-form ($\omega \in \Lambda_p$) and $\eta$ is a $q$-form ($\eta \in \Lambda_q$), then

$$\omega \wedge \eta = (-1)^{pq} \eta \wedge \omega$$

4. Let $n = 3$. Let two 1-forms be given by

$$\omega = \omega_i dx^i$$
$$\eta = \eta_j dx^j$$

Write out the wedge product of these two 1-forms explicitly. Compare the result to the components as the cross product of the corresponding covariant vectors.